EWCE 101

ANALYTICAL MECHANICS

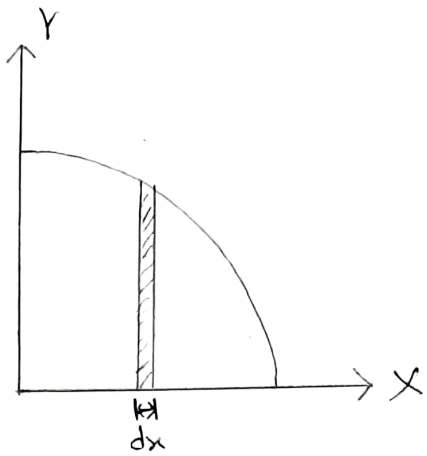
3 Credit Hour

Reference Book: Analytical Mechanics (Virgil Moring Faires)

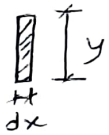
Department of Environmental, Water Resources, and Coastal Engineering (EWCE)

Military Institute of Science and Technology

Centroid of a Quarter Circle



Here, area of the small strip = $y \cdot dx$



we know, $\bar{y} = \frac{\int y\, dA}{A}$

area of quarter circle $= \frac{\pi R^2}{4}$ [Let, radius of the circle = R]

$$\int y\, dA = \int_0^R \left(\frac{y}{2}\right)(y\, dx)$$

$$= \frac{1}{2}\int_0^R y^2\, dx$$

[From eqn of circle, $x^2 + y^2 = R^2$ $\therefore y^2 = R^2 - x^2$]

$$= \frac{1}{2}\int_0^R (R^2 - x^2)\, dx$$

$$= \frac{1}{2}\left[R^2 x - \frac{x^3}{3}\right]_0^R$$

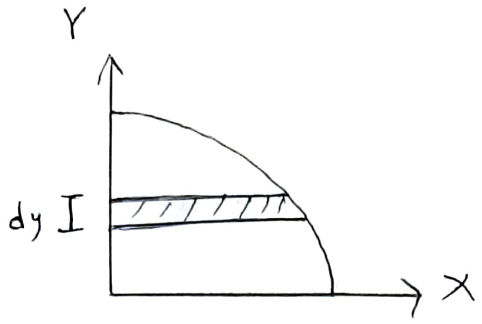
$$= \frac{1}{2}\left[\left(R^2 \cdot R - \frac{R^3}{3}\right) - 0\right]$$

$$= \frac{1}{2}\left[R^3 - \frac{R^3}{3}\right]$$

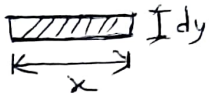
$$= \frac{1}{2} \times \frac{2R^3}{3} = \frac{R^3}{3}$$

$$\therefore \bar{y} = \frac{\int y\, dA}{A} = \frac{\frac{R^3}{3}}{\frac{\pi R^2}{4}} = \left(\frac{R^3}{3} \times \frac{4}{\pi R^2}\right) = \frac{4R}{3\pi}$$

$$= 0.424R$$



Here, area of small strip = $x \cdot dy$



we know, $\bar{x} = \frac{\int x\,dA}{A}$

area of quarter circle $= \frac{\pi R^2}{4}$

$$\int x\,dA = \int_0^R \left(\frac{x}{2}\right)(x\,dy) = \frac{1}{2}\int_0^R x^2\,dy$$

$$= \frac{1}{2}\int_0^R (R^2 - y^2)\,dy \quad \left[\because x^2 + y^2 = R^2,\ \therefore x^2 = R^2 - y^2\right]$$

$$= \frac{1}{2}\left[R^2 y - \frac{y^3}{3}\right]_0^R$$

$$= \frac{1}{2}\left[\left(R^2 \cdot R - \frac{R^3}{3}\right) - 0\right]$$

$$= \frac{1}{2}\left[R^3 - \frac{R^3}{3}\right]$$

$$= \frac{1}{2} \times \frac{2R^3}{3} = \frac{R^3}{3}$$

$$\therefore \bar{x} = \frac{\int x\,dA}{A}$$

$$= \frac{R^3/3}{\pi R^2/4} = \left(\frac{R^3}{3} \times \frac{4}{\pi R^2}\right) = \frac{4R}{3\pi} = 0.424R$$

Composite Figure: A composite figure is one made up of two or more basic figures, such as a triangle and a rectangle, a circular arc and a straight line etc.

$$A\bar{x} = \sum A'x'$$

$$A\bar{y} = \sum A'y'$$

A= total net area for which the centroid is desired

A'=one component part of the total area whose centroid is known to be a distance x'and y' from the axis of references.

