EWCE 101

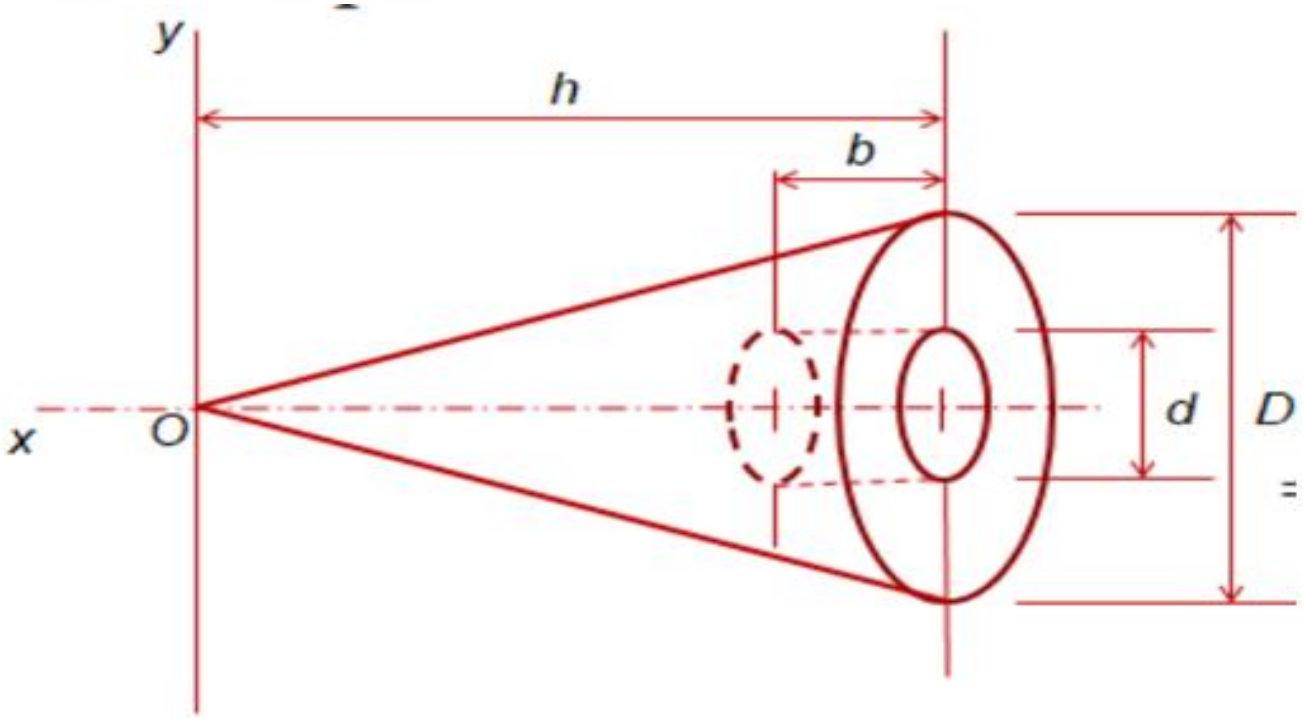
ANALYTICAL MECHANICS

3 Credit Hour

Department of Environmental, Water Resources, and Coastal Engineering (EWCE)

Military Institute of Science and Technology

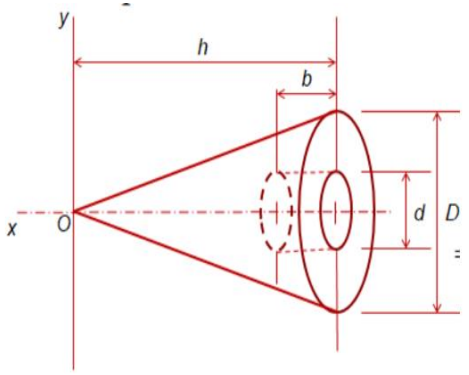
Q: A right circular cone of homogeneous material has a diameter D=20 in, an altitude h=30 in. A cylindrical hole, whose geometric axis is coincident with that of the cone, is bored in the base. This hole has a diameter d=8 in and a depth of b=6 in. Locate the center of gravity of the cone with the hole.



