Alexander-Sadiku Fundamentals of Electric Circuits

Chapter 12

Three-Phase Circuit

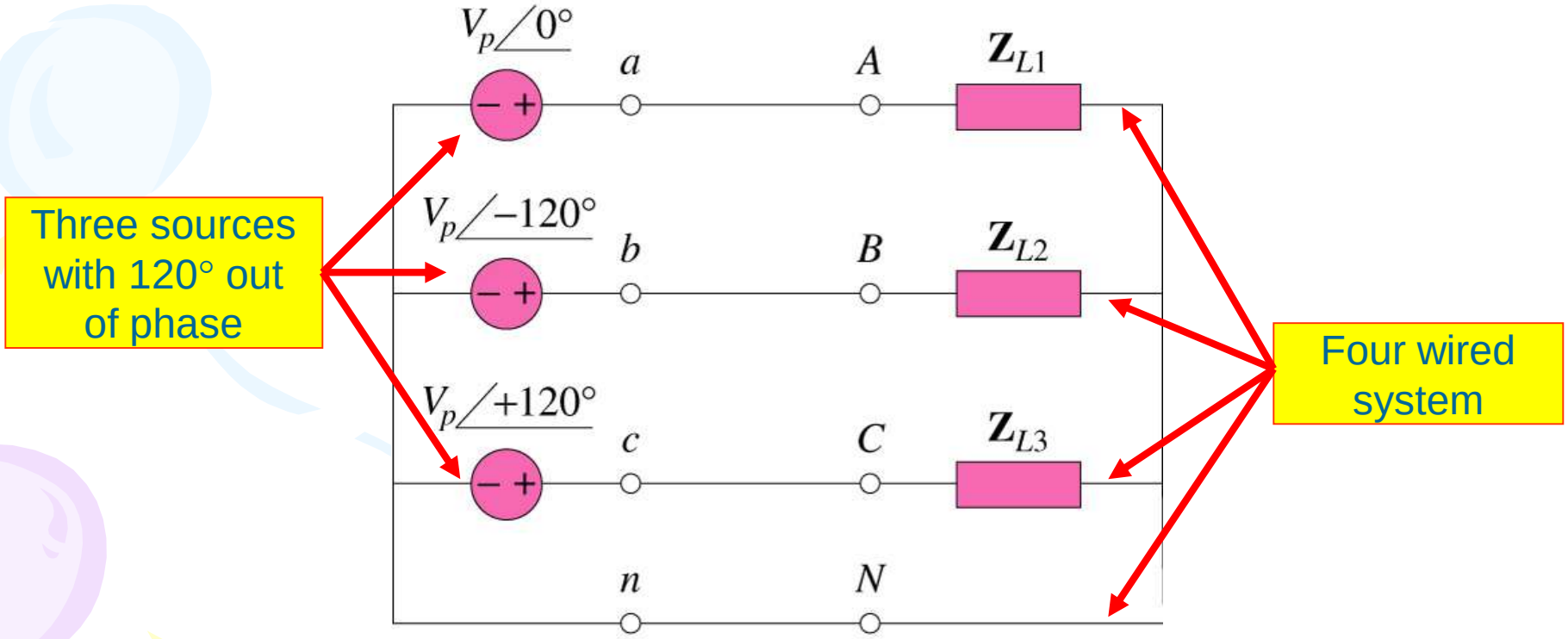
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Three-Phase Circuits
Chapter 12

12.1 What is a Three-Phase Circuit?
12.2 Balance Three-Phase Voltages
12.3 Balance Three-Phase Connection
12.4 Power in a Balanced System
12.5 Unbalanced Three-Phase Systems
12.6 Application – Residential Wiring

12.1 What is a Three-Phase Circuit?(1)

- It is a system produced by a generator consisting of **three sources** having the same amplitude and frequency but **out of phase** with each other by 120°.



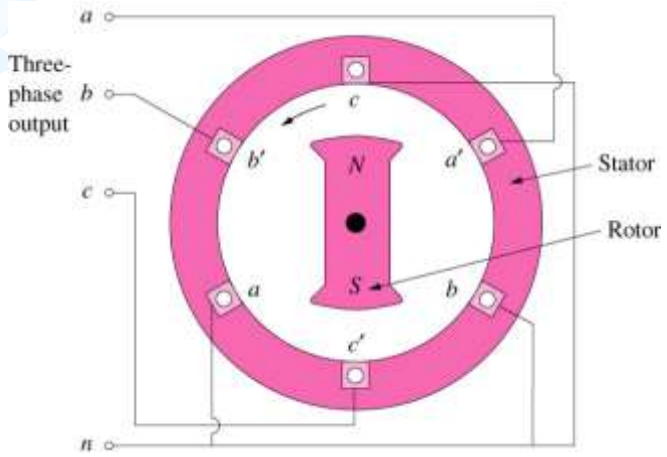
12.1 What is a Three-Phase Circuit?(2)

