Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 8
Second-Order Circuits

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Second-Order Circuits
Chapter 8

8.1 Examples of 2nd order RCL circuit

8.2 The source-free series RLC circuit

8.3 The source-free parallel RLC circuit

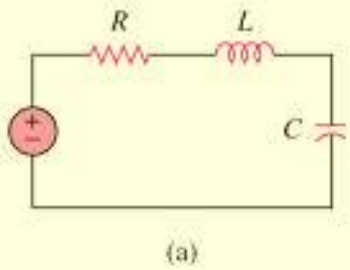
8.4 Step response of a series RLC circuit

8.5 Step response of a parallel RLC

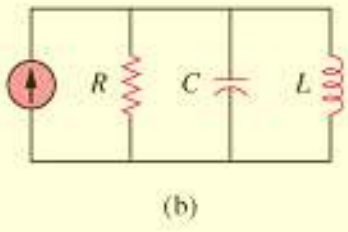
8.1 Examples of Second Order RLC circuits (1)

What is a 2nd order circuit?

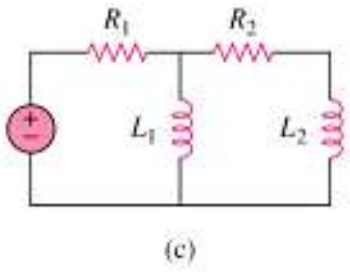
A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



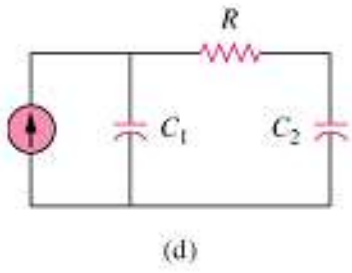
RLC Series



RLC Parallel

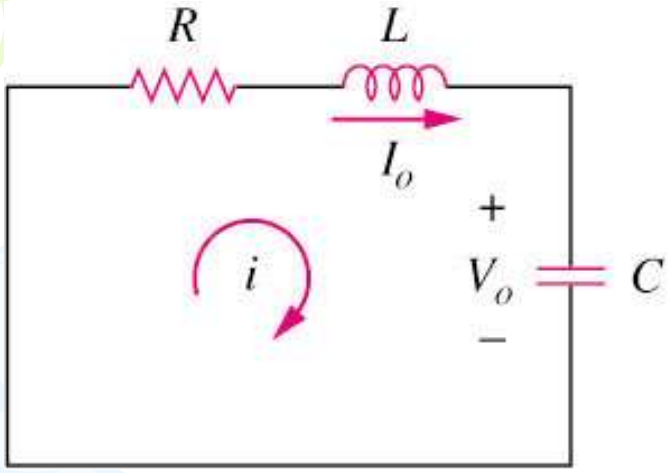


RL T-config



RC Pi-config

8.2 Source-Free Series RLC Circuits (1)



- The solution of the source-free series RLC circuit is called as the natural response of the circuit.
- The circuit is excited by the energy initially stored in the capacitor and inductor.

The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

How to derive and how to solve?

8.2 Source-Free Series RLC Circuits (2)

Method will be illustrated during the lecture

8.2 Source-Free Series RLC Circuits (3)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0$$

=> $$\frac{d^2i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0$$

General 2nd order Form

where $$\alpha=\frac{R}{2L} \quad and \quad \omega_0=\sqrt{\frac{1}{LC}}$$

The types of solutions for i(t) depend on the relative values of α and ω.

