Topics / EECE 167

- [x] Current divider law
- [x] Wye-Delta transformation
- [x] Nodal analysis
- [x] Mesh analysis
- [x] Super node
- [x] Super mesh
- [x] Superposition theorem
- [x] Thevenin's theorem
- [x] Norton's theorem
- [x] Maximum power transfer

DC

- [x] Phasor & impedence/admittance
- [x] Simple series & parallel circuits
- [x] Active power, reactive power, apparent power
- [x] Power factor & energy
- [ ] Complex power
- [ ] Phasor diagram

AC

- ☐ Impedence triangle, p
- ☑ power triangle
- ☐ Resonance in series & parallel circuits

Wye- Delta transformations

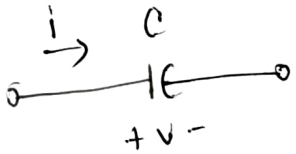
$$R_1 = \frac{R_aR_b + R_bR_c + R_cR_a}{R_b}$$

$$R_a = \frac{R_1R_2}{R_1+R_2+R_3}$$



Capacitors

$q = Cv$ → Capacitance

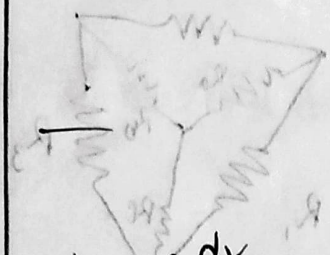


$$\Rightarrow \frac{dq}{dt} = i = C\frac{dv}{dt}; \quad v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$$

$v = L\frac{di}{dt} \Rightarrow di = \frac{1}{L} v\,dt$

$\Rightarrow i = \frac{1}{L}\int v\,dt + K_1$

inductor



$i = C\frac{dv}{dt} \Rightarrow \int dv = \frac{1}{C}\int i\,dt + K_2$

$\Rightarrow v = \frac{1}{C}\int i\,dt + K_2$

capacitor

$w = \frac{1}{2}Cv^2$

$w = \frac{1}{2}Li^2$

Phasor

Sinusoids

$$v(t) = V_m \sin(\omega t + \phi)$$

$5 \sin(4\pi t - 60°) \rightarrow$ amplitude: 5

phase: $-60°$

angular freq.: $\omega = 4\pi$ rads^{-1}

period: 0.5 s

freq.: 2 Hz

i_2 / $5 \sin(377t + 50°)$

i_1 / $5 \sin(377t + 205°)$

} i_1 leads i_2 by $155°$

Phasor : represents amplitude and phase in complex

$$z = x + jy = r(\cos\phi + j\sin\phi)$$

$$z = r\angle\phi$$

$$z = re^{j\phi}$$

$$r = \sqrt{x^2+y^2}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$i = 6\cos(50t - 40^\circ)\ A$$

$$\Rightarrow I = 6\angle -40^\circ\ A$$

$$v = -4\sin(30t + 50^\circ)\ V$$

$$= 4\cos(30t + 140^\circ)\ V$$

$$\Rightarrow V = 4\angle 140^\circ\ V$$

$$v(t) \Rightarrow V\angle\phi$$

$$\frac{dv}{dt} \Rightarrow j\omega V$$

$$\int v\,dt \rightarrow \frac{V}{j\omega}$$

Element	time domain	freq. domain
R	$v = iR$	$V = IR$
L	$v = L\frac{di}{dt}$	$V = \boxed{j\omega L}\, I$ ($\rightarrow X_L$)
C	$i = C\frac{dv}{dt}$	$V = \boxed{\frac{1}{C}\,\frac{1}{j\omega}}\, I$ ($\rightarrow X_C$)

short circuit → allows current to pass with low to no resistance.

