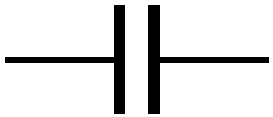
EGR 2201 Unit 8
Capacitors and Inductors

- Read Alexander & Sadiku, Chapter 6.
- Homework #8 and Lab #8 due next week.
- Quiz next week.

Two New Passive Circuit Elements

- Recall that resistors are called *passive elements* because they cannot generate electrical energy.
- The two other common passive elements are **capacitors** and **inductors**.
- Resistors *dissipate* energy as heat, but capacitors and inductors *store* energy, which can later be returned to the circuit.

Capacitors



- A **capacitor** is a passive device designed to store energy in its electric field.

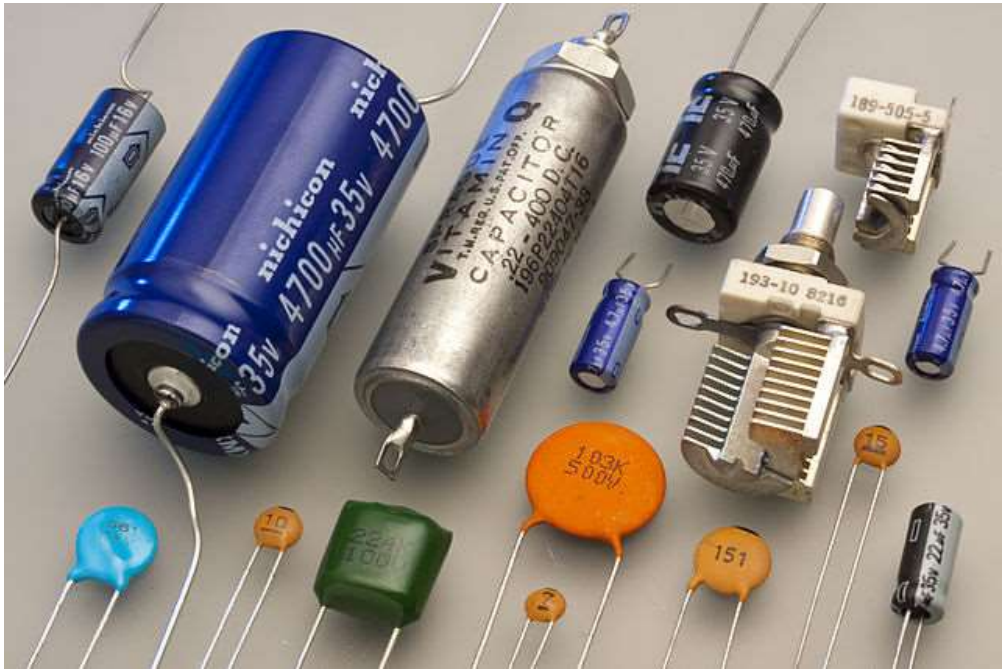
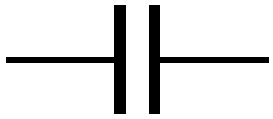
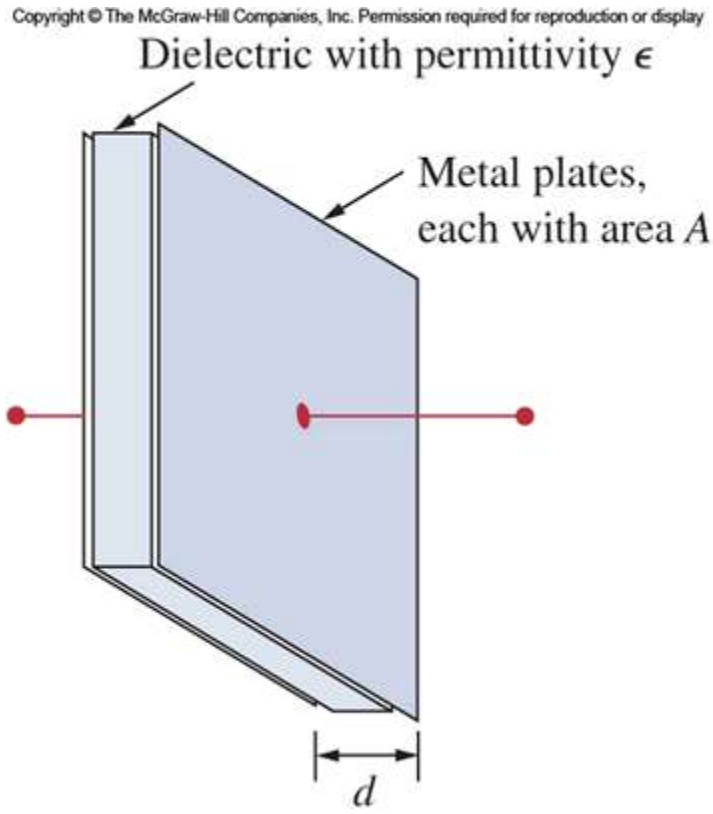


Image from Wikipedia.

Parallel-Plate Capacitor



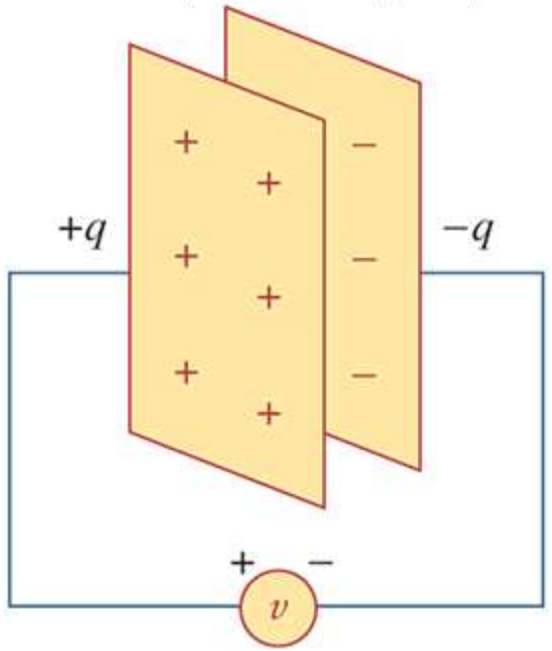
- A capacitor typically consists of two metal plates separated by an insulator.
- The insulator between the plates is called the **dielectric**.