8.2 Source-Free Series RLC Circuits (4)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0$$

1. If $\alpha > \omega_o$, over-damped case

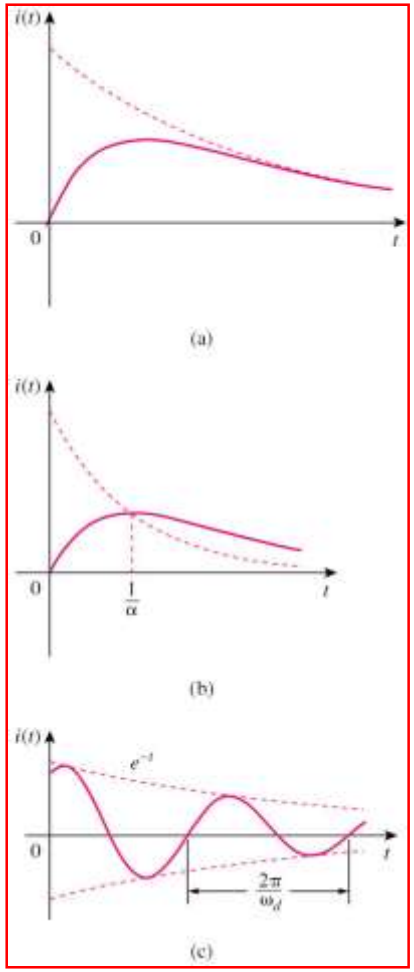
$$i(t)=A_1e^{s_1t}+A_2e^{s_2t} \quad \textbf{where} \quad s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^{\,2}}$$

2. If $\alpha = \omega_o$, critical damped case

$$i(t)=(A_2+A_1t)e^{-\alpha t} \quad \textbf{where} \quad s_{1,2}=-\alpha$$

3. If $\alpha < \omega_o$, under-damped case

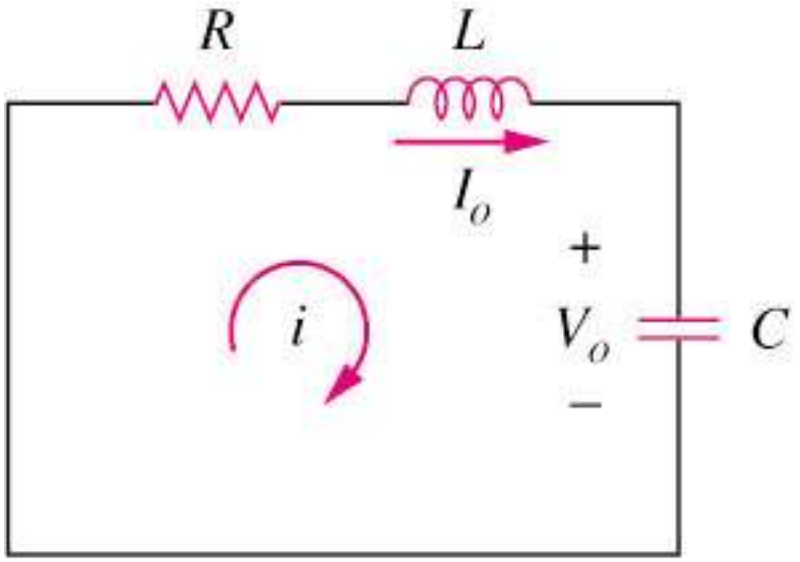
$$i(t)=e^{-\alpha t}(B_1\cos\omega_d t+B_2\sin\omega_d t) \quad \textbf{where} \quad \omega_d=\sqrt{\omega_0^2-\alpha^2}$$



8.2 Source-Free Series RLC Circuits (5)

Example 1

If R = 10 Ω, L = 5 H, and C = 2 mF in 8.8, find α, ω0, s1 and s2.



What type of natural response will the circuit have?

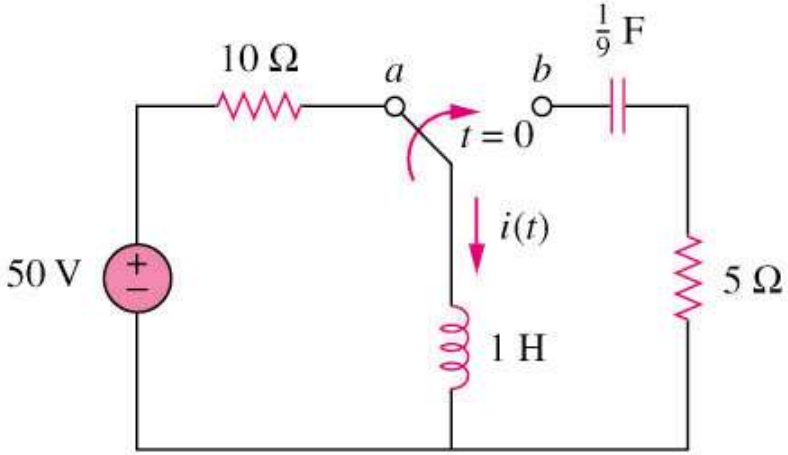
- Please refer to lecture or textbook for more detail elaboration.