Advantages:

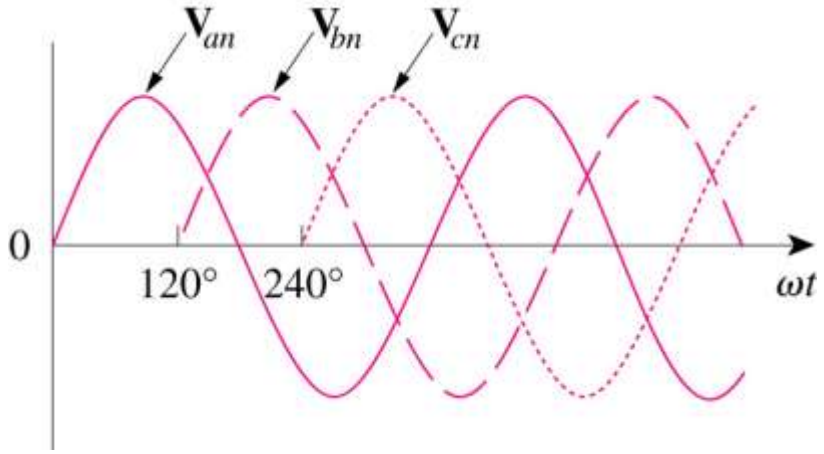
1. Most of the electric power is generated and distributed in three-phase.
2. The instantaneous power in a three-phase system can be constant.
3. The amount of power, the three-phase system is more economical that the single-phase.
4. In fact, the amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

12.2 Balance Three-Phase Voltages (1)

- A three-phase generator consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).



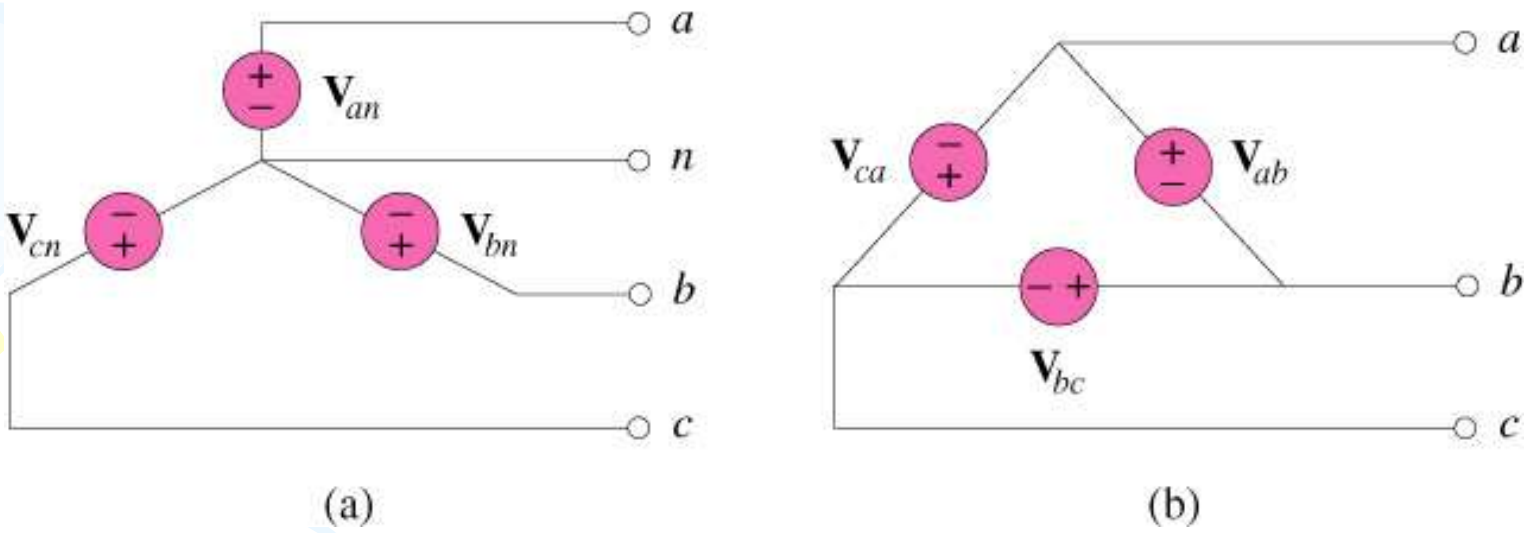
A three-phase generator



The generated voltages

12.2 Balance Three-Phase Voltages (2)

- Two possible configurations:



Three-phase voltage sources: (a) Y-connected ; (b) Δ-connected

12.2 Balance Three-Phase Voltages (3)

- ***Balanced phase voltages*** are equal in magnitude and are out of phase with each other by 120°.
- The ***phase sequence*** is the time order in which the voltages pass through their respective maximum values.
- A ***balanced load*** is one in which the phase impedances are equal in magnitude and in phase

12.2 Balance Three-Phase Voltages (4)

Example 1

Determine the phase sequence of the set of voltages.

$$v_{an} = 200\cos(\omega t + 10°)$$

$$v_{bn} = 200\cos(\omega t - 230°)$$

$$v_{cn} = 200\cos(\omega t - 110°)$$

12.2 Balance Three-Phase Voltages (5)

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\angle 10^\circ \text{ V}$$
$$\mathbf{V}_{bn} = 200\angle -230^\circ \text{ V}$$
$$\mathbf{V}_{cn} = 200\angle -110^\circ \text{ V}$$

We notice that $\mathbf{V_{an}}$ leads $\mathbf{V_{cn}}$ by 120° and $\mathbf{V_{cn}}$ in turn leads $\mathbf{V_{bn}}$ by 120°.

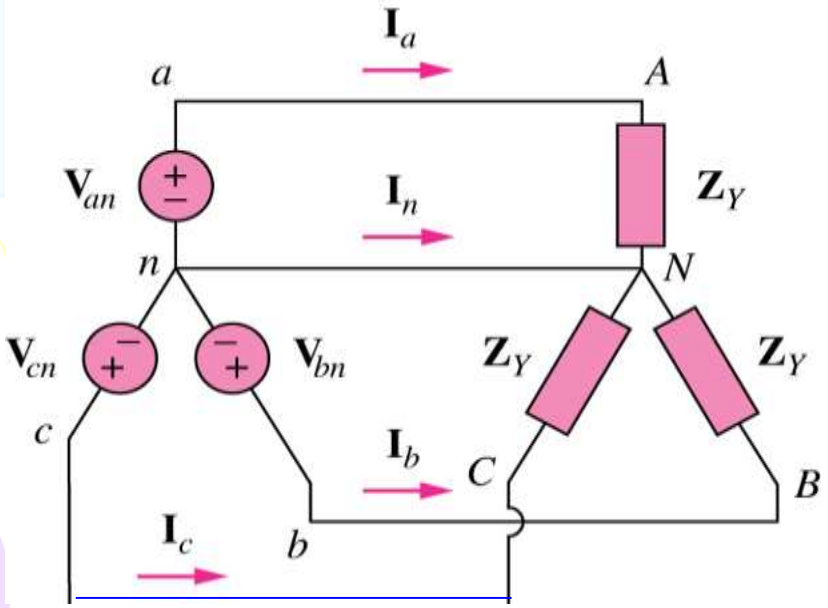
Hence, we have an acb sequence.

12.3 Balance Three-Phase Connection (1)

- Four possible connections
  1. Y-Y connection (Y-connected source with a Y-connected load)
  2. Y-Δ connection (Y-connected source with a Δ-connected load)
  3. Δ-Δ connection
  4. Δ-Y connection

12.3 Balance Three-Phase Connection (2)

- A balanced Y-Y system is a three-phase system with a balanced y-connected source and a balanced y-connected load.



$$V_L = \sqrt{3}V_p \text{, where}$$

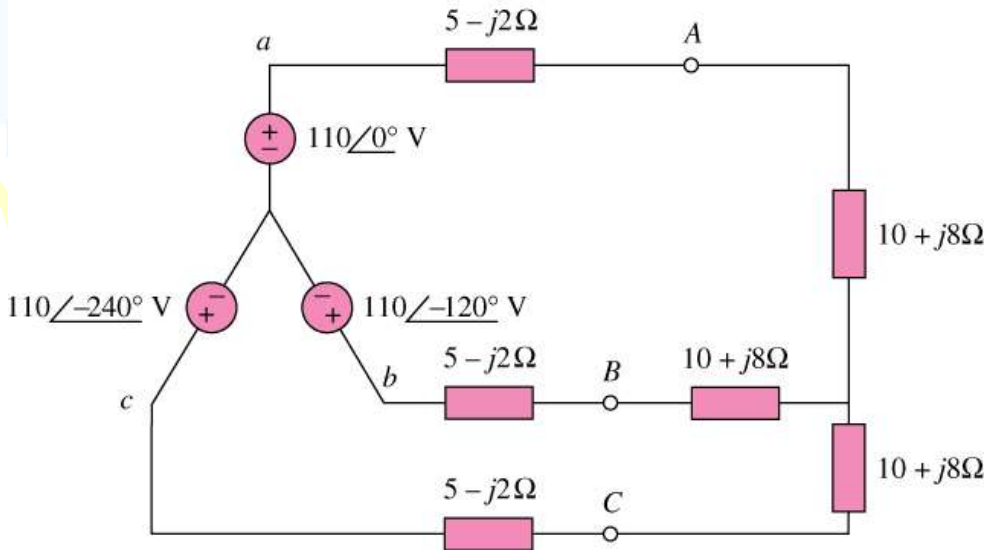
$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