Charging a Capacitor

- When a capacitor is connected across a voltage source, charge flows between the source and the capacitor's plates until the voltage across the capacitor is equal to the source voltage.
- In this process, one plate becomes positively charged, and the other plate becomes negatively charged.

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Units of Capacitance

- **Capacitance** is the measure of a capacitor's ability to store charge.
- Capacitance is abbreviated *C*.
- The unit of capacitance is the **farad** (F).
- Typical capacitors found in electronic equipment are in the picofarad (pF), nanofarad (nF), or microfarad (μF) range.

Labels on Capacitors

- Several different codes are used for the labels on capacitors to indicate their capacitance.
- In the most common code, the base unit is picofarads, and a 3-digit number (similar to the three color bands on resistors) designates the number of pF.
- Example: a code of 103 means 10,000 pF = 10 nF = 0.01 μF.

Measuring Capacitance

- We use an **LCR meter** (for inductance-capacitance-resistance meter) to measure a capacitor's capacitance.



User manual for this meter.

This button selects between inductance, capacitance, resistance.

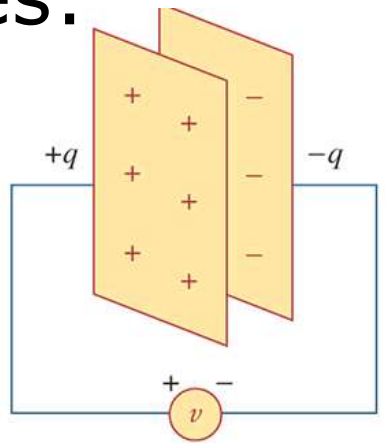
Update: Some Quantities and Their Units

Quantity	Symbol	SI Unit	Symbol for the Unit
Current	*I* or *i*	ampere	A
Voltage	*V* or *v*	volt	V
Resistance	*R*	ohm	Ω
Charge	*Q* or *q*	coulomb	C
Time	*t*	second	s
Energy	*W* or *w*	joule	J
Power	*P* or *p*	watt	W
Conductance	*G*	siemens	S
Capacitance	*C*	farad	F
Inductance	*L*	henry	H

Capacitance = Charge per Voltage

- Mathematically, capacitance is defined as the ratio of the charge stored on a capacitor's plate to the voltage across the two plates:

$$C = \frac{q}{v}$$

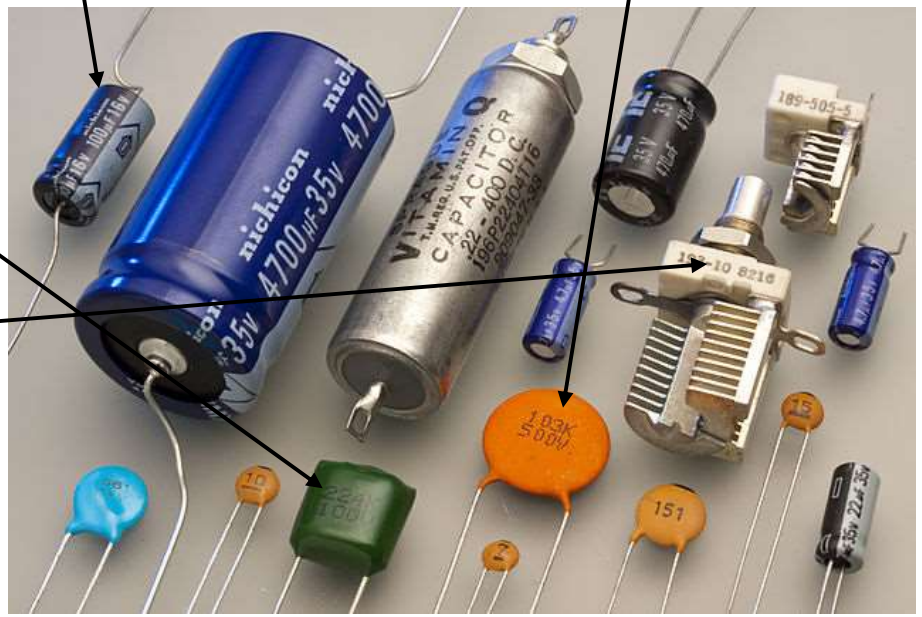


where C is in farads, q is in coulombs, and v is in volts.

- Thus one farad equals one coulomb per volt.

Capacitor Types

- Capacitors can be classified by the materials used for their dielectrics (such as air, paper, tantalum, ceramic, plastic film, mica, electrolyte).
- Each type has its own tradeoffs in practical use.
- Variable capacitors are also available.



Electrolytic Capacitors (1 of 2)

- Electrolytic capacitors are available in very large values, such as 100,000 μF.
- Unlike most other capacitors, they are polarized: one side must remain positive with respect to the other.
- Therefore . . .

Arrow printed on the case points toward negative lead.



Electrolytic Capacitors (2 of 2)

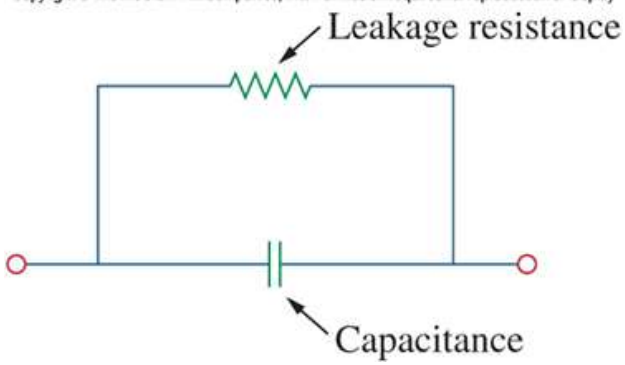
- You must insert electrolytic capacitors in the proper direction. Inserting them backwards can result in injury or in damage to equipment.



