Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 14
Frequency Response

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Frequency Response
Chapter 14

14.1 Introduction
14.2 Transfer Function
14.3 Series Resonance
14.4 Parallel Resonance
14.5 Passive Filters

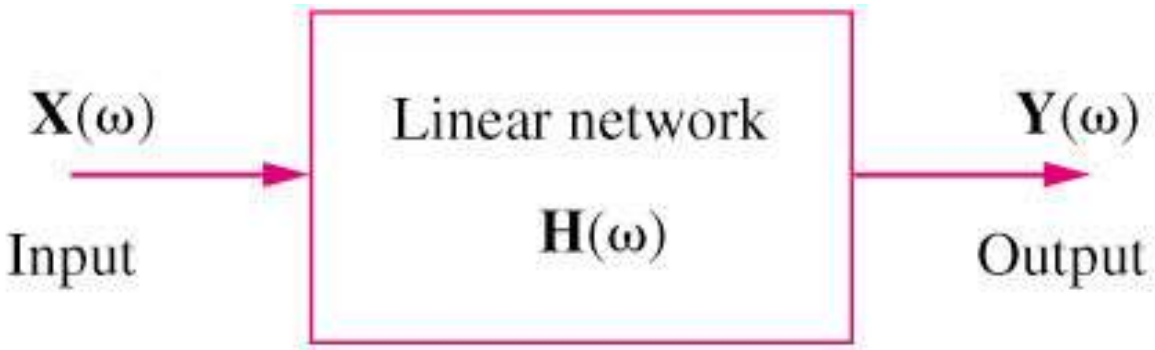
14.1 Introduction (1)

What is FrequencyResponse of a Circuit?

It is the variation in a circuit's behavior with change in signal frequency and may also be considered as the variation of the gain and phase with frequency.

14.2 Transfer Function (1)

- The transfer function H(ω) of a circuit is the **frequency-dependent ratio** of a phasor output Y(ω) (an element voltage or current) to a phasor input X(ω) (source voltage or current).



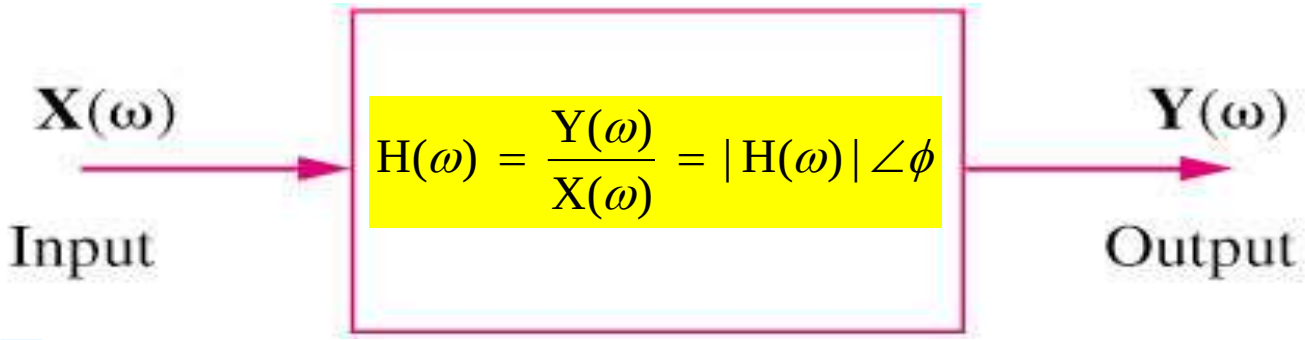
$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = |H(\omega)| \angle \phi$$

14.2 Transfer Function (2)

- Four possible transfer functions:

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Transfer Impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$