Answer: *underdamped*

8.2 Source-Free Series RLC Circuits (6)

Example 2

The circuit shown below has reached steady state at t = 0-.

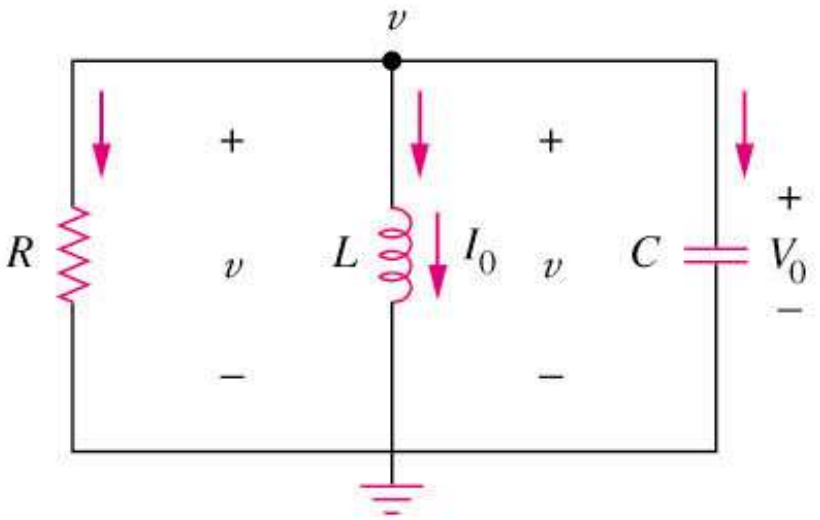


If the make-before-break switch moves to position b at t = 0, calculate i(t) for t > 0.

- Please refer to lecture or textbook for more detail elaboration.

Answer: $i(t) = e^{-2.5t}[5\cos 1.6583t - 7.538\sin 1.6583t]$ A

8.3 Source-Free Parallel RLC Circuits (1)



Let $$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)dt$$

$v(0) = V_0$

Apply KCL to the top node:

$$\frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v\,dt + C\frac{dv}{dt} = 0$$

Taking the derivative with respect to t and dividing by C

The 2nd order of expression

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

8.3 Source-Free Parallel RLC Circuits (2)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \textbf{where} \quad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

$$v(t) = (A_2 + A_1 t)e^{-\alpha t} \quad \textbf{where} \quad s_{1,2} = -\alpha$$