Capacitors Store Energy

- Recall that energy is dissipated as heat when current flows through a resistance.
- An ideal capacitor does not dissipate energy. Rather it stores energy, which can later be returned to the circuit.
- We can model a real, non-ideal capacitor by including a resistance in parallel with the capacitance.

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Capacitor Energy Equation

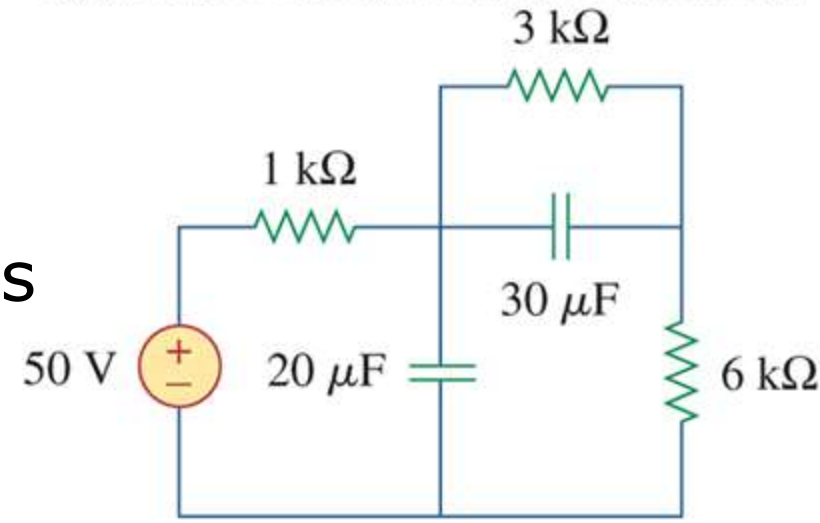
- The energy w stored in a capacitor is given by

$$w = \frac{1}{2} C v^2$$

  where *C* is the capacitor's capacitance and *v* is the voltage across the capacitor.
- Recall the units: w is in joules, C is in farads, and v is in volts.

DC Conditions in a Circuit with Capacitors

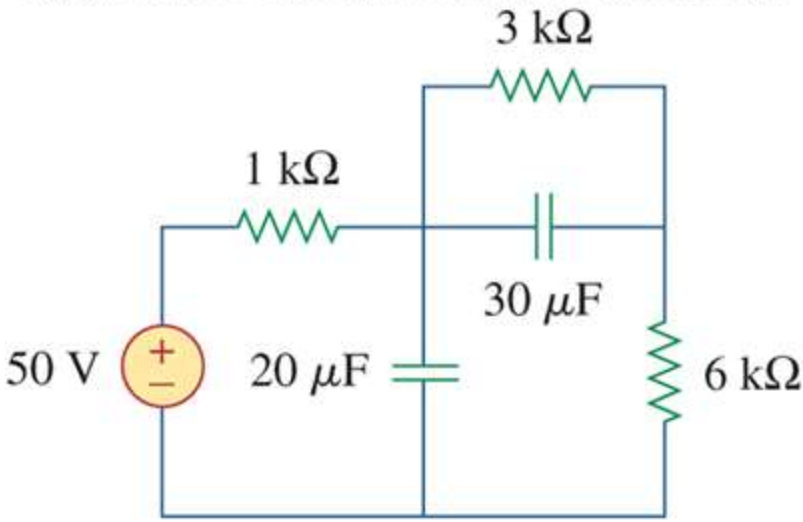
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- When power is first applied to a circuit like the one shown, voltages and currents change briefly as the capacitors "charge up."
- But once the capacitors are fully charged, all voltages and currents in the circuit have constant values.
- We use the term "**dc conditions**" to refer to these final constant values.

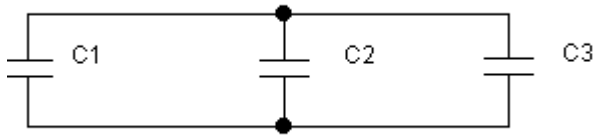
Under DC Conditions, Capacitors Act Like Opens

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- Under dc conditions, a capacitor acts like an open circuit.
- So to analyze a circuit containing capacitors *under dc conditions*, replace all capacitors with open circuits.
- Later we'll look at how to analyze such circuits during the "charging-up" time. (It's trickier!)

Capacitors in Parallel:
Equivalent Capacitance

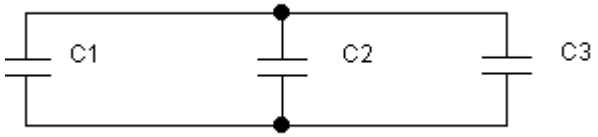


- The equivalent capacitance of capacitors in parallel is the sum of the individual capacitances:

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots + C_N$$

- Similar to the formula for resistors in series.

Capacitors in Parallel:
Voltage, Charge, and Energy

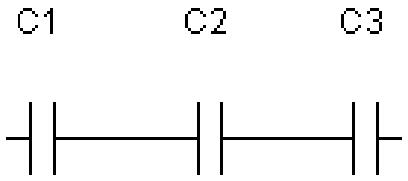


- Parallel-connected capacitors have the same voltage.
- If you know the voltage v across the capacitors, you can find each capacitor's charge and energy by applying the formulas

$$q = Cv \quad \text{and} \quad w = \frac{1}{2}Cv^2$$

to each capacitor.

Capacitors in Series:
Equivalent Capacitance



- The equivalent capacitance of capacitors in series is given by the reciprocal formula:

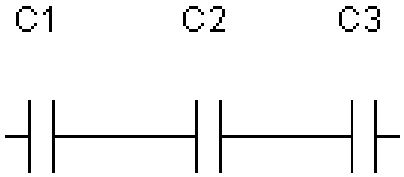
$$C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}}$$

- For **two** capacitors in series, we can use the product-over-sum rule:

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}$$

- Similar to the formulas for resistors in parallel.