Centroid of a Composite Figure Math Problems

Example: Locate the centroid of the section of an angle iron(Fig 01) whose dimensions are as shown in figure.

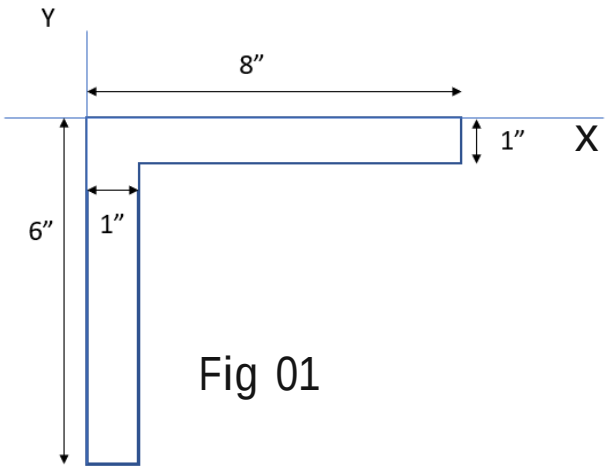
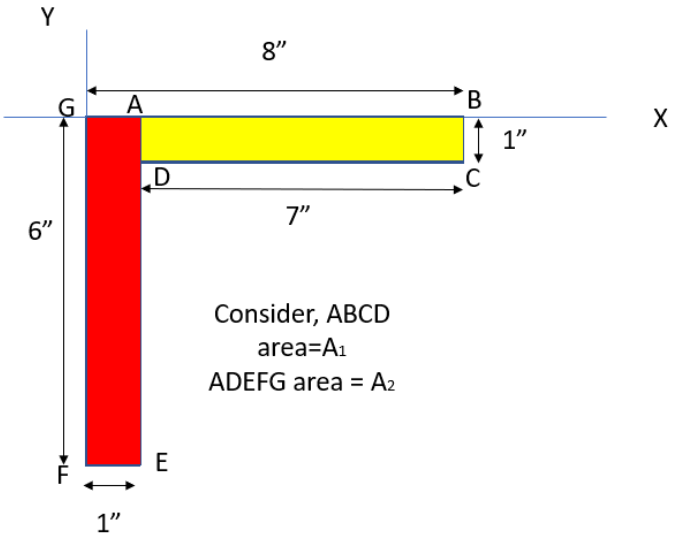


Fig 01



$$\Rightarrow x_1 = (8-7) + \frac{7}{2} = 1 + \frac{7}{2} = 4.5 \text{ in}$$

$$x_2 = \frac{(8-7)}{2} = \frac{1}{2} = 0.5 \text{ in}$$

$$y_1 = \frac{1}{2} = 0.5 \text{ in} \; ; \; y_2 = \frac{6}{2} = 3 \text{ in}$$

$$A_1 = 1 \times 7 = 7 \text{ in}^2 \; ; \; A_2 = (6 \times 1) = 6 \text{ in}^2$$

$$\therefore \bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{(7 \times 4.5) + (6 \times 0.5)}{7+6}$$

$$= 2.65 \text{ in}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(7 \times 0.5) + (6 \times 3)}{7+6}$$

$$= 1.65 \text{ in}$$

In obtaining $\bar{y}$, downward direction is considered positive.

Example: Locate the centroid of the section(Fig:02) as shown below.