12.3 Balance Three-Phase Connection (3)

Example 2

Calculate the line currents in the three-wire Y-Y system shown below:



Ans

$$I_a = 6.81\angle -21.8^\circ \text{ A}$$

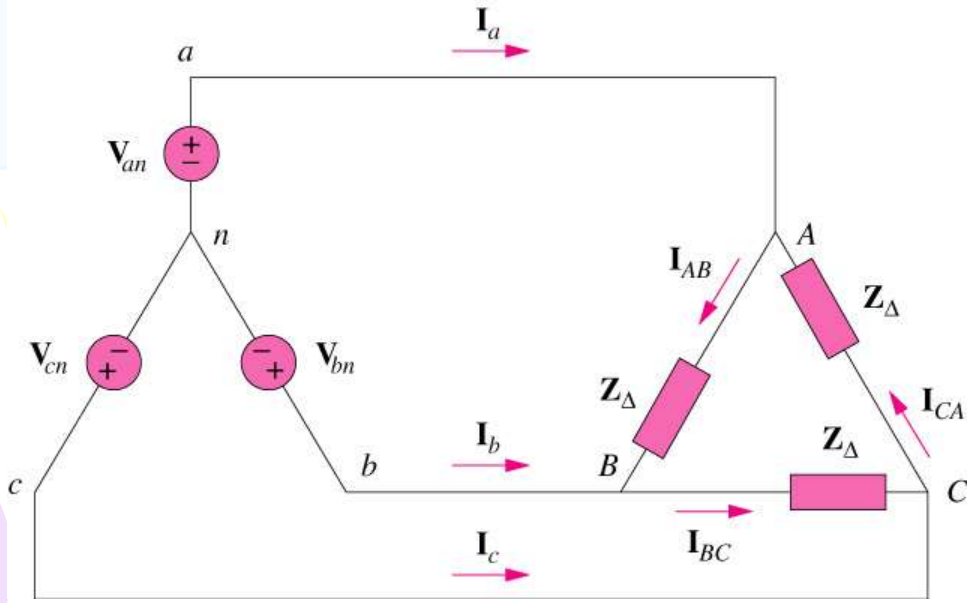
$$I_b = 6.81\angle -141.8^\circ \text{ A}$$

$$I_c = 6.81\angle 98.2^\circ \text{ A}$$

*Refer to in-class illustration, textbook

12.3 Balance Three-Phase Connection (4)

- A balanced Y-Δ system is a three-phase system with a balanced y-connected source and a balanced Δ-connected load.



$$I_L = \sqrt{3} I_p \text{ , where}$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

12.3 Balance Three-Phase Connection (5)

Example 3

A balanced *abc*-sequence Y-connected source with ($V_{an} = 100\angle 10°$) is connected to a Δ-connected load (8+j4)Ω per phase. Calculate the phase and line currents.

Solution

Using single-phase analysis,

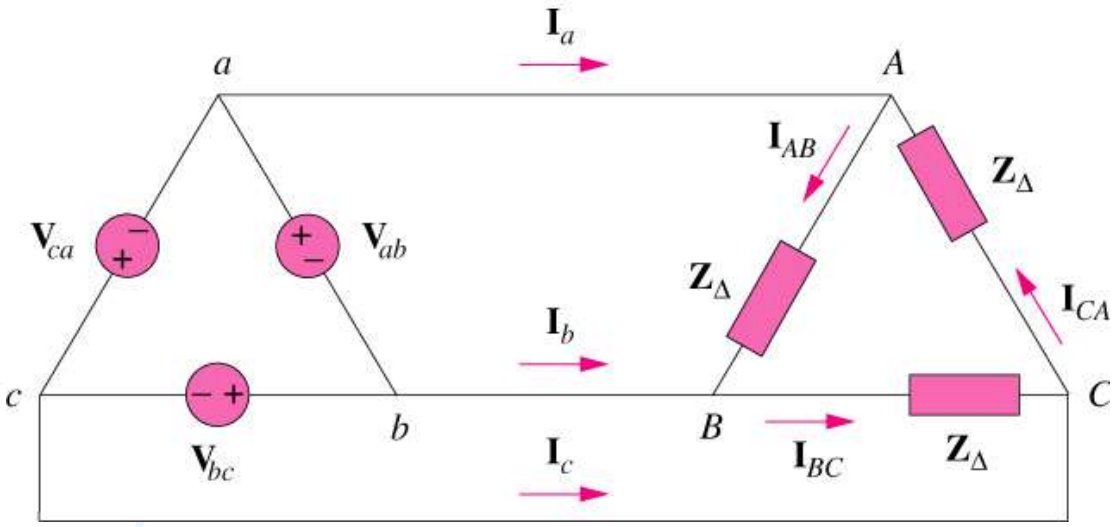
$$I_a = \frac{V_{an}}{Z_\Delta / 3} = \frac{100\angle 10°}{2.981\angle 26.57°} = 33.54\angle -16.57° \text{ A}$$