Capacitors in Series:
Charge, Voltage, and Energy



- Series-connected capacitors have the same charge:

$$q_1 = q_2 = q_3 = \ldots$$

- If you know the capacitor's charges, you can find each capacitor's voltage and energy by applying the formulas

$$v = \frac{q}{C} \quad \text{and} \quad w = \frac{1}{2}Cv^2$$

to each capacitor.

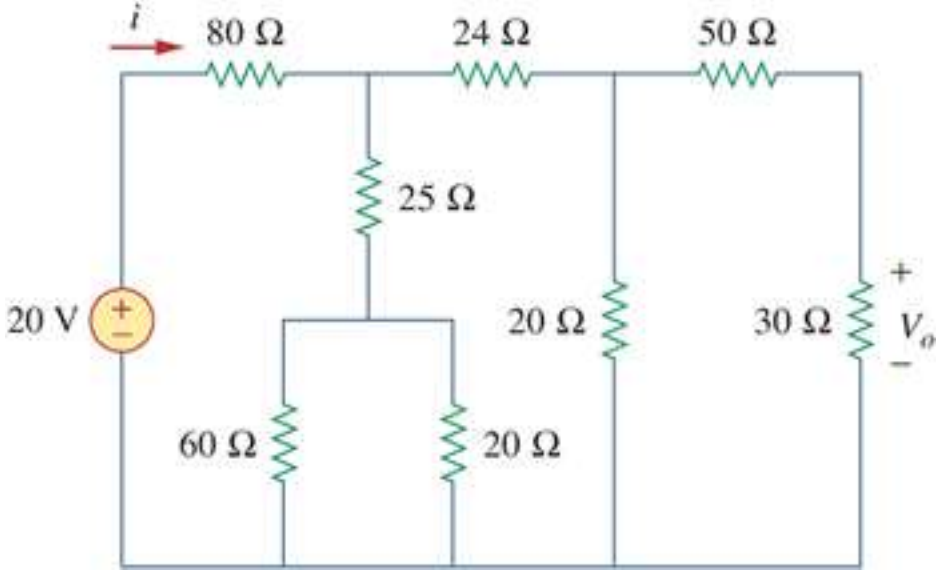
Series-Parallel Capacitors

- For series-parallel capacitor circuits:

1. Combine series and parallel capacitors to obtain progressively simpler equivalent circuits.
2. Then work backwards, using $C = \frac{q}{v}$ and remembering how charge is distributed among series capacitors and parallel capacitors.

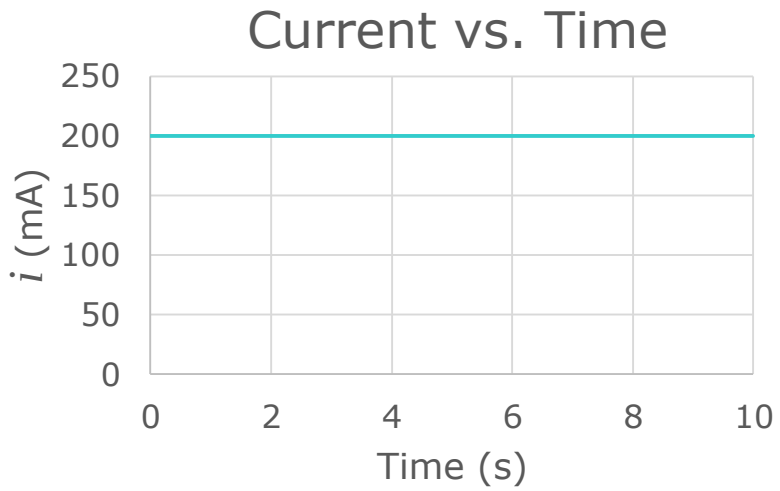
Constant Voltages and Currents

- In circuits that we've analyzed up to now, voltages and currents have been constant as time passes.
  - Example: In this circuit, the current i is constant (200 mA) and the voltage V_o is constant (600 mV).



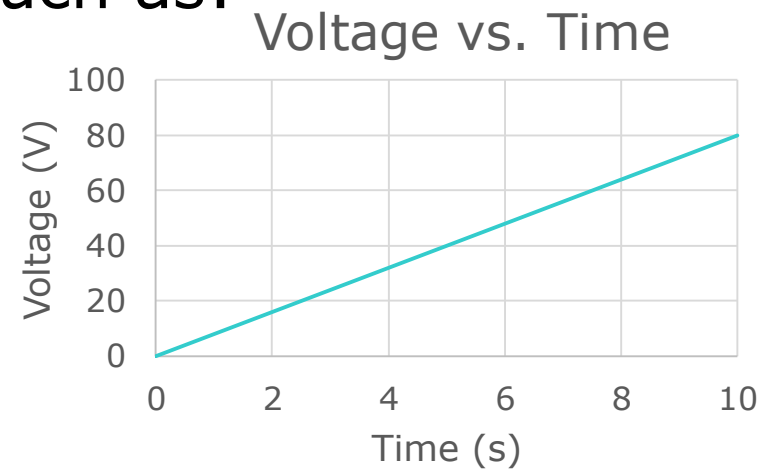
Graphs of Constant Values Versus Time

- Up to now we haven't used graphs of voltage versus time or of current versus time. With constant voltages and currents, such graphs wouldn't be very interesting.
  - Example: Here's a graph of the current i versus time for the circuit on the previous slide.