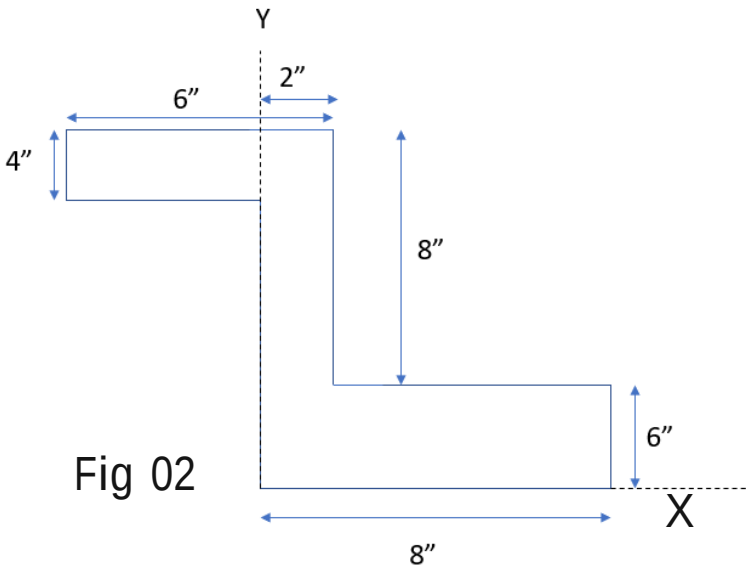
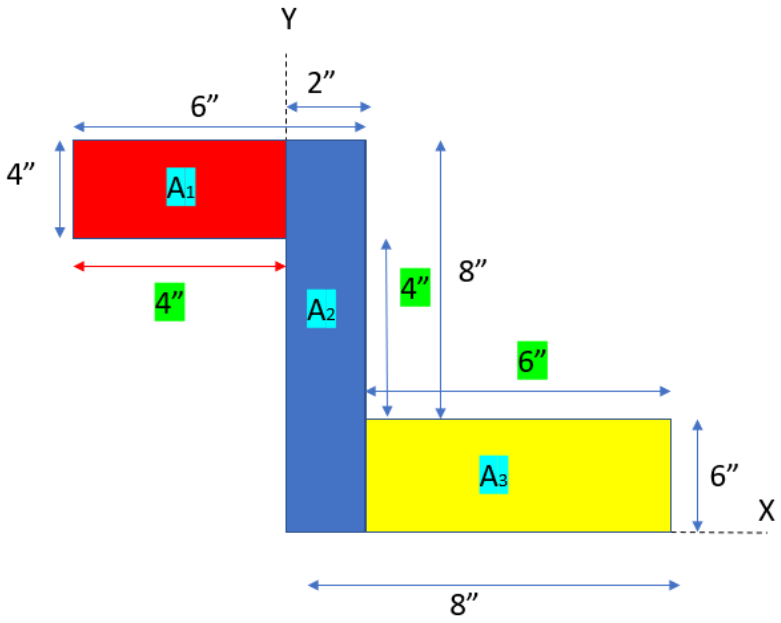


Fig 02



$A_1 = (4\times4) = 16\ in^2$; $A_2 = 2\times(6+8) = 28\ in^2$; $A_3 = 6\times6 = 36\ in^2$

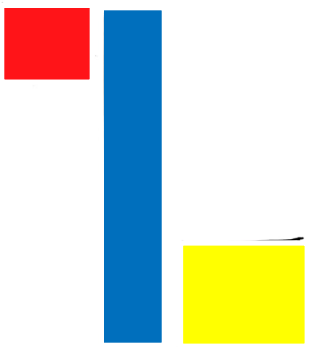
$x_1 = -\frac{4}{2} = -2\ in$; $x_2 = \frac{2}{2} = 1\ in$; $x_3 = 2+\frac{6}{2} = 5\ in$

$y_1 = 6+4+\frac{4}{2} = 12\ in$; $y_2 = \frac{6+8}{2} = 7\ in$; $y_3 = \frac{6}{2} = 3\ in$

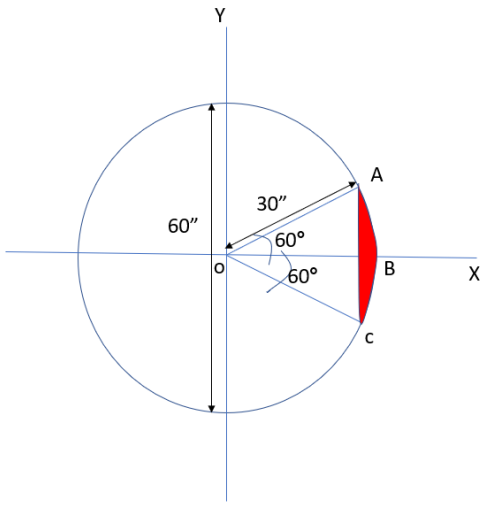
$$\bar{x} = \frac{\{16\times(-2)\}+\{28\times1\}+\{36\times5\}}{16+28+36} = 2.2\ in$$

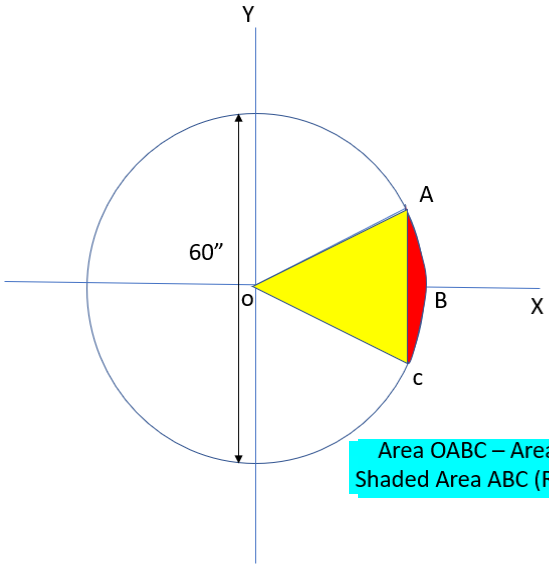
$$\bar{y} = \frac{\{16\times12\}+\{28\times7\}+\{36\times3\}}{16+28+36}$$