Power: $\Sigma P = 0$

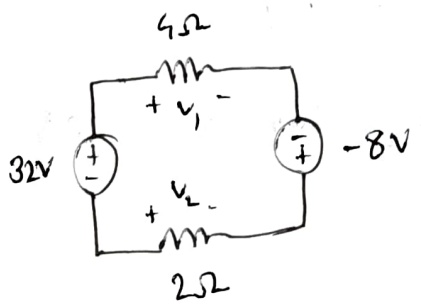
$2i - 3i = 20 \Rightarrow i = -20\,A$

$v_1 =$

$-32 + 4i - 8 - 2i = 0$

$\Rightarrow i = 20\,A$

$v_1 = 5,\ v_2 = -10$



$-32 + v_1 + 8 - v_2 = 0$

$\Rightarrow -32 + 4i + 8 + 2i = 0$

$\Rightarrow i = 4,\ v_1 = 16$

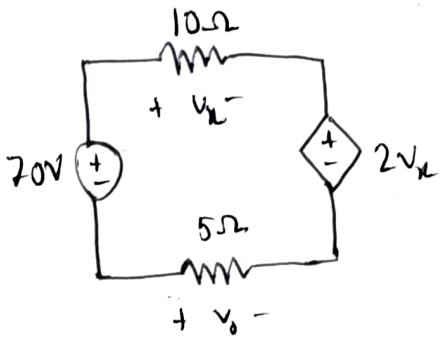
$-32 + 4i + 8 + 2i$

$-32 + v_1 - 8 - v_2 = 0$

$\Rightarrow -32 + 4i - 8 + 2i = 0$

$\Rightarrow i = 6.67\,A,\ v_1 =$

$-70+10i+2v_x+5i=0$ — (i)



$v_x = 10i$ — (ii)

$v_o = -5i$ — (iii)

(i) $\Rightarrow -70+10i+20i+25i=0$

$\therefore i = 2A$

$v_x = 20V$, $v_o = -10V$

$i_o = 0.5i_o + 3$

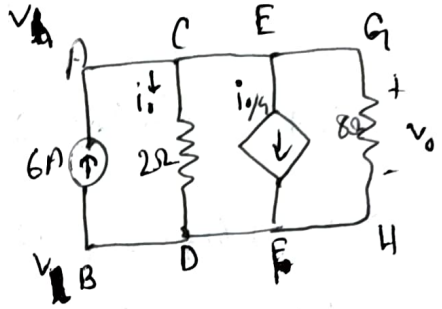
$\therefore i_o = 6A$, $v_o = 4 \times 6 = 24V$