Given,

$D = 20\text{ in}, h = 30\text{ in}$

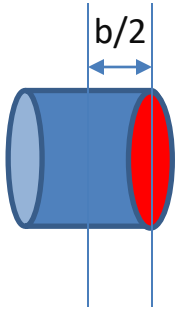
$d = 8\text{ in}, b = 6\text{ in}$



$$V_1' = V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{D}{2}\right)^2 \times h$$

$$= \frac{1}{3} \times 3.1416 \times \left(\frac{20}{2}\right)^2 \times 30$$

$$= 3141.6\text{ in}^3$$



$$V_2' = V_{hole} = \pi r^2 b$$

$$= \pi\left(\frac{d}{2}\right)^2 b = \frac{\pi \times (8)^2 \times 6}{4} = 301.6\text{ in}^3$$

$$x_1' = \frac{3}{4}h = \left(\frac{3}{4} \times 30\right) = 22.5\text{ in} \quad [\text{from the y axis}]$$

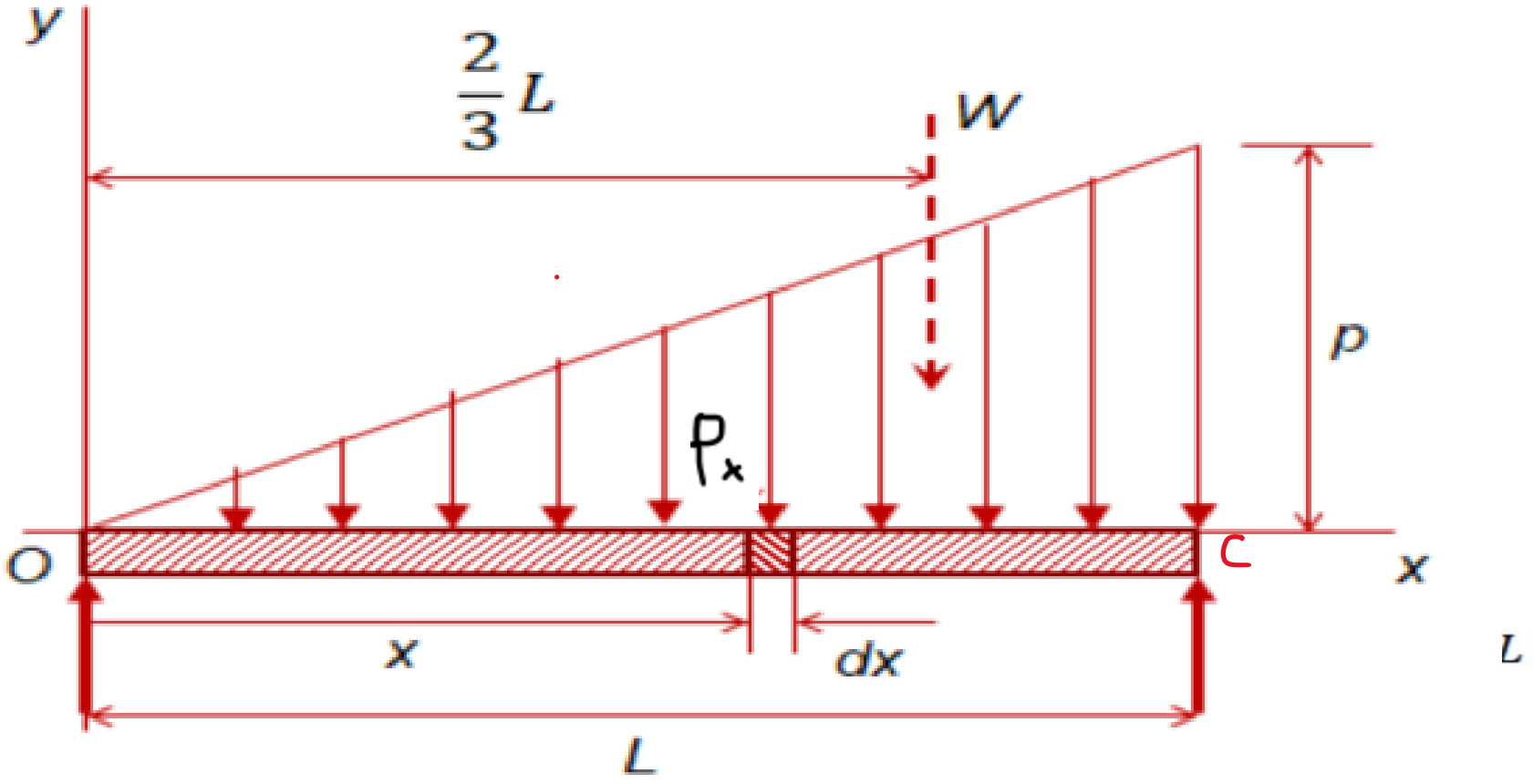
$$x_2' = h - \frac{b}{2} = 30 - \frac{6}{2} = 27\text{ in} \quad [\text{from the y axis}]$$