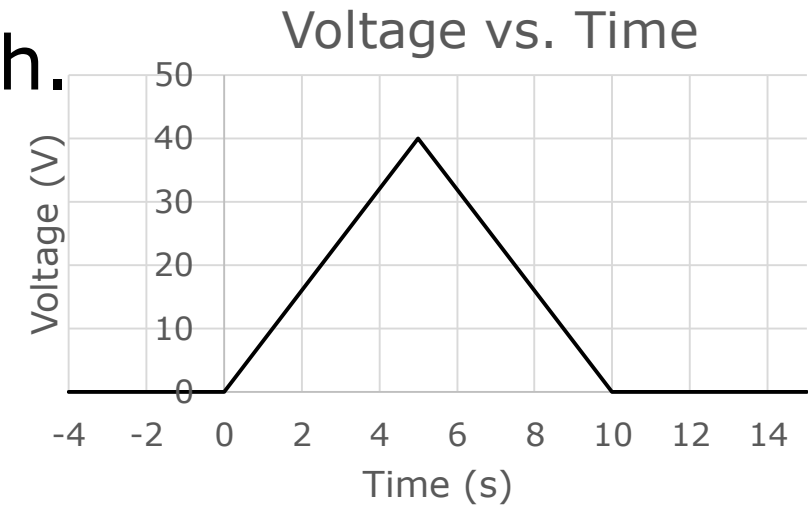
Changing Voltages and Currents

- In many cases, voltages and currents in a circuit change as time passes.
- We have two ways of describing these changing values:
  1. Using an equation, such as $v(t) = 8t$ V.
  2. Using a graph, such as:



A More Complicated Example

- Consider this graph.



- To describe it using equations, write:

$$v(t) = \begin{cases} 0\ \text{V}, & t < 0\ \text{s} \\ 8t\ \text{V}, & 0 \le t < 5\ \text{s} \\ 80 - 8t\ \text{V}, & 5 \le t < 10\ \text{s} \\ 0\ \text{V}, & 10\ \text{s} \le t \end{cases}$$

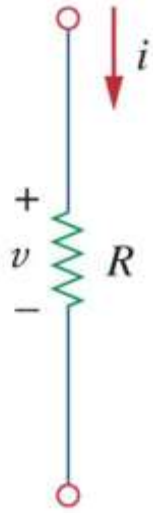
Current-Voltage Equations

- Key equations for any circuit element are the equations that relate the element's current to its voltage.
- For resistors, these are purely algebraic equations, as given by Ohm's law, which we'll review on the next slide.
- But for capacitors and inductors, the equations involve derivatives and integrals.

Review of Equations for a Resistor

- Recall that for a resistor, we have

$$i = \frac{v}{R}$$



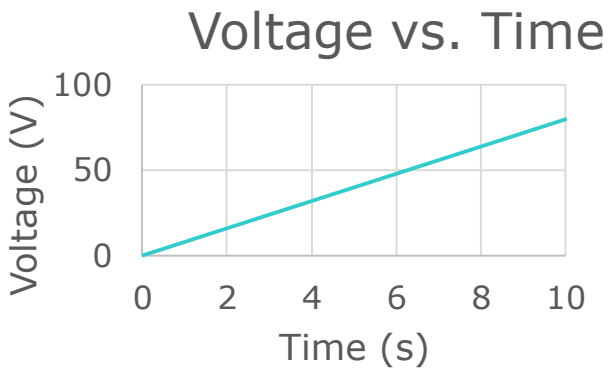
- Let's call that the **current-voltage equation** for a resistor.
- And a resistor's **voltage-current equation** is

$$v = iR$$

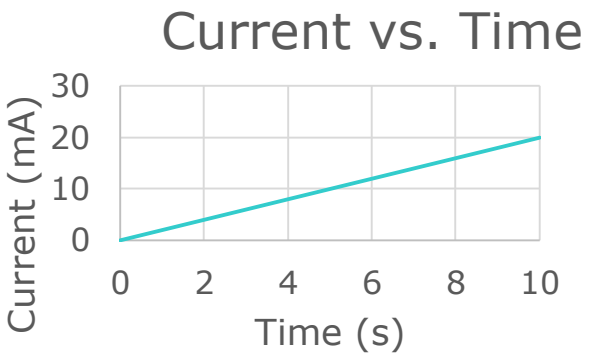
- These equations involve only algebraic operations (division and multiplication).
- Both equations assume the passive sign convention (current flows into the positive end). Otherwise you must insert a − sign.

Changing Voltages and Currents in Resistors

- Since a resistor's voltage and current are directly proportional to each other, it's easy to find one when given the graph or equation of the other.
- Example: Suppose a 4-kΩ resistor's voltage is $v(t) = 8t$ V:



- Then the resistor's current is $i(t) = 2t$ mA:



Changing Voltages and Currents in Resistors: A More Complicated Example (1 of 2)

- Since a resistor's voltage and current are directly proportional to each other, it's easy to write the equation for one when given the equation for the other.
- Example: Suppose a 2-kΩ resistor's voltage is given by:

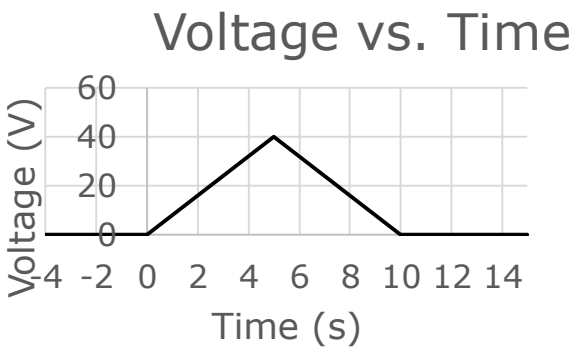
$$v(t) = \begin{cases} 0\ \text{V}, & t < 0\ \text{s} \\ 8t\ \text{V}, & 0 \le t < 5\ \text{s} \\ 80 - 8t\ \text{V}, & 5 \le t < 10\ \text{s} \\ 0\ \text{V}, & 10\ \text{s} \le t \end{cases}$$

- Then the resistor's current is given by:

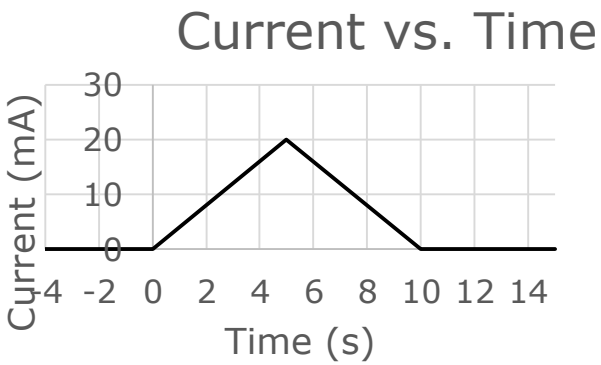
$$i(t) = \begin{cases} 0\ \text{mA}, & t < 0\ \text{s} \\ 4t\ \text{mA}, & 0 \le t < 5\ \text{s} \\ 40 - 4t\ \text{mA}, & 5 \le t < 10\ \text{s} \\ 0\ \text{mA}, & 10\ \text{s} \le t \end{cases}$$

Changing Voltages and Currents in Resistors: A More Complicated Example (2 of 2)

- Since a resistor's voltage and current are directly proportional to each other, it's easy to graph either one when given the graph of the other.
- Example: Suppose a 2-kΩ resistor's voltage is as shown.



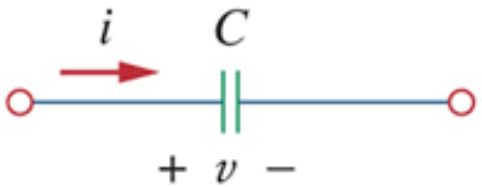
- Then the resistor's current looks like this:



Current-Voltage Relationship for a Capacitor

- Using the formula for the charge stored in a capacitor ($q = Cv$), we can find the current-voltage relationship.
- Taking the derivative with respect to time gives:

$$i = C\frac{dv}{dt}$$



- This equation assumes the passive sign convention (current flows into the positive end). Otherwise you must insert a − sign.

Math Review: Some Derivative Rules

- $\frac{d}{dt}(c) = 0$
- $\frac{d}{dt}(t^n) = nt^{n-1}$
- $\frac{d}{dt}(\sin(\omega t)) = \omega\cos(\omega t)$
- $\frac{d}{dt}(\cos(\omega t)) = -\omega\sin(\omega t)$
- $\frac{d}{dt}(e^{at}) = ae^{at}$

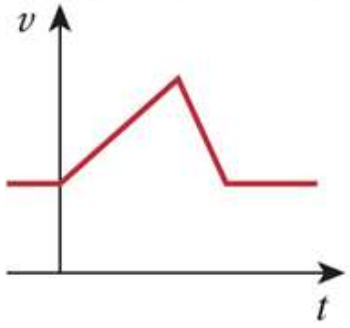
where a, c, n, and ω are constants.

- See pages A-17 to A-19 in textbook for more derivative rules.

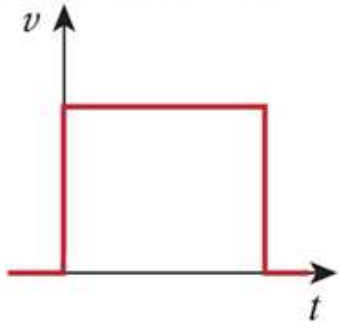
No Abrupt Voltage Changes for Capacitors

- A capacitor's voltage cannot change "abruptly" or "instantaneously."
- By this we mean that the graph of a capacitor's voltage cannot be vertical, as in the right-hand graph.

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Allowed



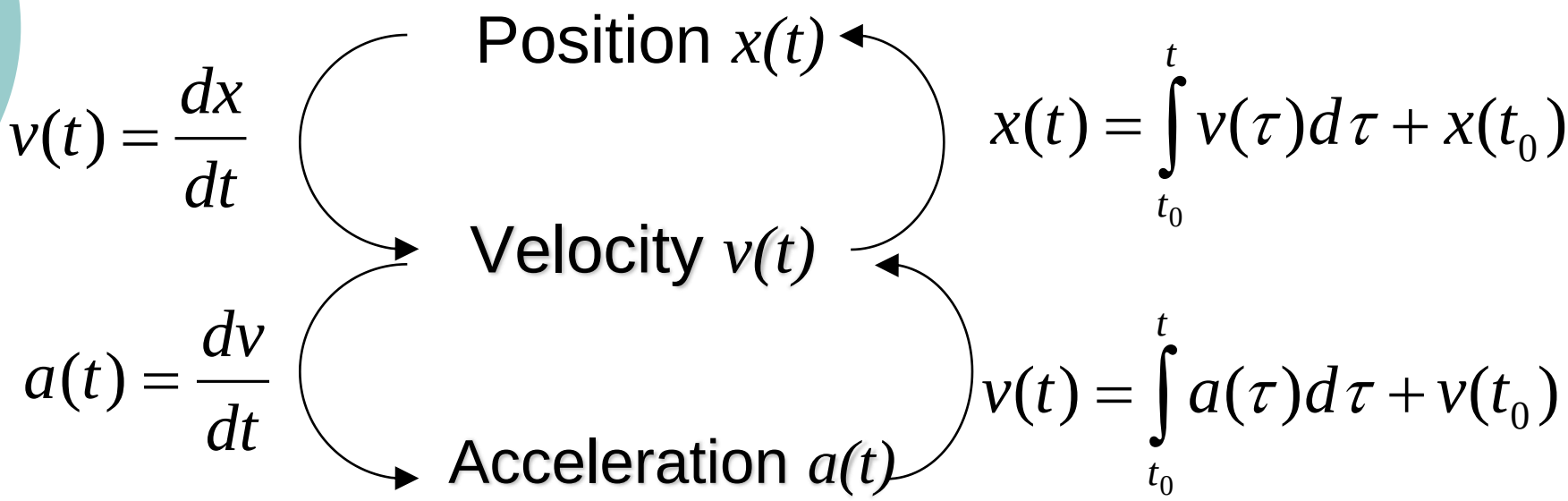
Not Allowed!

