EWCE 101

ANALYTICAL MECHANICS

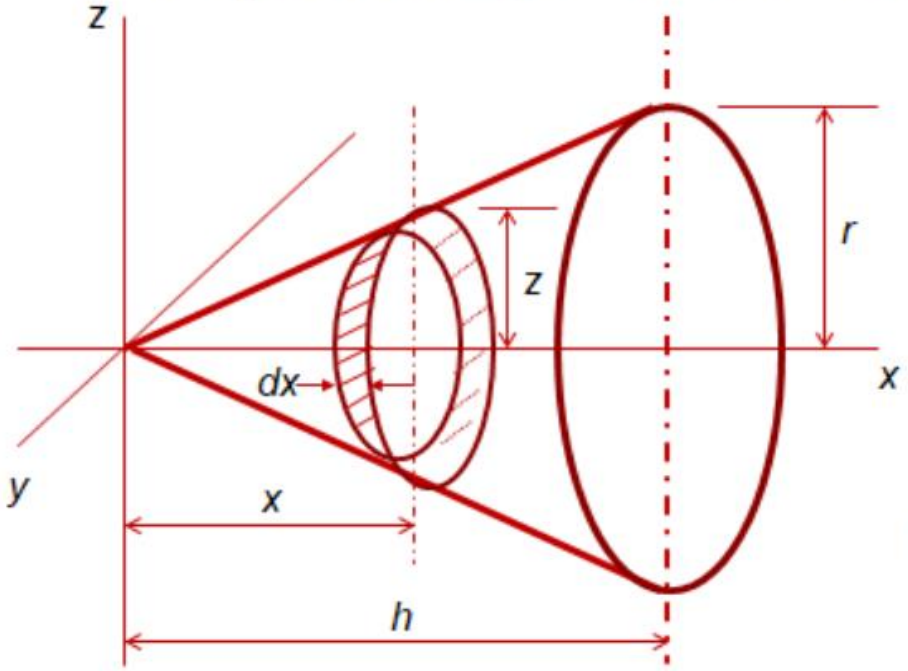
3 Credit Hour

Lecturer
Department of Environmental, Water Resources, and Coastal Engineering (EWCE)

Military Institute of Science and Technology

Q: Find the centroid of a right circular cone, whose altitude is h and whose base has a radius r.

Example 120: Right Circular Cone



$$dV = \pi z^2\,dx$$

$$\frac{z}{r} = \frac{x}{h} \qquad z = \frac{r}{h}x$$

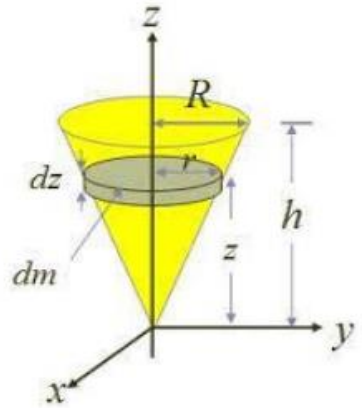
$$V = \int dV = \int_0^h \pi z^2\,dx$$

$$= \frac{\pi r^2}{h^2}\int_0^h x^2\,dx = \frac{\pi r^2 h}{3}$$

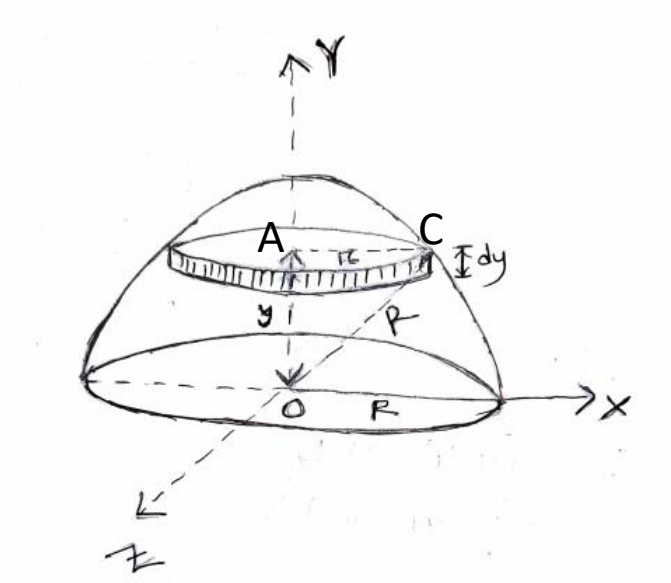
$$V\bar{x} = \int x\,dV = \int_0^h x\pi z^2\,dx$$