Other line currents are obtained using the *abc* phase sequence

*Refer to in-class illustration, textbook

12.3 Balance Three-Phase Connection (6)

- A balanced Δ-Δ system is a three-phase system with a balanced Δ -connected source and a balanced Δ -connected load.



12.3 Balance Three-Phase Connection (7)

Example 4

A balanced Δ-connected load having an impedance 20-j15 Ω is connected to a Δ-connected positive-sequence generator having ($V_{ab} = 330\angle 0° \text{ V}$). Calculate the phase currents of the load and the line currents.

Ans:

The phase currents

$$I_{AB} = 13.2\angle 36.87° \text{ A}; I_{BC} = 13.2\angle -81.13° \text{ A}; I_{AB} = 13.2\angle 156.87° \text{ A}$$

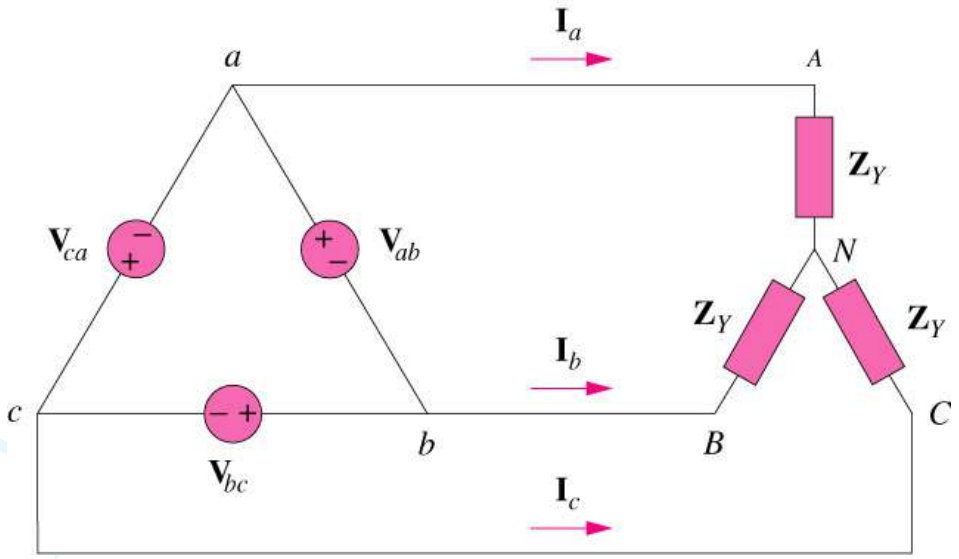
The line currents

$$I_a = 22.86\angle 6.87° \text{ A}; I_b = 22.86\angle -113.13° \text{ A}; I_c = 22.86\angle 126.87° \text{ A}$$

*Refer to in-class illustration, textbook

12.3 Balance Three-Phase Connection (8)

- A balanced Δ-Y system is a three-phase system with a balanced y-connected source and a balanced y-connected load.



12.3 Balance Three-Phase Connection (9)

Example 5

A balanced Y-connected load with a phase impedance 40+j25 Ω is supplied by a balanced, positive-sequence Δ-connected source with a line voltage of 210V. Calculate the phase currents. Use V_{ab} as reference.

Answer

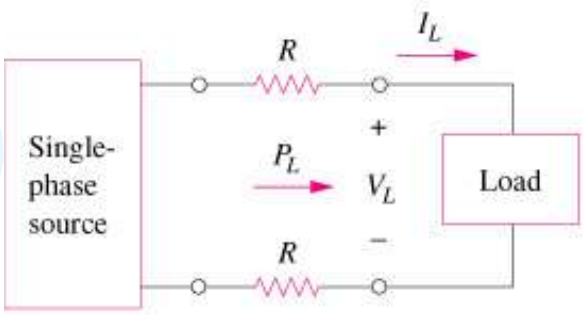
The phase currents

$$\mathrm{I}_{AN} = 2.57\angle -62^\circ \text{ A};$$
$$\mathrm{I}_{BN} = 2.57\angle -178^\circ \text{ A};$$
$$\mathrm{I}_{CN} = 2.57\angle 58^\circ \text{ A};$$