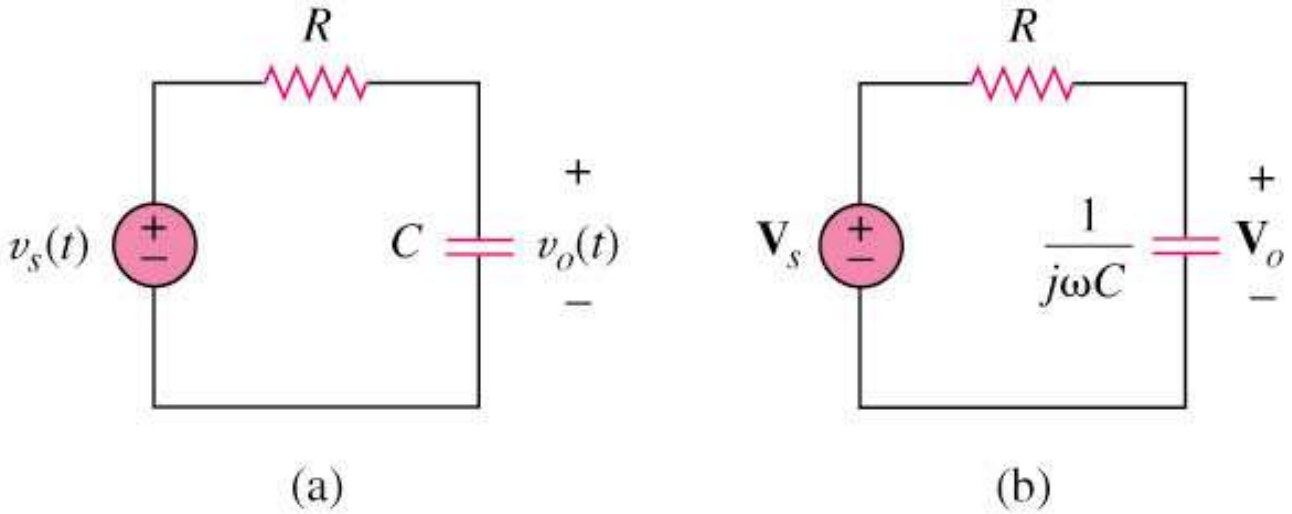
$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

14.2 Transfer Function (3)

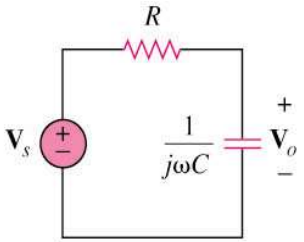
Example 1

For the RC circuit shown below, obtain the transfer function Vo/Vs and its frequency response.
Let $v_s = V_m \cos\omega t$.



14.2 Transfer Function (4)

Solution:



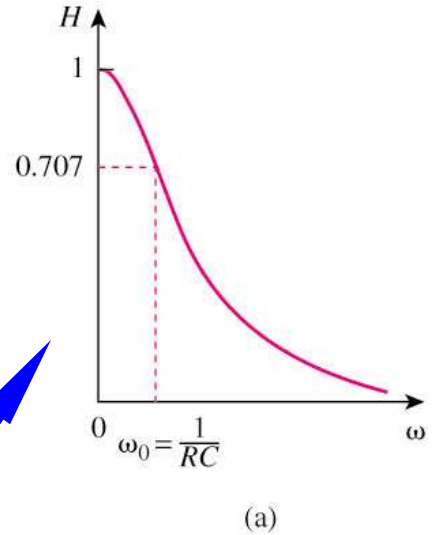
The transfer function is

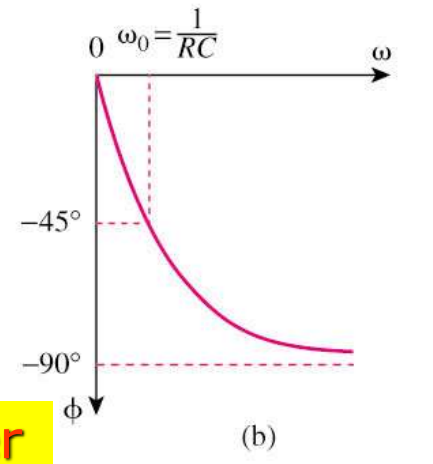
$$H(\omega) = \frac{V_o}{V_s} = \frac{\frac{1}{j\omega C}}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