- Why not? Because $\frac{dv}{dt} = \infty$ for a vertical line, so $i = C\frac{dv}{dt}$ means we would need an infinite current, which is impossible.

Math Review: Differentiation and Integration

- Recall that differentiation and integration are inverse operations.
- Therefore, any relationship between two quantities that can be expressed in terms of derivatives can also be expressed in terms of integrals.

Example: Position, Velocity, & Acceleration



Voltage-Current Relationship for a Capacitor

- By integrating the current-voltage equation, $i(t) = C\frac{dv}{dt}$, we can find the voltage-current equation for a capacitor:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

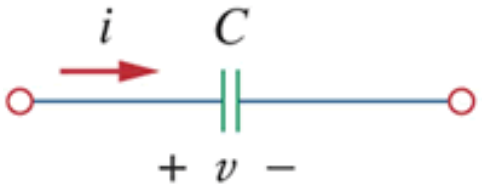


Table 6.1 (on page 232)

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TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

†*Passive sign convention is assumed.*

Inductors



- An **inductor** is a passive device designed to store energy in its magnetic field.

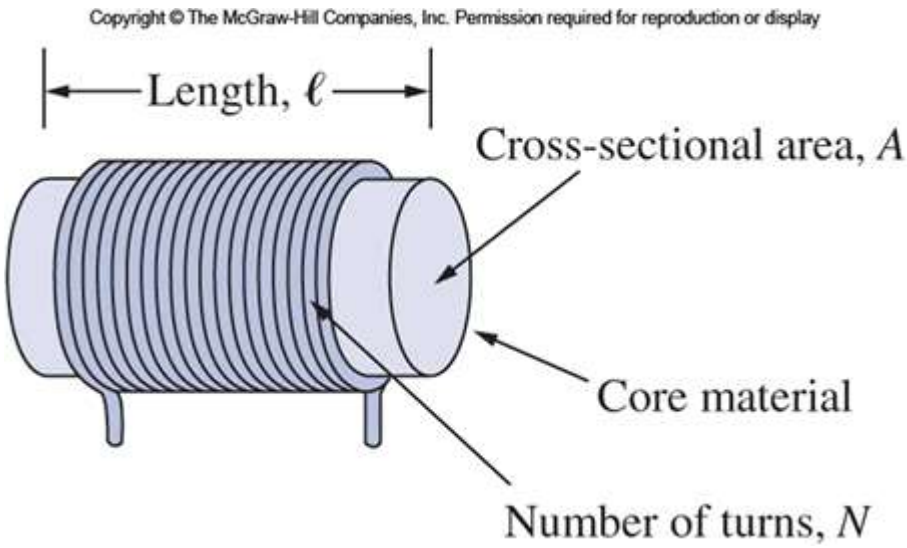


Image from Wikipedia.

Building an Inductor



- An inductor typically consists of a cylindrical coil of wire wound around a core, which is a rod usually made of an iron alloy.



Inductance

- When the current in a coil increases or decreases, a voltage is induced across the coil that depends on the rate at which the current is changing.
- The polarity of the voltage is such as to oppose the change in current.
- This property is called **self-inductance**, or simply **inductance**.

Units of Inductance

- Inductance is abbreviated *L*.
- The unit of inductance is the **henry** (H).
- Typical inductors found in electronic equipment are in the microhenry (μH) or millihenry (mH) range.

Update: Some Quantities and Their Units

Quantity	Symbol	SI Unit	Symbol for the Unit
Current	*I* or *i*	ampere	A
Voltage	*V* or *v*	volt	V
Resistance	*R*	ohm	Ω
Charge	*Q* or *q*	coulomb	C
Time	*t*	second	s
Energy	*W* or *w*	joule	J
Power	*P* or *p*	watt	W
Conductance	*G*	siemens	S
Capacitance	*C*	farad	F
Inductance	*L*	henry	H

Inductor Types

- Inductors are classified by the materials used for their cores.
- Common core materials are air, iron, and ferrites.
- Variable inductors are also available.

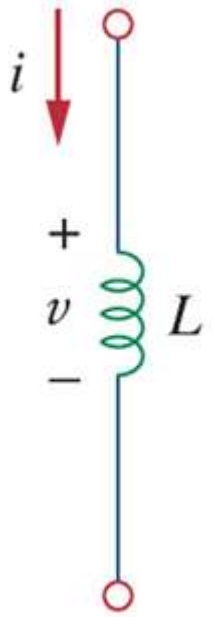
Chokes and Coils

- Inductors used in high-frequency (ac) circuits are often called **chokes**.
- Inductors are also sometimes simply called **coils**.

Voltage-Current Relationship for an Inductor

- The voltage across an inductor is proportional to the rate of change of the current through it:

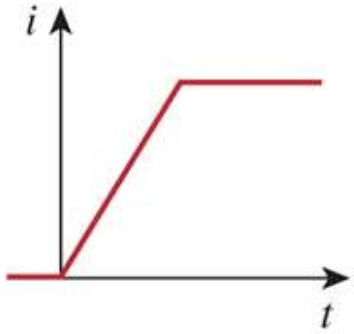
$$v = L\frac{di}{dt}$$



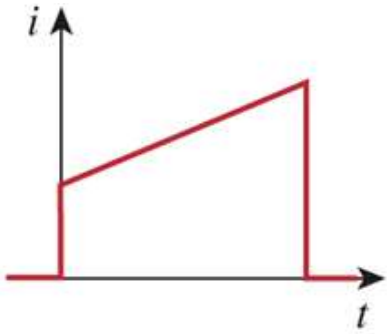
- This equation assumes the passive sign convention (current flows into the positive end).

No Abrupt Current Changes for Inductors

- An inductor's current cannot change "abruptly" or "instantaneously."
- By this we mean that the graph of an inductor's current cannot be vertical, as in the right-hand graph.