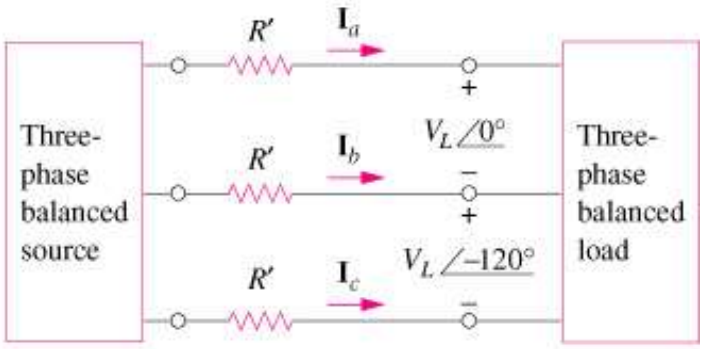
*Refer to in-class illustration, textbook

12.4 Power in a Balanced System (1)

- Comparing the power loss in (a) a single-phase system, and (b) a three-phase system



(a)



(b)

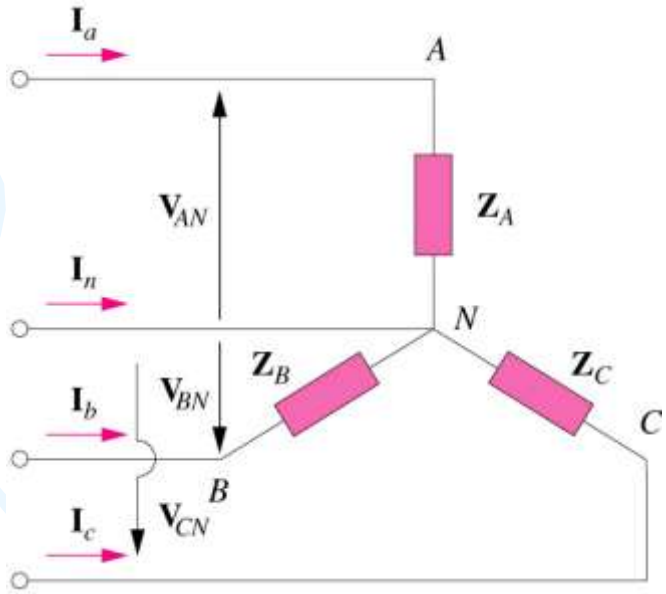
$$P'_{loss} = 2R\frac{P_L^2}{V_L^2}, \text{single - phase}$$

$$P'_{loss} = R'\frac{P_L^2}{V_L^2}, \text{three - phase}$$

- If same power loss is tolerated in both system, three-phase system use only 75% of materials of a single-phase system

12.5 Unbalanced Three-Phase Systems (1)

- An unbalanced system is due to unbalanced voltage sources or an unbalanced load.



$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A},\ \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B},\ \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C},$$

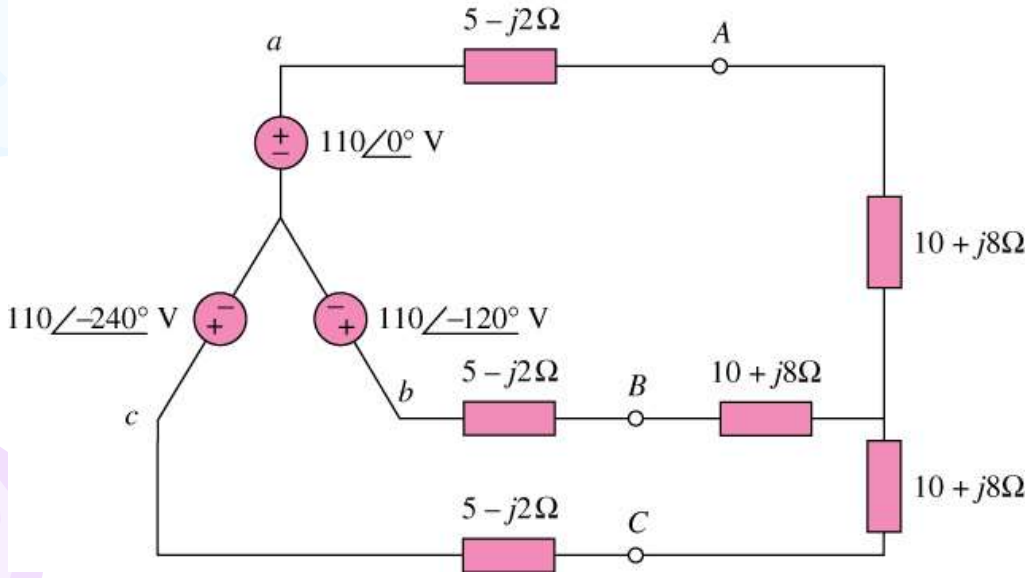
$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$