$$= 6.2\ in$$



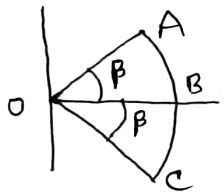
Example: Locate the centroid of the shaded area as shown in figure.





Area OABC – Area OAC = Shaded Area ABC (Red Color)

$\Rightarrow$





area of sector (OABC) $= \pi r^2 \times \frac{2\beta}{360}$ [formula]

$= 3.1416 \times (30)^2 \times \frac{2 \times 60}{360}$

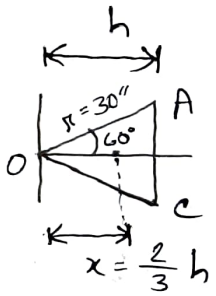
$= 942.48 \text{ in}^2$

[Formula]

$x_1 = \frac{2r \sin\beta}{3\beta} = \frac{2 \times 30 \times \sin 60^\circ}{3 \times 60 \times \frac{\pi}{180}} = 16.54 \text{ inch}$ [calc in degree mode]

$A_2 = \left(\frac{1}{2} \times 30 \cos 60 \times 30 \sin 60\right) \times 2$ [OAC]

$= 389.71 \text{ in}^2$



$x_2 = \frac{2}{3} h = \frac{2}{3} \times 30 \cos 60^\circ$

formula

$= 10 \text{ in}$

$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{(942.48 \times 16.54) - (389.71 \times 10)}{942.48 - 389.71}$

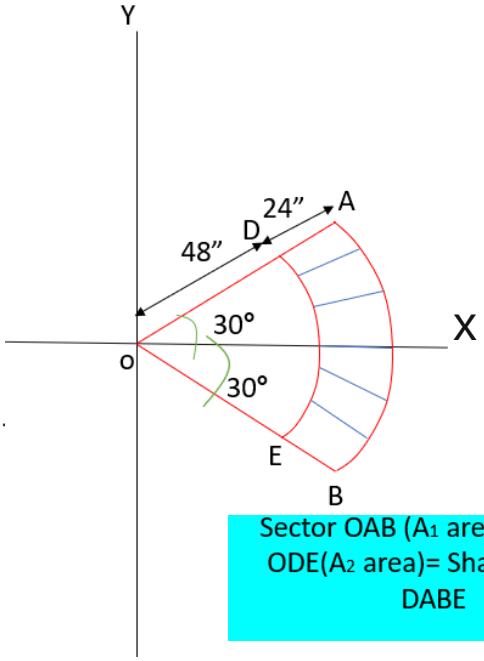
$= 21.15 \text{ inch}$

$\cos 60^\circ = \frac{h}{30}$

$\therefore h = 30 \cos 60^\circ$

Here, x is the axis of symmetry, so $\bar{y} = 0$

Example: Locate the centroid of the shaded area as shown in figure.



Sector OAB (A_1 area)– Sector ODE(A_2 area)= Shaded Area DABE

sector OAB

$$\Rightarrow A_1 = \pi r_1^2 \times \frac{60^\circ}{360^\circ} = 3.1416 \times (72)^2 \times \frac{60}{360} = 2714.342 \text{ in}^2$$

formula

$$\bar{x}_1 = \frac{2r_1 \sin\beta}{3\beta} = \frac{2 \times 72 \times \sin 30}{3 \times 30 \times \frac{\pi}{180}} = 45.83 \text{ inch}$$