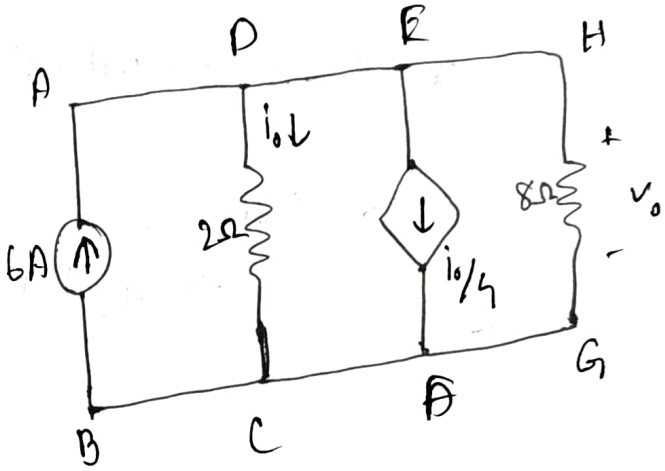
2.7/

~~$6 = i_o + \frac{i_o}{4} + \frac{v_o}{8}$~~

~~$\frac{i_o}{4} + i_o = 6 \Rightarrow i_o = 4.8\,A$~~

$6 = i_o + \frac{i_o}{4} + (6 - i_o - \frac{i_o}{4})$

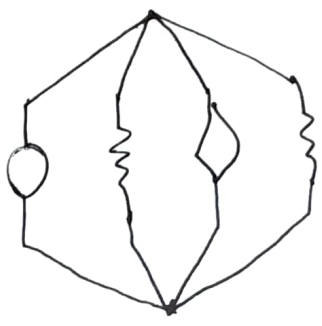




$6 = \frac{v_o}{2} + \frac{v_o}{8} + \frac{v_o}{8}$

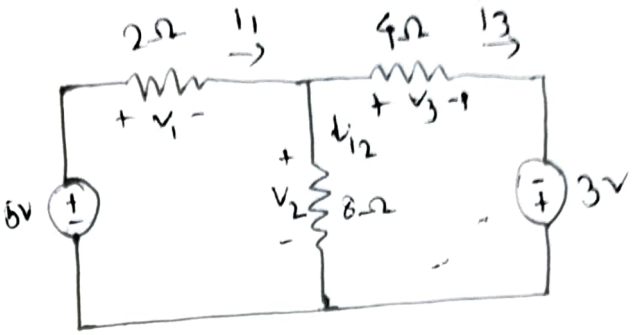
$\therefore v_o = 8\,V$

$i_o = 4\,V$



2.81

$i_1 - i_2 - i_3 = 0$



$-5 + 2i_1 + 8i_1 - 8i_3 = 0$

$\Rightarrow 10i_1 - 8i_3 = 0 + 5$

$8i_3 - 8i_2 + 4i_3 - 3 = 0$

$\Rightarrow 12i_3 - 8i_2 = 3$

$i_1 = 1.5A$

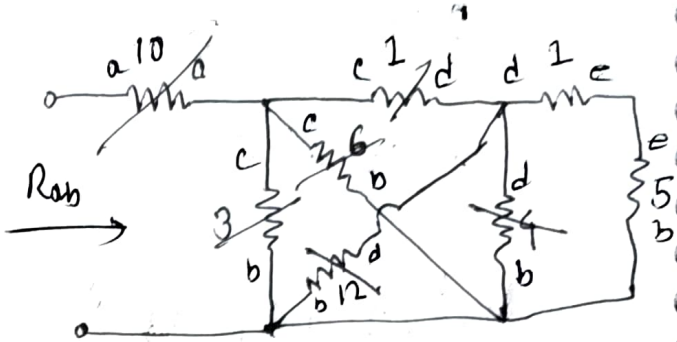
$i_2 = 0.25A$

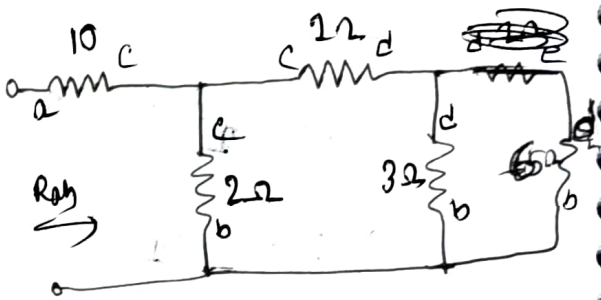
$I_3 = 1.25A$

$$v_n = \frac{R_n}{\Sigma R} V$$

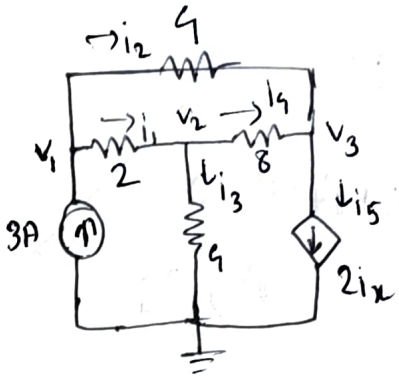
$$i_1 = \frac{R_2}{\Sigma R} i$$

$$R_{ab} = 10 + \frac{3\times6}{3+6} + 1 + \frac{12\times4}{12+4} + 1 + 5$$





KCL at node 1:



$; i_x = i_1$

$$3 = \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4}$$

$$\Rightarrow 12 = 2v_1 - 2v_2 + v_1 - v_3$$

$$\Rightarrow 12 = 3v_1 - 2v_2 - v_3$$

KCL at node 2:

$$\frac{v_1 - v_2}{2} = \frac{v_2}{4} + \frac{v_2 - v_3}{8}$$

$$\Rightarrow 4v_1 - 4v_2 - 2v_2 - v_2 - v_3 = 0$$

$$\Rightarrow 4v_1 - 7v_2 - v_3 = 0$$

KCL at node 3:

$$\frac{v_2 - v_3}{8} + \frac{v_1 - v_3}{4} = 2i_1 = 2\,\frac{v_1 - v_2}{2}$$

$$\Rightarrow v_2 - v_3 + 2v_1 - 2v_3 - v_1 - v_2 = 0$$

$$\Rightarrow v_1 - 3v_3 = 0$$

$v_1 = 7.41\,V$

$v_2 = 3.88\,V$

$v_3 = 2.47\,V$

Voltage divider rule:

$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_1 = \frac{L_1}{L_1 + L_2} V$$

$$V_1 = \frac{C_2}{C_1 + C_2} V$$

Current divider rule:

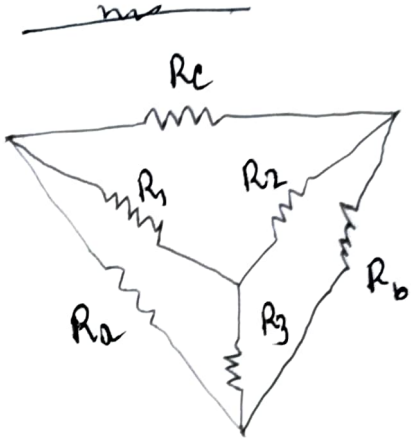
$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_1 = \frac{L_2}{L_1 + L_2} I$$

$$I_1 = \frac{C_1}{C_1 + C_2} I$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

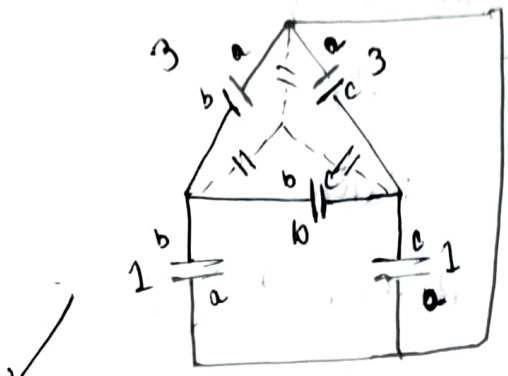
$$R_1 = \frac{R_aR_b}{R_a + R_b + R_c}$$