- To calculate power in an unbalanced three-phase system requires that we find the power in each phase.
- The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

12.3 Unbalanced Three-Phase Systems (2)

Example 6

Determine the total average power, reactive power, and complex power at the source and at the load



Ans

At the source:

S_s = -(2087 + j834.6) VA

P_a = -2087W

P_r = -834.6VAR

At the load:

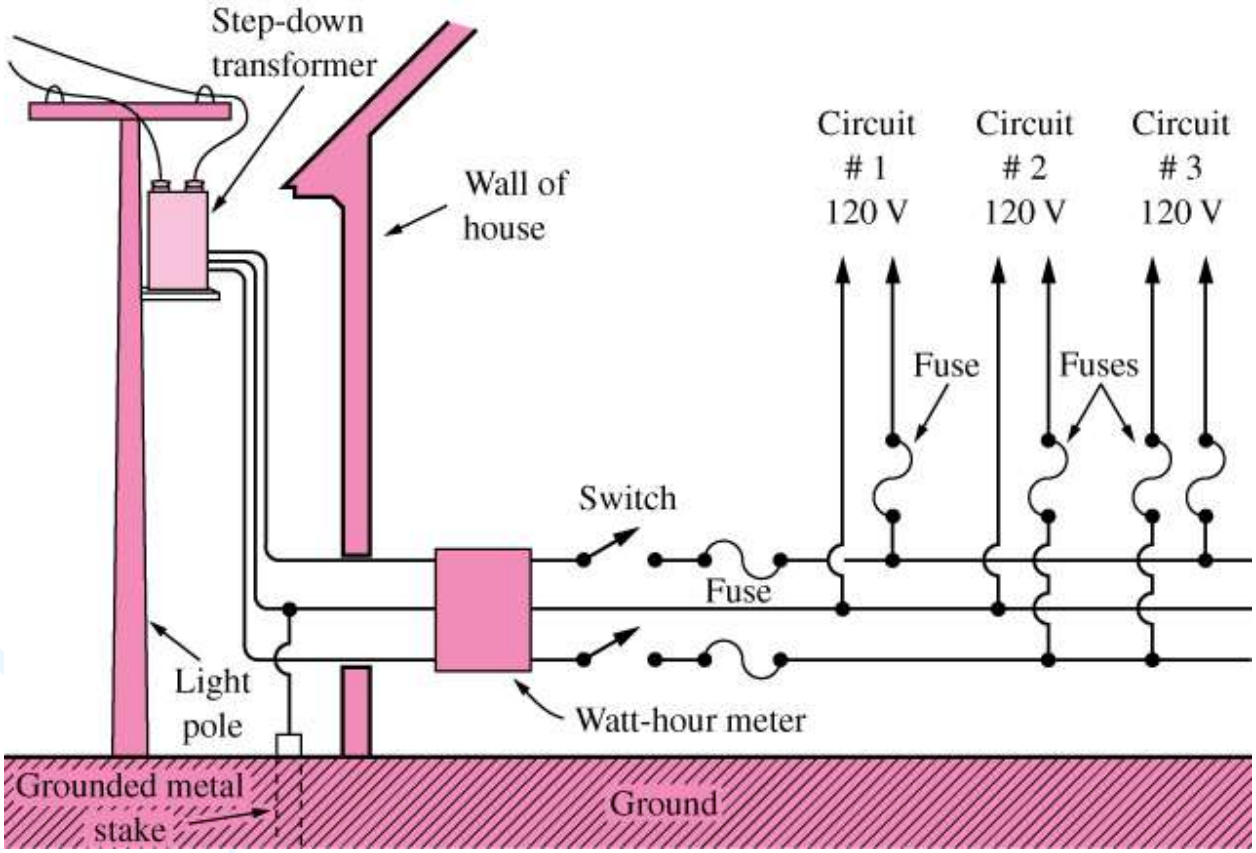