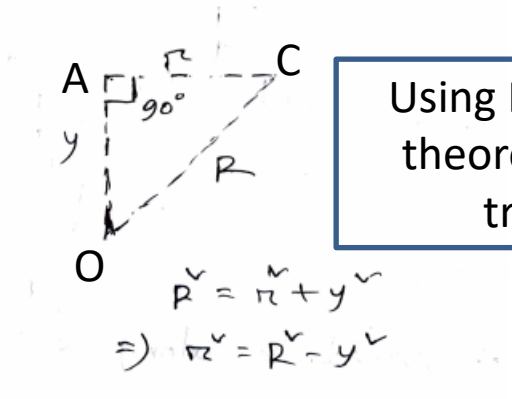
$$= \frac{\pi r^2}{h^2}\int_0^h x^3\,dx = \frac{\pi r^2 h^2}{4}$$

$$\bar{x} = \frac{\int x\,dV}{V} = \frac{\pi r^2 h^2}{4} \div \frac{\pi r^2 h}{3} = \frac{3}{4}h$$



Q: Determine the centroid of a hemispherical part of a body.





Using Pythagorus theorem in ACO triangle

Here, r is the radius of strip and R is the radius of hemisphere

consider an element disc of radius r and dy thickness at a distance y distance from the ox axis.

∴ volume of elemental disc

$$dv = \pi r^2 dy$$

$$\Rightarrow dv = \pi (R^2 - y^2) dy$$

$\therefore$ Total volume $= \int dv = V$

$$= \int_0^R \pi \left(R^2 - y^2\right) dy$$

$$= \pi \left[R^2 y - \frac{y^3}{3}\right]_0^R$$

$$= \pi \left[R^3 - \frac{R^3}{3}\right]$$

$$= \pi \left[\frac{2R^3}{3}\right]$$

$$= \frac{2\pi R^3}{3}$$

$\therefore \int dv = V = \frac{2}{3}\pi R^3$

$$\int y\,dv = \int_0^R y \cdot \pi\left(R^2 - y^2\right) dy$$

$$= \int_0^R \pi \left(yR^2 - y^3\right) dy$$

$$= \pi \left[R^2 \cdot \frac{y^2}{2} - \frac{y^4}{4}\right]_0^R$$

$$= \pi \left[R^2 \cdot \frac{R^2}{2} - \frac{R^4}{4}\right]$$

$$= \frac{\pi R^4}{4}$$

$$\therefore \bar{y} = \frac{\int y\,dv}{V} = \frac{\pi R^4}{4} \div \frac{2}{3}\pi R^3$$

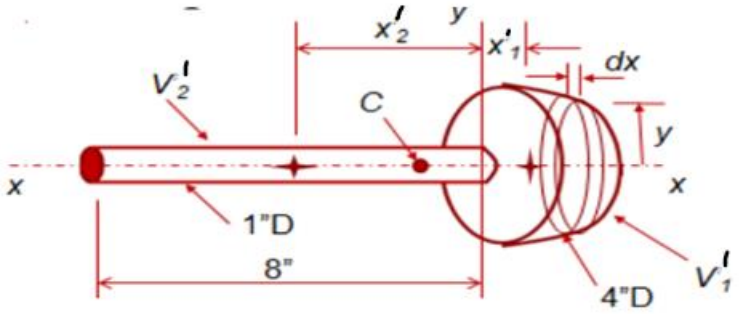
$$= \frac{\pi R^4}{4} \times \frac{3}{2\pi R^3}$$

$$= \frac{3}{8}R$$

$\therefore$ Symmetric along y axis so $\bar{x} = 0$.

Centroid of Composite Volume (Examples)

Q: A homogeneous rod and a hemispherical part of the same material are welded together, as shown in fig. Where is the center of gravity of this composite body?



Formula from previous slide

$$x_1' = \frac{3}{8}R = \frac{3}{8} \times 2'' = \frac{3}{4}''$$

$$x_2' = \frac{-8}{2} = -4''$$

$$V_1' = \frac{\left(\frac{4}{3}\right)\pi R^3}{2} = \frac{\frac{4}{3}\pi (2)^3}{2} = 16.76\ in^3$$

$$V_2' = \frac{\pi D^2}{4} \cdot L = \frac{\pi (1)^2}{4} \times (8) = 6.28\ in^3$$

$$\therefore V = V_1' + V_2' = (16.76 + 6.28) = 23.04\ in^3$$

$$\bar{x} = \frac{V_1' x_1' + V_2' x_2'}{V_1 + V_2} = \frac{(16.76 \times 0.75) + \{6.28 \times (-4)\}}{16.76 + 6.28}$$

$$= -0.595\ in$$

Here, radius of homogeneous rod = 0.5”
Radius of hemispherical part = 2”