The magnitude is $|H(\omega)| = \dfrac{1}{\sqrt{1 + (\omega/\omega_o)^2}}$

The phase is $\phi = -\tan^{-1}\dfrac{\omega}{\omega_o}$

$\omega_o = 1/RC$



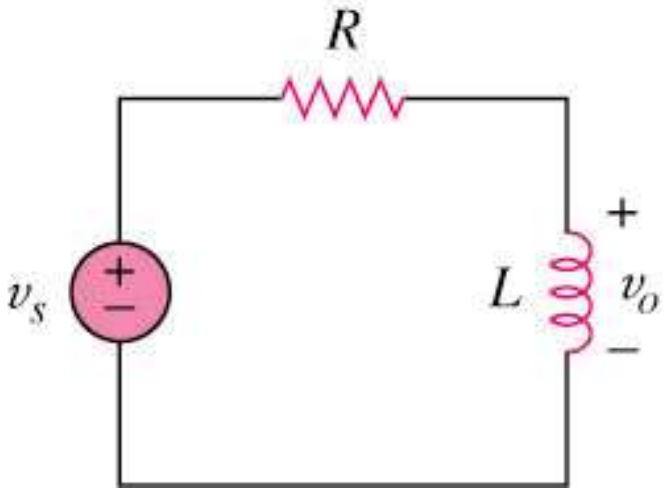


Low Pass Filter

14.2 Transfer Function (5)

Example 2

Obtain the transfer function Vo/Vs of the RL circuit shown below, assuming $v_s = V_m \cos\omega t$. Sketch its frequency response.

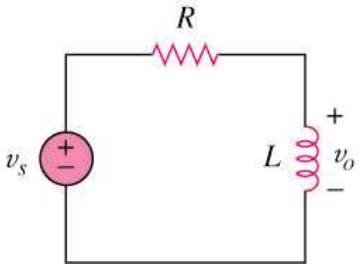


14.2 Transfer Function (6)

Solution:

The transfer function is

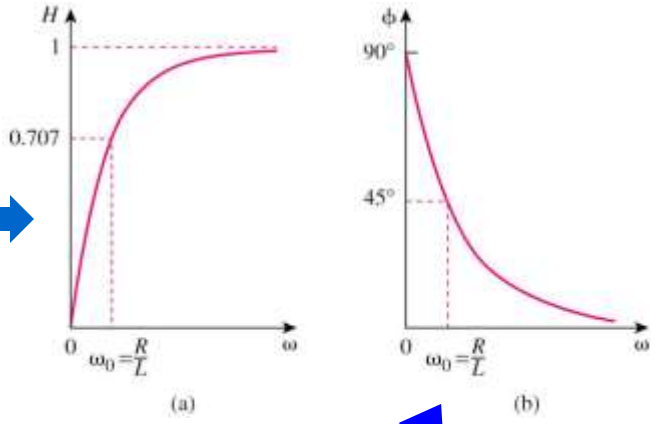
$$H(\omega) = \frac{V_o}{V_s} = \frac{j\omega L}{R + j\omega L} = \frac{1}{1 + \frac{R}{j\omega L}}$$



The magnitude is

$$H(\omega) = \frac{1}{\sqrt{1 + (\frac{\omega_o}{\omega})^2}}$$

High Pass Filter

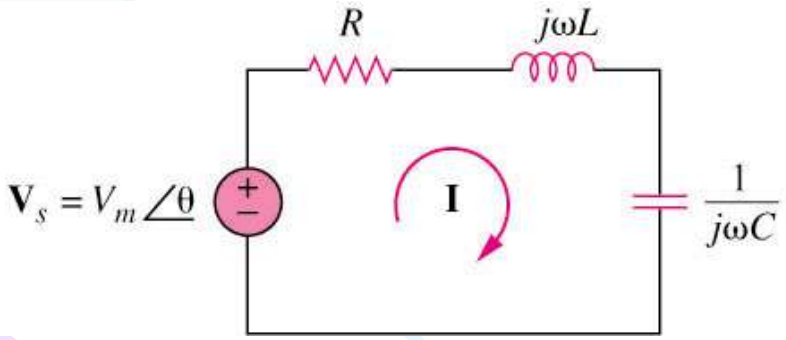


The phase is $\phi = \angle 90° - \tan^{-1}\frac{\omega}{\omega_o}$

$$\omega_o = R/L$$

14.3 Series Resonance (1)

Resonance is a condition in an RLC circuit in which the capacitive and inductive reactance are equal in magnitude, thereby resulting in purely resistive impedance.