S_L = (1392 + j1113) VA

P_a = 1392W

P_r = 1113VAR

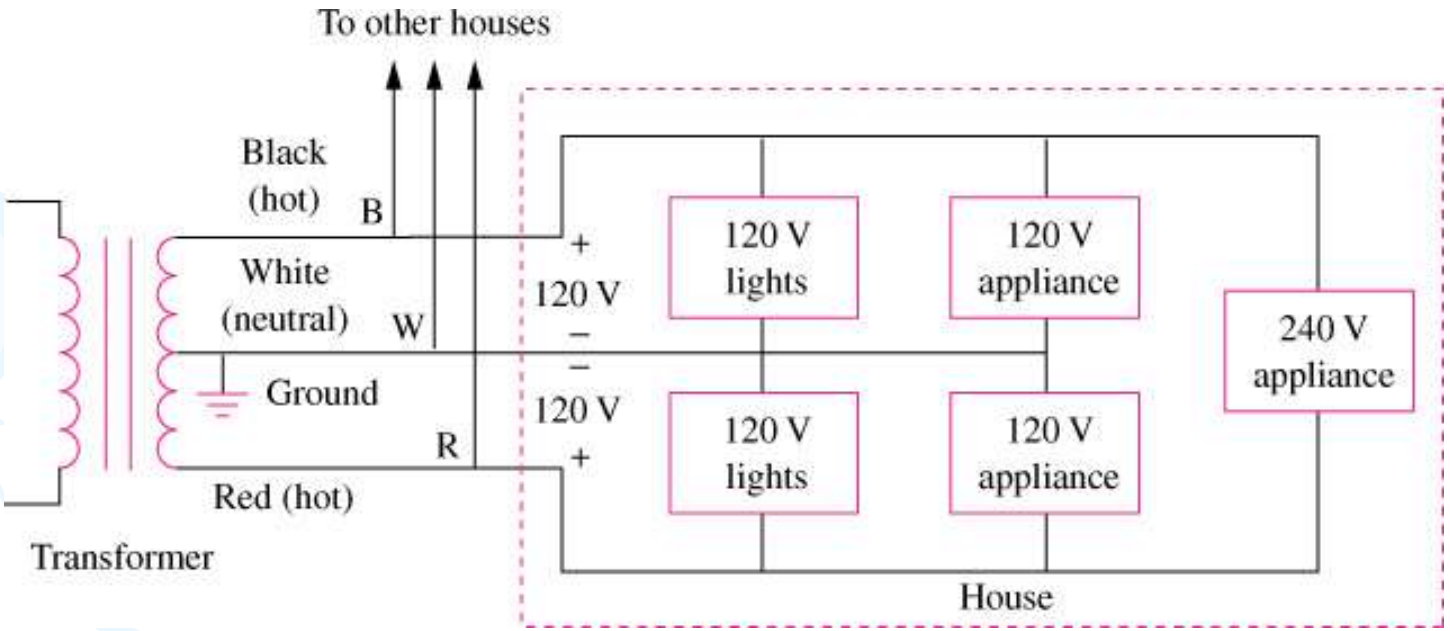
*Refer to in-class illustration, textbook

12.6 Application – Residential Wiring (1)



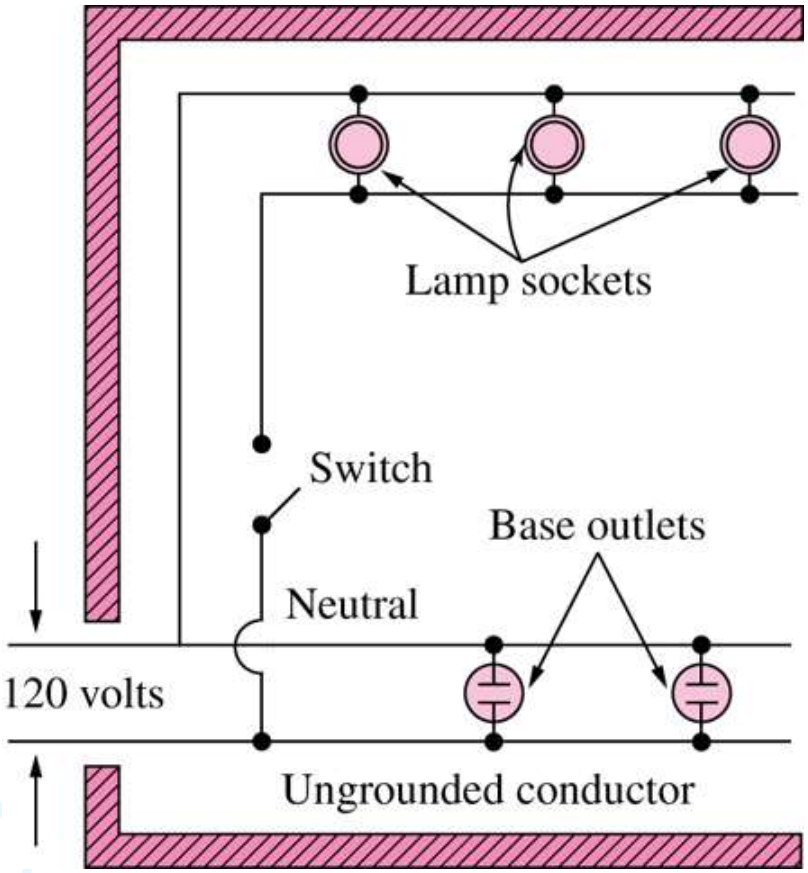
A 120/240 household power system

12.6 Application – Residential Wiring (2)



Single-phase three-wire residential wiring

12.6 Application – Residential Wiring (3)



A typical wiring diagram of a room