EWCE 101

ANALYTICAL MECHANICS

3 Credit Hour

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Analytic Mechanics

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Syllabus of Part B

1. Centroids
2. Moment of Inertia of Areas
3. Moments of Inertia of Masses
4. Impulse and Momentum

Centre of mass : It is the point at which the distribution of mass is equal in all directions and does not depend on gravitational field.

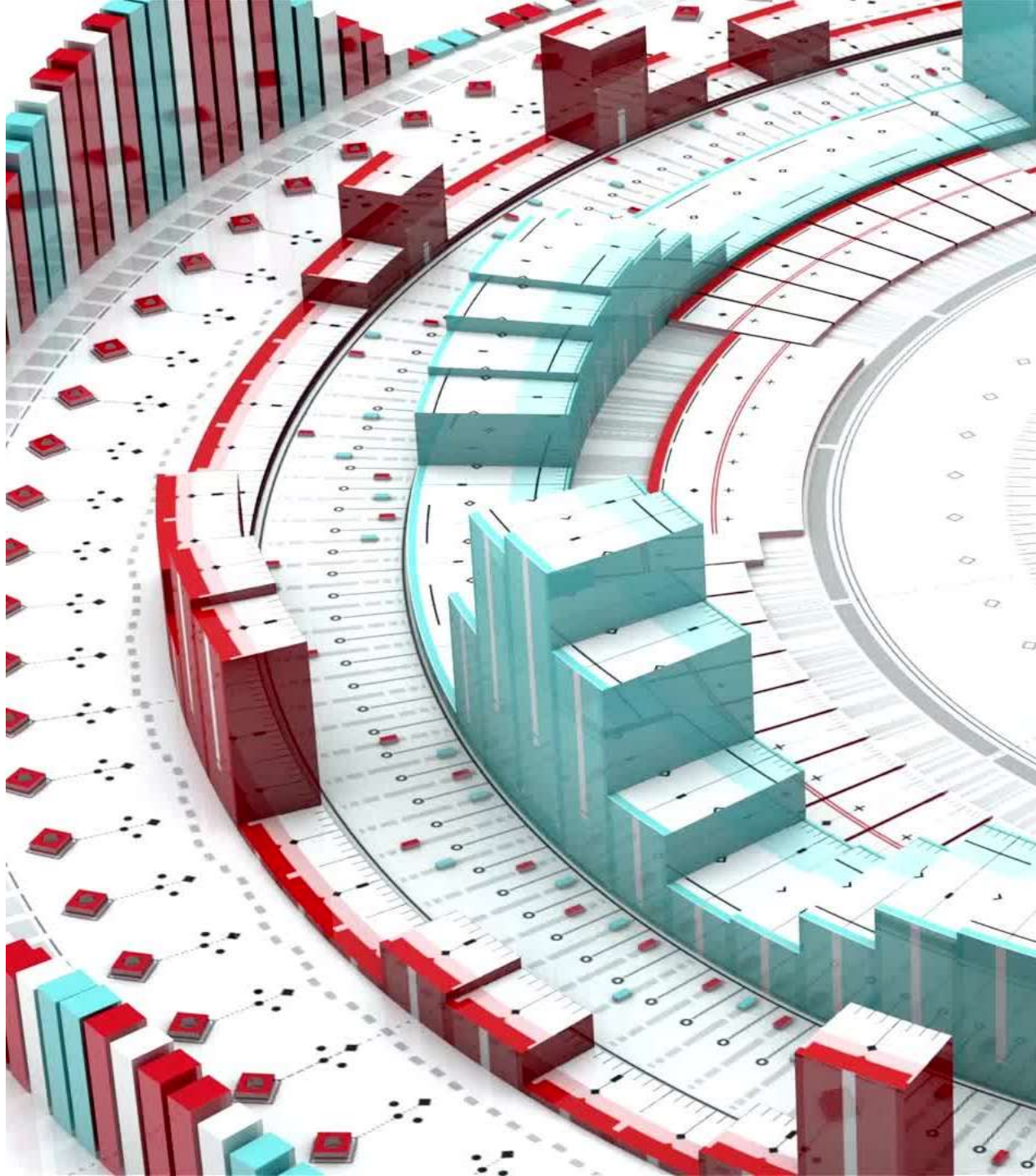
Centre of gravity: It is the point at which the distribution of weight is equal in all directions and it does depend on gravitational field. The centre of mass and centre of gravity are in same position if the gravitational field in which the object exists stays uniform.