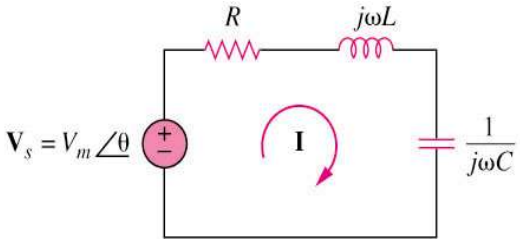
$$Z = R + j(\omega L - \frac{1}{\omega C})$$

Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s} \qquad or$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

14.3 Series Resonance (2)



$$Z = R + j(\omega L - \frac{1}{\omega C})$$

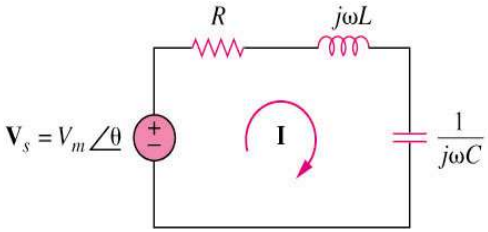
The features of series resonance:

The impedance is purely resistive, Z = R;

- The supply voltage Vs and the current I are in phase, so cos θ = 1;
- The magnitude of the transfer function H(ω) = Z(ω) is minimum;
- The inductor voltage and capacitor voltage can be much more than the source voltage.

14.3 Series Resonance (3)

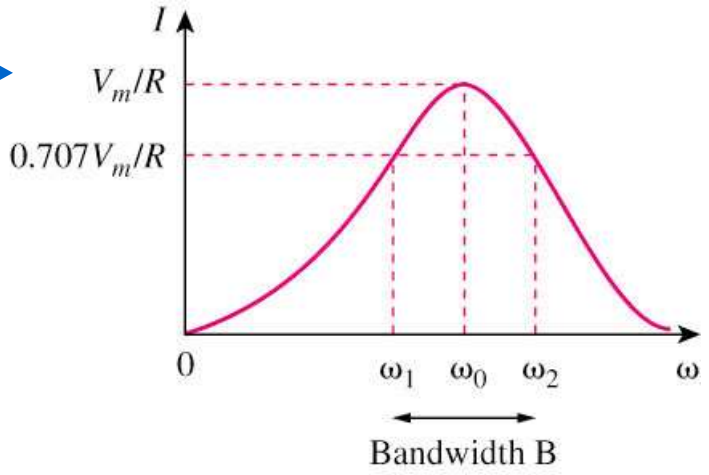
Bandwidth B



$$Z = R + j(\omega L - \frac{1}{\omega C})$$

The frequency response of the resonance circuit current is

$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



The average power absorbed by the RLC circuit is

$$P(\omega) = \frac{1}{2} I^2 R$$

The highest power dissipated occurs at resonance:

$$P(\omega_o) = \frac{1}{2} \frac{V_m^2}{R}$$

14 3 Series Resonance (4)

Half-power frequencies ω_1 and ω_2 are frequencies at which the dissipated power is half the maximum value:

$$P(\omega_1) = P(\omega_2) = \frac{1}{2}\frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{4R}$$

The half-power frequencies can be obtained by setting Z equal to $\sqrt{2}$ R.

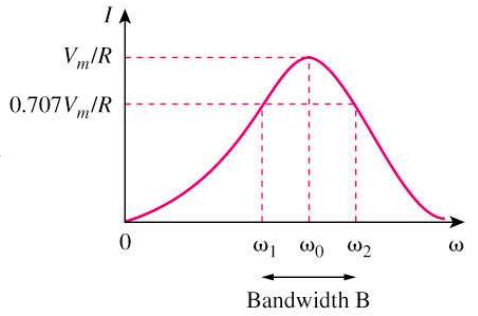
$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