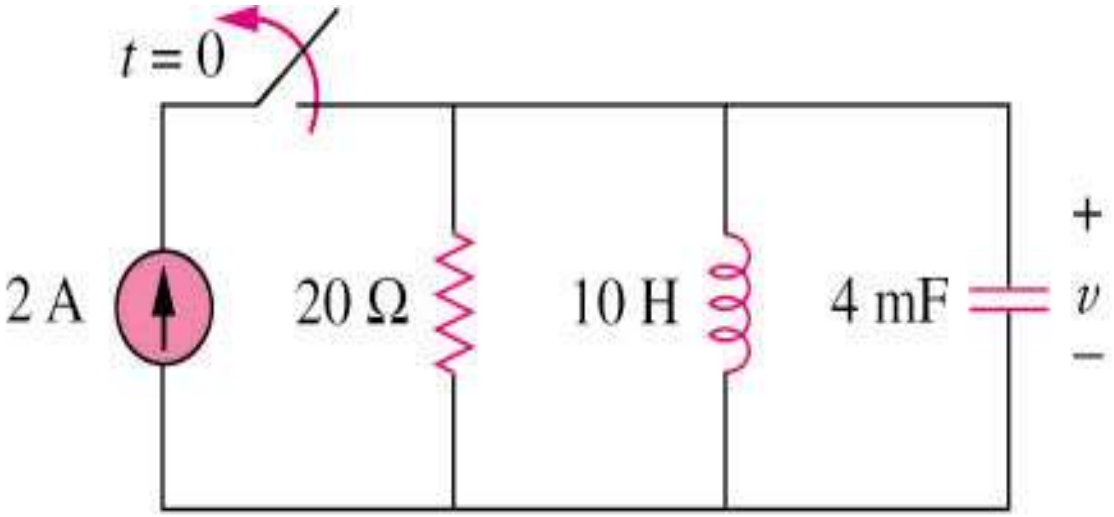
3. If $\alpha < \omega_o$, under-damped case

$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad \textbf{where} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

8.3 Source-Free Parallel RLC Circuits (3)

Example 3

Refer to the circuit shown below. Find v(t) for t > 0.

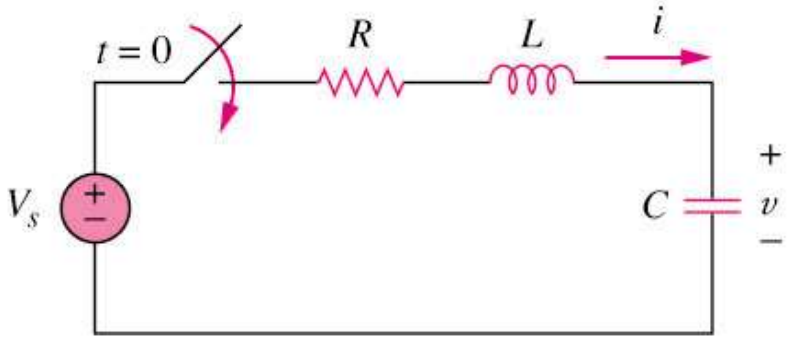


- Please refer to lecture or textbook for more detail elaboration.

Answer: $v(t) = \mathbf{66.67(e^{-10t} - e^{-2.5t})\ V}$

8.4 Step-Response Series RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{v_s}{LC}$$

The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters).
- Different circuit variable in the equation.

8.4 Step-Response Series RLC Circuits (2)

The solution of the equation should have two components: the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$v(t) = v_t(t) + v_{ss}(t)$$

- The transient response v_t is the same as that for source-free case

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over-damped)}$$

$$v_t(t) = (A_1 + A_2 t) e^{-\alpha t} \quad \text{(critically damped)}$$

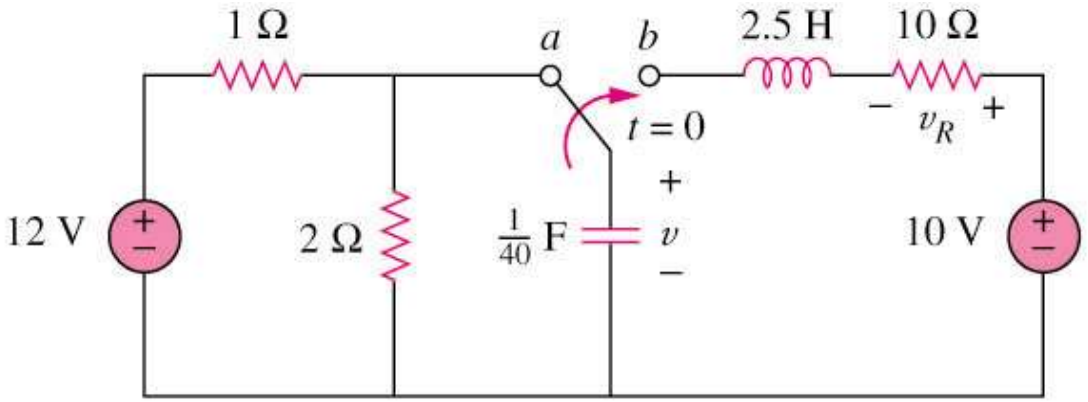
$$v_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad \text{(under-damped)}$$

- The steady-state response is the final value of v(t).
  - $v_{ss}(t) = v(\infty)$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - v(0) and dv(0)/dt.