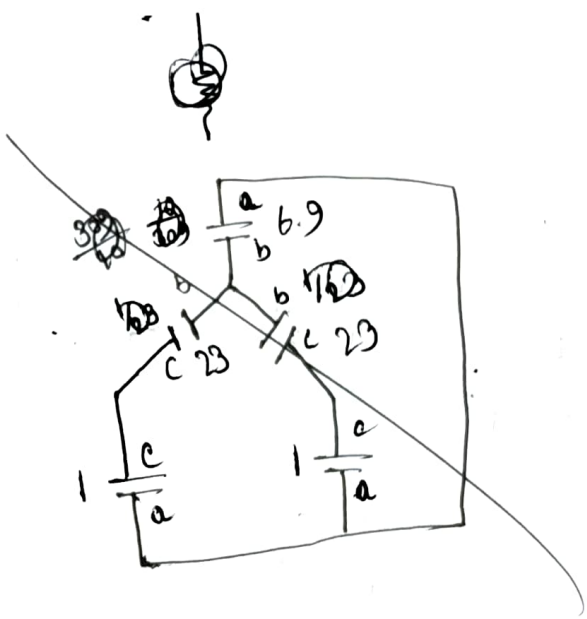


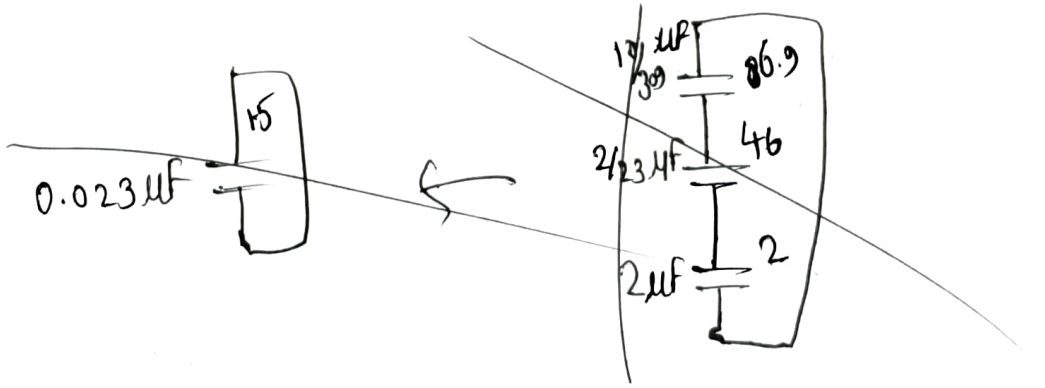
also applicable for inductors

$$\frac{1}{C_a} = \frac{\frac{1}{C_1}\frac{1}{C_2} + \frac{1}{C_2}\frac{1}{C_3} + \frac{1}{C_3}\frac{1}{C_1}}{\frac{1}{C_2}}$$

$$\frac{1}{C_1} = \frac{\frac{1}{C_a}\frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$


3
a
b
a
c
3
11
b
c
b
b
c
1
a
1
a


a
b 6.9
b
c 23
c 23
c
a
1
a


15
0.023 μF
μF
86.9
46
2 μF
2

$\sin(\omega t - 10^\circ) = \cos(-\omega t + 100)$

$\cos(90 - \omega t + 10)$ $\sin(\theta) = \cos(90 + -\theta)$

$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 230^\circ)$

$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t + 100^\circ)$

$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 130^\circ) = 10\cos(\omega t + 230^\circ)$

$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(-\omega t + 100^\circ)$

$= 12\cos(\omega t - 100^\circ)$

$v = 6\sin(\omega t + 10^\circ)\ V$

$= 6\cos(\omega t - 80^\circ)\ V$

$\bar{V} = 6\angle -80^\circ\ V$

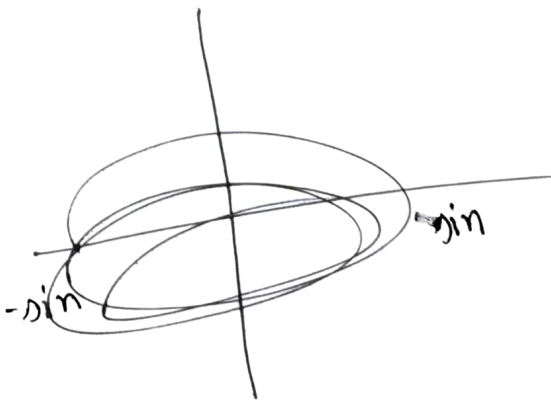
$\bar{I} = 3\angle -80^\circ\ A$

$i = 3\cos(\omega t - 80^\circ)\ A$

$i = -4\sin(10t + 10^\circ)$

$= 4\cos(10t + 100^\circ)$

$\bar{I} = 4\angle 100^\circ\ A$



- sin
cos
- cos
sin

$$\bar{V} = j8e^{-j20^\circ}$$

$$= j8 \angle -20^\circ$$

$$= (1\angle 90^\circ)(8\angle -20^\circ)$$

$$= 8\angle 70^\circ$$

Instantaneous power

$$p(t) = v(t) \times i(t)$$

$$= \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i)$$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$I_{rms} = \frac{9}{\sqrt{2}}$ A