Centroid: The centroid is the geometric centre point of the object.

Mass is the amount of "matter" in an object (though "matter" may be difficult to define), whereas weight is the force exerted on an object by gravity.

Centre of Gravity

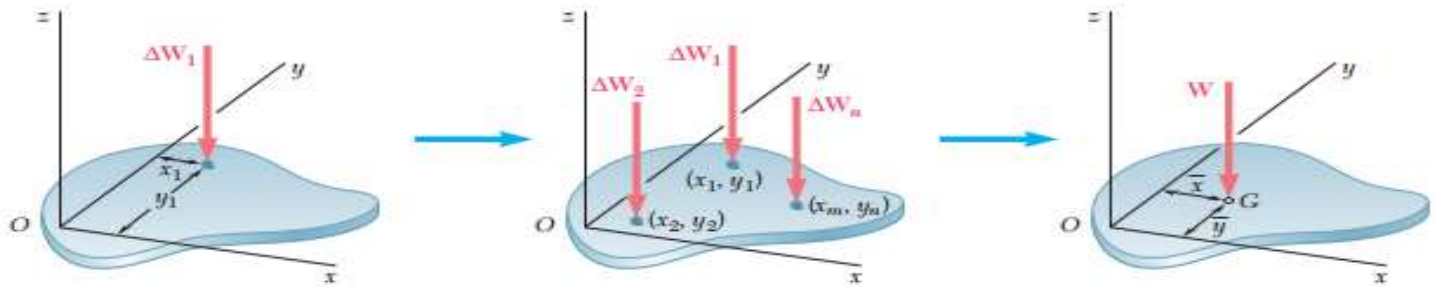
- It is a theoretical point in the body where the body's total weight is thought to be concentrated. It is necessary to know the centre of gravity because it predicts the behaviour of a moving body when acted on by gravity. It is also useful in designing static structures such as buildings and bridges.



5.1A Center of Gravity of a Two-Dimensional Body

Let us first consider a flat horizontal plate (Fig. 5.1). We can divide the plate into n small elements. We denote the coordinates of the first element by x_1 and y_1, those of the second element by x_2 and y_2, etc. The forces exerted by the earth on the elements of the plate are denoted, respectively, by $\Delta\mathbf{W}_1, \Delta\mathbf{W}_2, \ldots, \Delta\mathbf{W}_n$. These forces or weights are directed toward the center of the earth; however, for all practical purposes, we can assume them to be parallel. Their resultant is therefore a single force in the same direction. The magnitude W of this force is obtained by adding the magnitudes of the elemental weights.

$$\Sigma F_z: \qquad W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n$$



(a) Single element of the plate (b) Multiple elements of the plate (c) Center of gravity

$$\bar{x} = \frac{\int x\,dW}{W} \qquad \bar{y} = \frac{\int y\,dW}{W}$$

Fig. 5.1 The center of gravity of a plate is the point where the resultant weight of the plate acts. It is the weighted average of all the elements of weight that make up the plate.

To obtain the coordinates $\bar{x}$ and $\bar{y}$ of point G where the resultant $\mathbf{W}$ should be applied, we note that the moments of $\mathbf{W}$ about the y and x axes are equal to the sum of the corresponding moments of the elemental weights:

$$\begin{aligned} \Sigma M_y: &\quad \bar{x}W = x_1\Delta W_1 + x_2\Delta W_2 + \cdots + x_n\Delta W_n \\ \Sigma M_x: &\quad \bar{y}W = y_1\Delta W_1 + y_2\Delta W_2 + \cdots + y_n\Delta W_n \end{aligned} \tag{5.1}$$

Solving these equations for $\bar{x}$ and $\bar{y}$ gives us

$$\bar{x} = \frac{x_1\Delta W_1 + x_2\Delta W_2 + \cdots + x_n\Delta W_n}{W}$$

$$\bar{y} = \frac{y_1\Delta W_1 + y_2\Delta W_2 + \cdots + y_n\Delta W_n}{W}$$

5.1B Centroids of Areas and Lines

In the case of a flat homogeneous plate of uniform thickness, we can express the magnitude ΔW of the weight of an element of the plate as

$$\Delta W = \gamma t\, \Delta A$$

where γ = specific weight (weight per unit volume) of the material
t = thickness of the plate
ΔA = area of the element

Similarly, we can express the magnitude W of the weight of the entire plate as

$$W = \gamma t A$$

where A is the total area of the plate.