Bandwidth B

$$B = \omega_2 - \omega_1$$



14.3 Series Resonance (5)

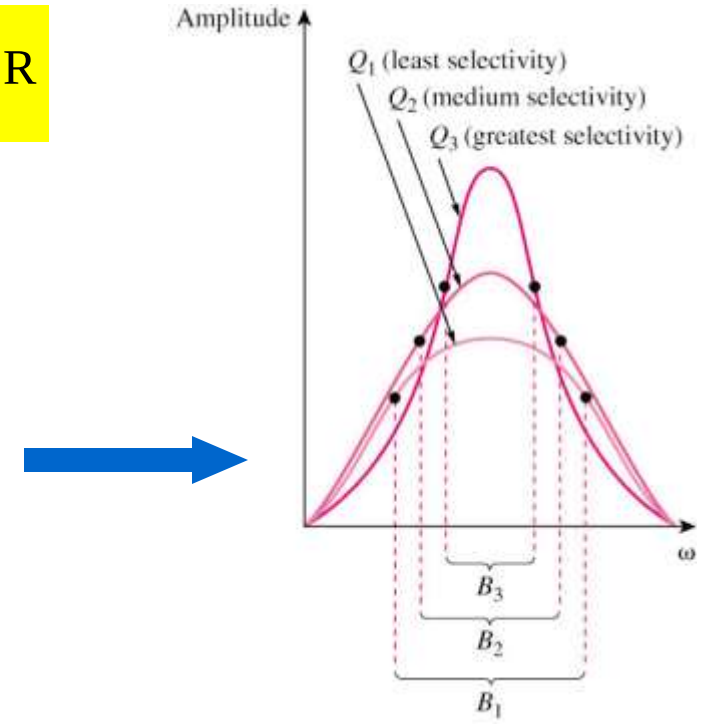
Quality factor,

$$Q = \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

The relationship between the B, Q and ω_o:

$$B = \frac{R}{L} = \frac{\omega_o}{Q} = \omega_o^2 CR$$

- The quality factor is the ratio of its resonant frequency to its bandwidth.
- If the bandwidth is narrow, the quality factor of the resonant circuit must be high.
- If the band of frequencies is wide, the quality factor must be low.



14.3 Series Resonance (6)

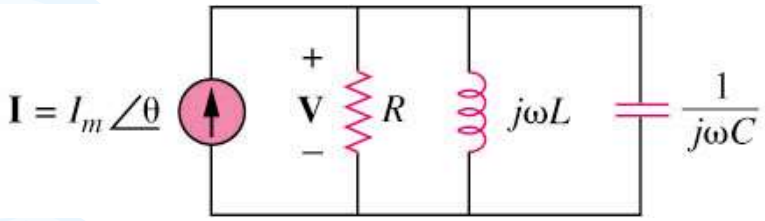
Example 3

A series-connected circuit has R = 4 Ω
and L = 25 mH.

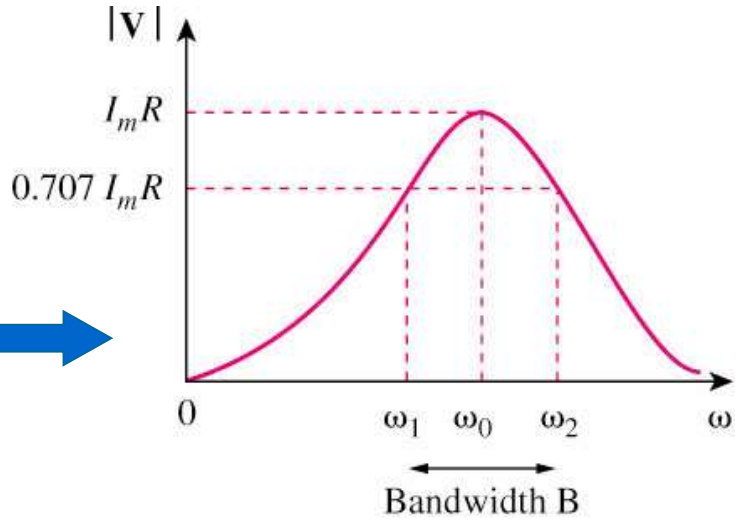
a. Calculate the value of C that will produce a quality factor of 50.
b. Find ω_1 and ω_2, and B.
c. Determine the average power dissipated at $\omega = \omega_o, \omega_1, \omega_2$. Take V_m= 100V.

14.4 Parallel Resonance (1)

It occurs when imaginary part of Y is zero



$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$



Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s} \quad \text{or} \quad f_o = \frac{1}{2\pi\sqrt{LC}} Hz$$