$V_{rms} = \frac{9 \times 20}{\sqrt{2}}$ V

$P =$

$I^2R = I_{rms}^2 R$

$= \frac{R}{T}\int_0^T i^2 dt$

$\Rightarrow I_{rms}^2 = \frac{1}{T}\int_0^T i^2 dt$

P → avg. power

$P = I_{rms} V_{rms} \cos(\theta_v - \theta_i)$

$= S \cos(\theta_v - \theta_i)$

S → apparent power

$\cos(\theta_v - \theta_i)$ → power factor

$\Rightarrow \frac{P}{S}$ = power factor

$P = Vi$

$\therefore i = 0.2\,A$

$P_5 = 10$

$P_{10} =$

$P = i^2R$

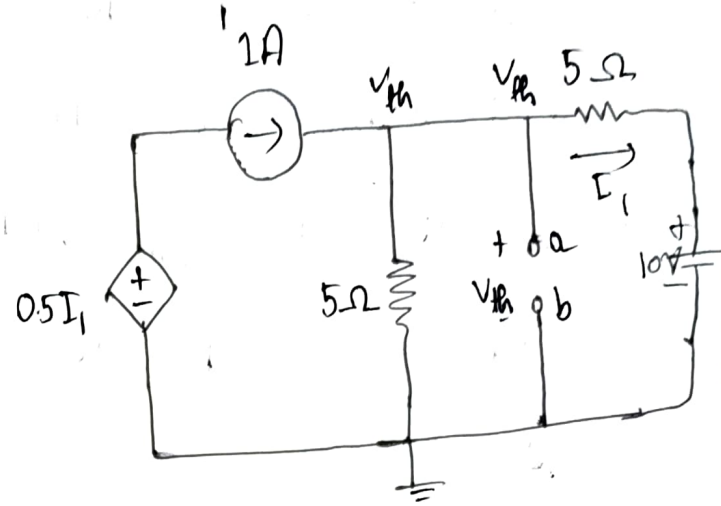
$\therefore i = \sqrt{2}\,A$

$\therefore I_{rms} = \sqrt{2}\,A$

$P_5 = 10, \quad P_{10} = I_{rms}^2 \times 10$

$= 20\,W$

$$I_1 = \frac{V_{th} - 10}{5}$$



$$-2 + I_5 + I_1 = 0$$

$$\Rightarrow -1 + \frac{V_{th}}{5} + \frac{V_{th} - 10}{5} = 0$$

$$\Rightarrow V_{th} = 7.5\,V$$

$$R_{th} = 5 \parallel 5 = 2.5\,\Omega$$

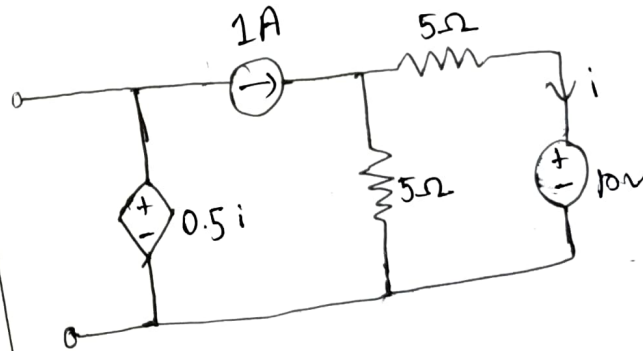
$i_1 = -2A$;

~~$V_{th_1} =$~~

~~$5 + 5 + 10$~~

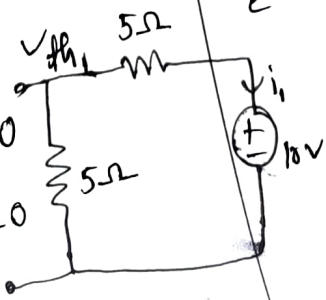
$$\frac{V_{th_1} - 10}{5} = -1$$

$$\therefore V_{th_1} = 5\,V$$



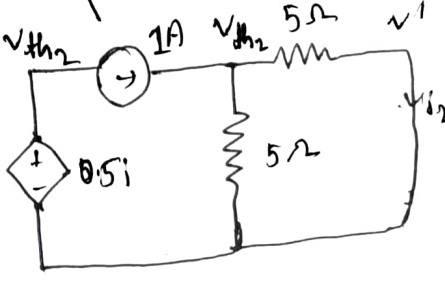
$$-1 + \frac{V_{th_2}}{5} + \frac{V_{th_2} - V_1'}{5} = 0 \Rightarrow 2V_{th_2} - V_1' = 5$$

~~$V_{th_2} = 2.5\,V$~~



$$5V_{th_2} + 5(V_{th_2} - V_1') = 0$$

$$\Rightarrow 10V_{th_2} - 5V_1' = 0$$



$$\frac{-1 + \frac{V_{th_2} - V_1'}{10}}{+}$$