8.4 Step-Response Series RLC Circuits (3)

Example 4

Having been in position for a long time, the switch in the circuit below is moved to position b at t = 0. Find v(t) and $v_R(t)$ for t > 0.



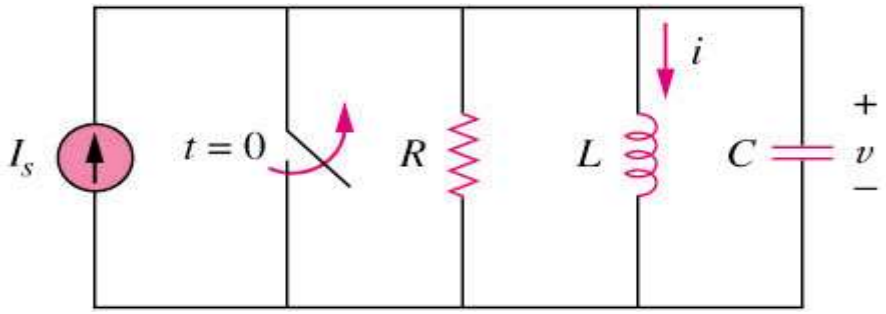
- Please refer to lecture or textbook for more detail elaboration.

Answer: $v(t) =$ ***{10 + [(−2cos3.464t − 1.1547sin3.464t)e−2t]} V***

$v_R(t) =$ ***[2.31sin3.464t]e^{-2t} V***

8.5 Step-Response Parallel RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).
- Different circuit variable in the equation.

8.5 Step-Response Parallel RLC Circuits (2)

The solution of the equation should have two components: the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$i(t) = i_t(t) + i_{ss}(t)$$

- The transient response i_t is the same as that for source-free case

$$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over-damped)}$$

$$i_t(t) = (A_1 + A_2 t) e^{-\alpha t} \quad \text{(critical damped)}$$

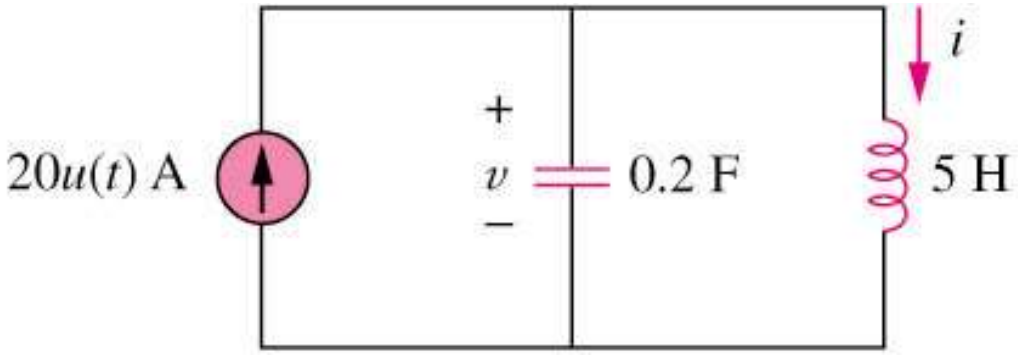
$$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad \text{(under-damped)}$$

- The steady-state response is the final value of i(t).
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - i(0) and di(0)/dt.

8.5 Step-Response Parallel RLC Circuits (3)

Example 5

Find i(t) and v(t) for t > 0 in the circuit shown in circuit shown below:



- Please refer to lecture or textbook for more detail elaboration.

Answer: ***v(t) = Ldi/dt = 5x20sint = 100sint V***