14.4 Parallel Resonance (2)

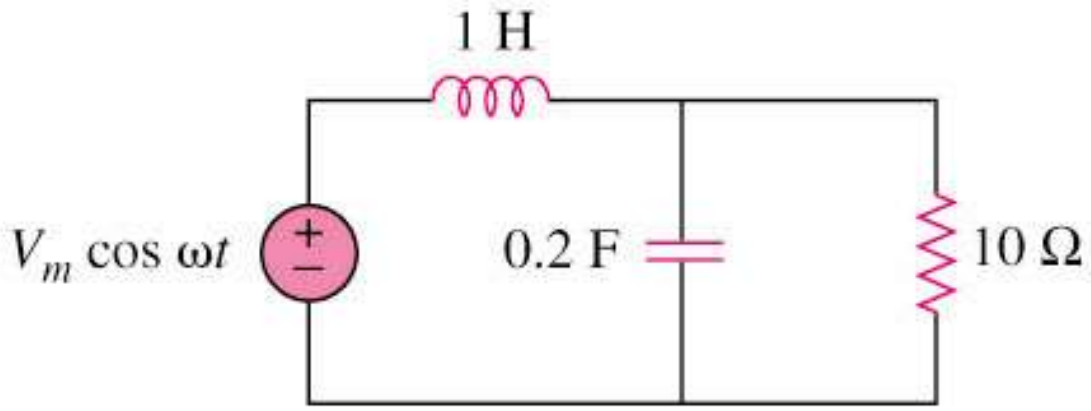
Summary of series and parallel resonance circuits:

characteristic	*Series circuit*	*Parallel circuit*
ω_o	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Q	$\frac{\omega_o L}{R}$ or $\frac{1}{\omega_o RC}$	$\frac{R}{\omega_o L}$ or $\omega_o RC$
B	$\frac{\omega_o}{Q}$	$\frac{\omega_o}{Q}$
$\omega_1,\ \omega_2$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$
$Q \geq 10, \omega_1,\ \omega_2$	$\omega_o \pm \frac{B}{2}$	$\omega_o \pm \frac{B}{2}$

14.4 Parallel Resonance (3)

Example 4

Calculate the resonant frequency of the circuit in the figure shown below.



Answer: $\omega = \dfrac{\sqrt{19}}{2} = 2.179 \text{ rad/s}$

14.5 Passive Filters (1)

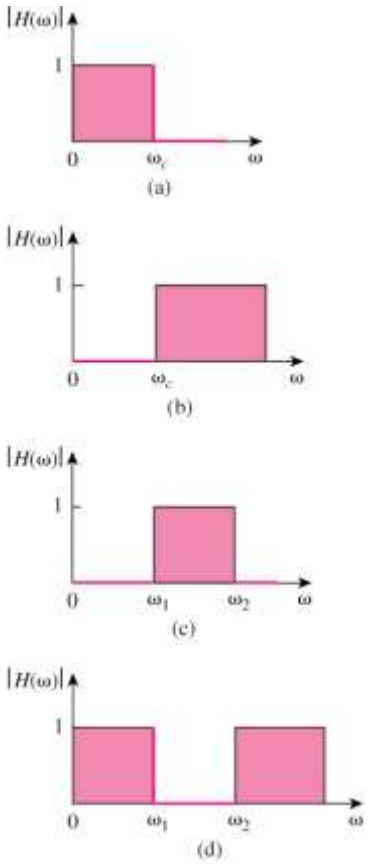
- A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.
- Passive filter consists of only passive element R, L and C.
- There are four types of filters.

Low Pass

High Pass

Band Pass

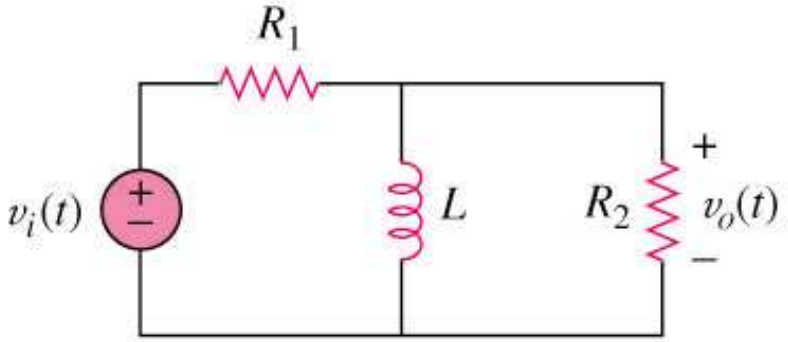
Band Stop



14.5 Passive Filters (2)

Example 5

For the circuit in the figure below, obtain the transfer function Vo(ω)/Vi(ω). Identify the type of filter the circuit represents and determine the corner frequency. Take R1=100Ω =R2 and L =2mH.



Answer: $\omega = 25$ krad/s