Allowed



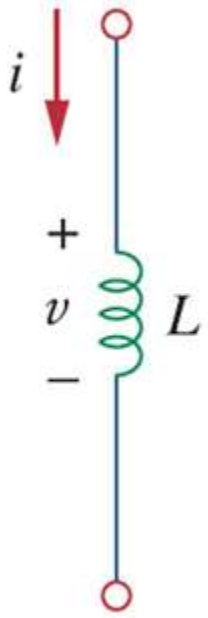
Not Allowed!

- Why not? Because $\frac{di}{dt} = \infty$ for a vertical line, so $v = L\frac{di}{dt}$ means we would need an infinite voltage, which is impossible.

Current-Voltage Relationship for an Inductor

- By integrating the voltage-current equation, $v(t) = L\frac{di}{dt}$, we can find the current-voltage equation for an inductor:

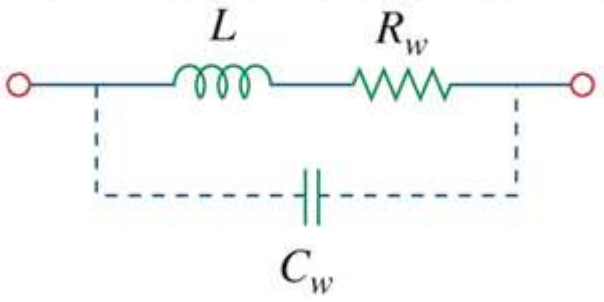
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$



Inductors Store Energy

- Recall that energy is dissipated as heat when current flows through a resistance.
- An ideal inductor does not dissipate energy. Rather it stores energy, which can later be returned to the circuit.
- We can model a real, non-ideal inductor by including a resistance in series with the inductance (and, for greater accuracy, a parallel capacitance).

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Inductor Energy Equation

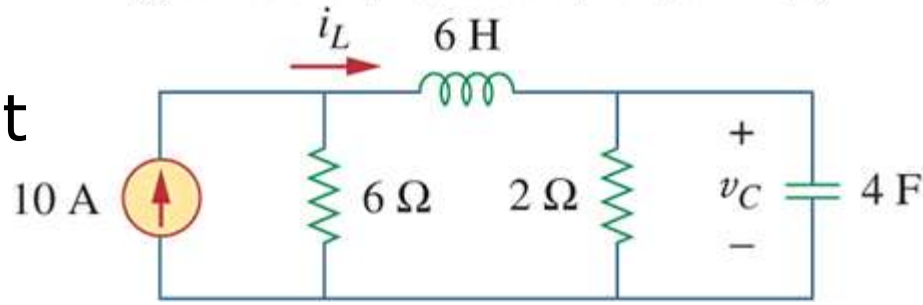
- The energy *w* stored in an inductor is given by

$$w = \frac{1}{2} L i^2$$

  where L is the inductor's inductance and i is the current through the inductor.
- Recall the units: w is in joules, L is in henries, and i is in amperes.

DC Conditions in a Circuit with Inductors or Capacitors

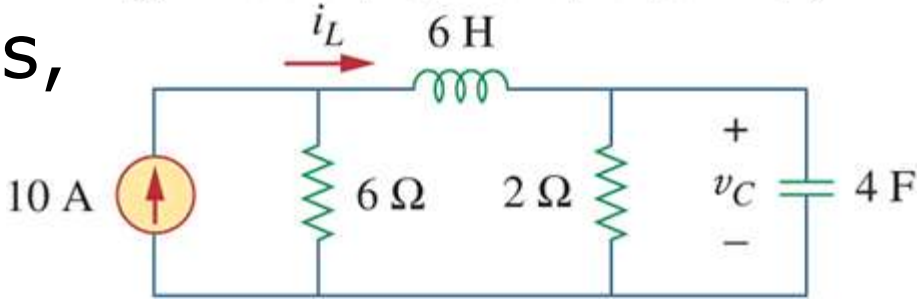
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- When power is first applied to a dc circuit with inductors or capacitors, voltages and currents change briefly as the inductors and capacitors become energized.
- But once they are fully energized, all voltages and currents in the circuit have constant values.
- Recall that we use the term "dc conditions" to refer to these final constant values.

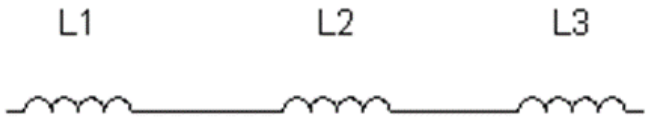
Under DC Conditions, Inductors Act Like Shorts

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- Under dc conditions, an inductor acts like a short circuit.
- So to analyze a circuit containing inductors *under dc conditions*, replace all inductors with short circuits.
- Later we'll look at how to analyze such circuits during the time while the inductors and capacitors are being energized. (It's trickier!)

Inductors in Series:
Equivalent Inductance

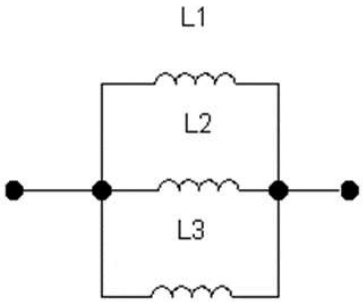


- The equivalent inductance of inductors in series is the sum of the individual inductances:

$$L_{\text{eq}} = L_1 + L_2 + L_3 + \ldots + L_N$$

- Similar to the formula for resistors in series.

Inductors in Parallel:
Equivalent Inductance



- The equivalent capacitance of inductors in parallel is given by the reciprocal formula:

$$L_{\text{eq}} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}}$$

- For **two** inductors in parallel, we can use the product-over-sum rule:

$$L_{\text{eq}} = \frac{L_1 L_2}{L_1 + L_2}$$

- Similar to the formulas for resistors in parallel.

Table 6.1 (on page 232)

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TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

†*Passive sign convention is assumed.*