If U.S. customary units are used, the specific weight γ should be expressed in lb/ft^3, the thickness t in feet, and the areas ΔA and A in square feet. Then ΔW and W are expressed in pounds. If SI units are used, γ should be expressed in N/m^3, t in meters, and the areas ΔA and A in square meters; the weights ΔW and W are then expressed in newtons.[†]

Substituting for ΔW and W in the moment equations (5.1) and dividing throughout by γt, we obtain

$$\Sigma M_y: \quad \bar{x}A = x_1\,\Delta A_1 + x_2\,\Delta A_2 + \cdots + x_n\,\Delta A_n$$
$$\Sigma M_x: \quad \bar{y}A = y_1\,\Delta A_1 + y_2\,\Delta A_2 + \cdots + y_n\,\Delta A_n$$

If we increase the number of elements into which the area A is divided and simultaneously decrease the size of each element, in the limit we obtain

Centroid of an area *A*

$$\bar{x}A = \int x\, dA \qquad \bar{y}A = \int y\, dA \tag{5.3}$$

Or, solving for $\bar{x}$ and $\bar{y}$, we obtain

$$\bar{x} = \frac{\int x\, dA}{A} \qquad \bar{y} = \frac{\int y\, dA}{A} \tag{5.3′}$$

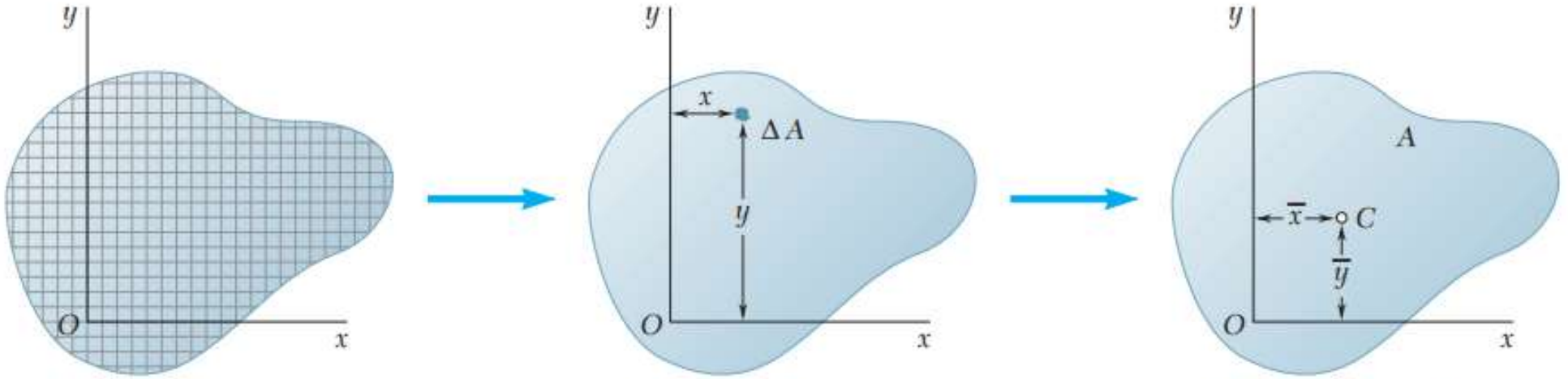
These equations define the coordinates $\bar{x}$ and $\bar{y}$ of the center of gravity of a homogeneous plate. The point whose coordinates are $\bar{x}$ and $\bar{y}$ is also known as the **centroid** C **of the area** A of the plate (Fig. 5.3). If the plate is not homogeneous, you cannot use these equations to determine the center of gravity of the plate; they still define, however, the centroid of the area.

Specific weight or weight density:

The specific weight, γ, of a fluid is defined as weight of fluid to it's volume.

$\gamma = W/V = mg/V = (m/V)g = \rho g$

where g is the gravitational acceleration. Basically, the specific weight represents weight per unit volume. Sometimes it is also referred to as the weight density. The SI unit for the specific weight is N/m^3 while the English unit is lb/ft^3.