(calculator in degree mode)

sector ODE

$$A_2 = \pi r_2^2 \times \frac{60}{360} = 3.1416 \times (48)^2 \times \frac{60}{360} = 1206.37 \text{ in}^2$$

$$\bar{x}_2 = \frac{2r_2 \sin\beta}{3\beta} = \frac{2 \times 48 \times \sin 30}{3 \times 30 \times \frac{\pi}{180}} = 30.55 \text{ inch}$$

formula

(calculator in degree mode)

$$\therefore \bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

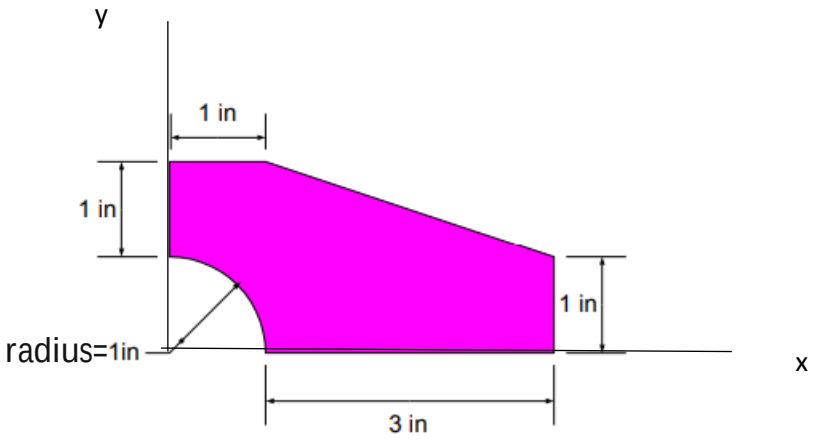
$$= \frac{(2714.3424 \times 45.83) - (1206.37 \times 30.6)}{2714.3424 - 1206.37}$$

$$= 58.5 \text{ inch}$$

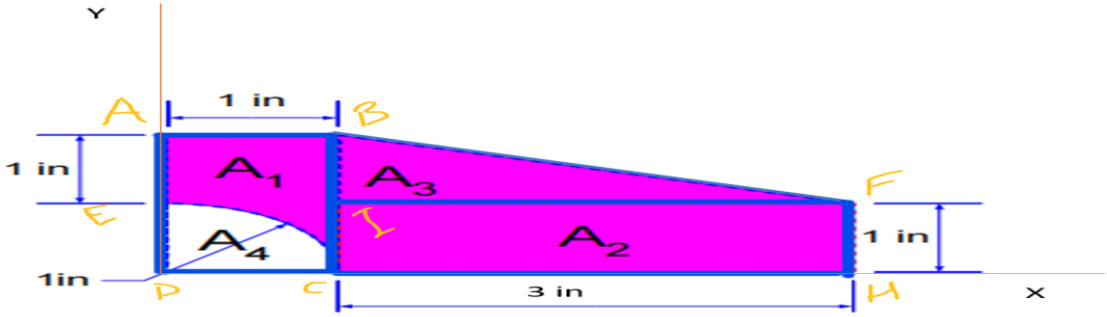
[In the fig, x is the axis of symmetry]

so, $\bar{y} = 0$

Ques: Determine the centroid of the following figure:



Solution:



Here ABCD = A_1 , FHCI=A_2, BIF=A_3 , ECD (circle) = A_4

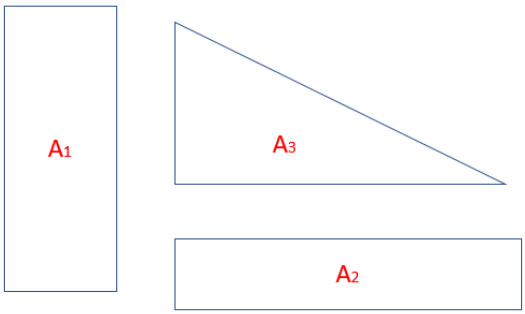
	Area (in^2)	x_i (in)	Area × x_i (in^3)	y_i (in)	Area × y_i (in^3)
A_1	(1×2) =2	$\frac{1}{2}$=0.5	(2×0.5) =1	$\frac{2}{2}$=1	(2×1) = 2
A_2	(3×1) =3	$1+\frac{3}{2}=2.5$	(3×2.5) =7.5	$\frac{1}{2}$ =0.5	(3×0.5) =1.5
A_3	(0.5×3×1) =1.5	$1+\frac{3}{3}=2$	(1.5×2) =3	$1+\frac{1}{3}=1.33$	(1.5×1.33) =2
A_4	$\frac{\pi r^2}{4}=\frac{\pi x(1)^2}{4}$=0.7854	0.424×1=0.424	(0.7854×0.424)=0.333	0.424×1=0.424	(0.7854×0.424)=0.333

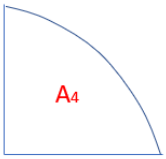
$$\bar{x}=\frac{1+7.5+3-0.333}{2+3+1.5-0.7854}$$

= 1.9541 in

$$\bar{y}=\frac{2+1.5+2-0.333}{2+3+1.5-0.7854}$$

=0.904 in





A_4 is a quarter circle, that's why it's area is divided by 4 in the table