(*a*) Divide area into elements (*b*) Element ΔA at point x, y (*c*) Centroid located at

$$\bar{x} = \frac{\int x\, dA}{A} \qquad \bar{y} = \frac{\int y\, dA}{A}$$

Fig. 5.3 The centroid of an area is the point where a homogeneous plate of uniform thickness would balance.

Centroid of a line L

$$\bar{x}L = \int x\,dL \qquad \bar{y}L = \int y\,dL$$

Solving for $\bar{x}$ and $\bar{y}$ gives us

$$\bar{x} = \frac{\int x\,dL}{L} \qquad \bar{y} = \frac{\int y\,dL}{L}$$

Mass Center and Centroid

$$\bar{x} = \frac{\int x\,dW}{W} = \frac{\int x\,dW/g}{W/g} = \frac{\int x dm}{m} \qquad \bar{y} = \frac{\int y dm}{m} \qquad \bar{z} = \frac{\int z dm}{m}$$

Center of Mass

$$\bar{x} = \frac{\int x\,dm}{m} = \frac{\int x\rho\,dV}{\rho V} = \frac{\int x dV}{V} \qquad \bar{y} = \frac{\int y dV}{V} \qquad \bar{z} = \frac{\int z dV}{V}$$

Centroid of Volume

$$\bar{x} = \frac{\int x dA}{A} \qquad \bar{y} = \frac{\int y dA}{A} \qquad \bar{z} = \frac{\int z dA}{A}$$

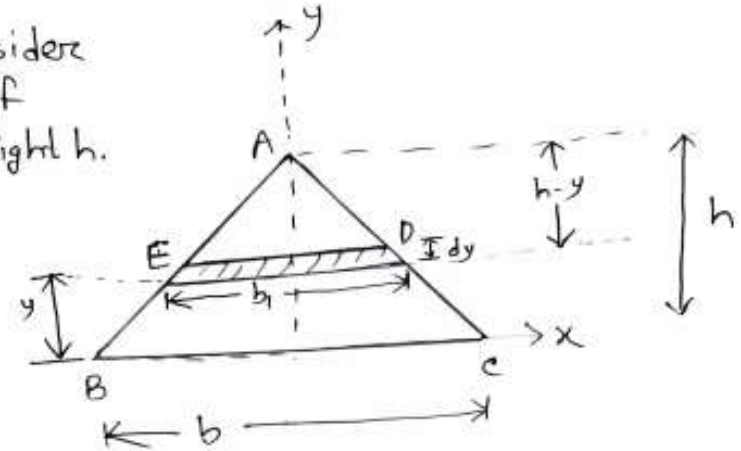
Centroid of Area

$$\bar{x} = \frac{\int x dL}{L} \qquad \bar{y} = \frac{\int y dL}{L} \qquad \bar{z} = \frac{\int z dL}{L}$$

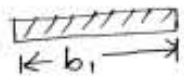
Centroid of Line

Centroid of a Triangle:

Let us consider a triangle of base b & height h.



Area of element, $dA = b_1 dy$



from similare triangle: (ΔAED & ΔABC)

$$\frac{b_1}{h-y} = \frac{b}{h}$$

$$\Rightarrow b_1 = b\left(1-\frac{y}{h}\right)$$

[In this figure, y is considered as axis of symmetry]

$$\therefore \int dA = A = \int_0^h b\left(1-\frac{y}{h}\right)dy$$

$$= b\left[y - \frac{y^2}{2h}\right]_0^h$$

$$= b\left[h - \frac{h^2}{2h}\right] = \frac{1}{2}bh$$

$$\int y\,dA = \int_0^h y\, b\left(1-\frac{y}{h}\right)dy$$

$$= b\int_0^h \left(y - \frac{y^2}{h}\right)dy$$

$$= b\left[\frac{y^2}{2} - \frac{y^3}{3h}\right]_0^h$$

$$= b\left[\frac{h^2}{2} - \frac{h^3}{3h}\right]$$

$$= b\left[\frac{3h^2 - 2h^2}{6}\right]$$

$$= \frac{bh^2}{6}$$

Now,

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$= \frac{\frac{bh^2}{6}}{\frac{1}{2}bh}$$

$$= \left(\frac{bh^2}{6} \times \frac{2}{bh}\right)$$

$$= \frac{h}{3}$$ (From the base)