$$V = V_1' - V_2' = 3141.6 - 301.6 = 2840\text{ in}^3$$

$$\bar{x} = \frac{V_1' x_1' - V_2' x_2'}{V_1' - V_2'} = \frac{(3141.6 \times 22.5) - (301.6 \times 27)}{2840}$$

$$= 22.02\text{ in}$$

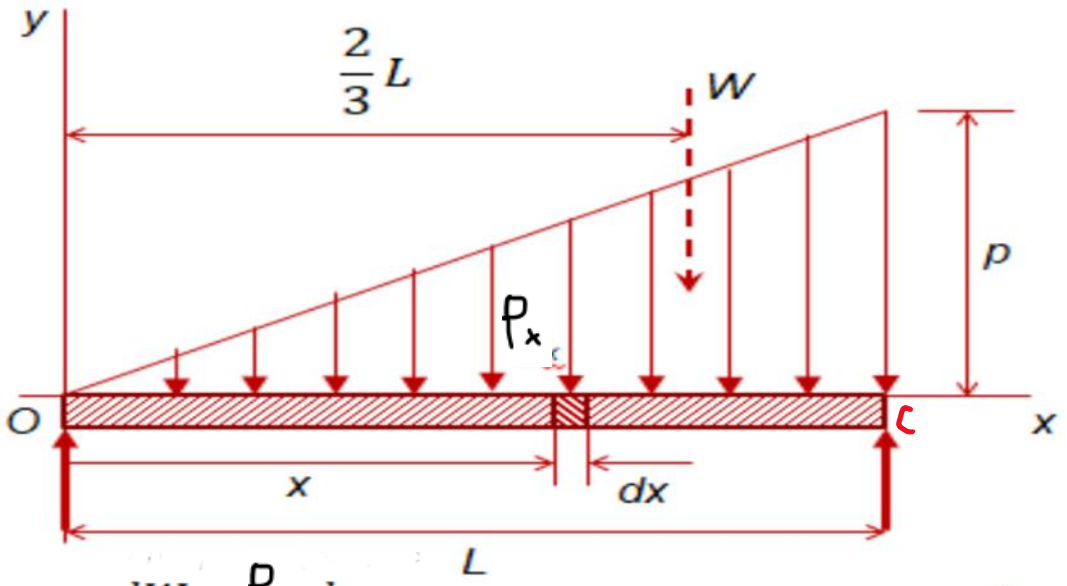
Q: In the fig OC represents a beam with a load varying from zero to O at a maximum of p psi at C. Let the beam have a unit depth perpendicular to the plane of the paper with no change in the loading in this direction at a particular section of the beam. Locate the center of pressure.



At a distance x from O, the pressure P_x on a differential length of the beam dx is virtually constant. Hence, the differential load on this length is $dW = P_x\, dA = P_x \cdot (dx \times 1) = P_x \cdot dx$ & the total load on the beam is $\int P_x\, dx$. From similar triangles,

$$\frac{P_x}{P} = \frac{x}{L}$$

$$\therefore P_x = \frac{Px}{L}$$



$$dW = P_x\, dx$$

$$W = \int P_x \cdot dx$$

$$= \frac{p}{L}\int_0^L x\, dx = \frac{pL}{2}$$

$$\bar{x} = \frac{\int x\, dw}{W}$$

$$= \frac{x\int dw}{\frac{PL}{2}}$$

$$= \frac{x \cdot \frac{P}{L}\int x\, dx}{\frac{PL}{2}}$$

$$= \frac{\frac{P}{L}\int_0^L x^r\, dx}{\frac{PL}{2}}$$

$$= \frac{P}{L} \times \left[\frac{x^3}{3}\right]_0^L \times \frac{2}{PL}$$

$$= \frac{P}{L} \times \frac{L^3}{3} \times \frac{2}{PL}$$

$$= \frac{2L}{3}$$

126. Theorems of Pappus and Guldinus.* **(a) *Theorem I.*** If a surface of revolution is generated by a plane curve revolving about any non-intersecting axis in its plane, the area of the generated surface *is equal to* the length of the curve *times* the distance traveled by the centroid of the curve while the surface is being generated.

PROOF. Consider any differential element dL of some curve MN, Fig. 346, which is revolved through an angle of θ radians ($\leq 2\pi$ radians) about the x axis. The differential area generated by the element dL is then $\theta y\, dL$, where the length of the arc swept out by the element dL is θy. The area generated by the curve MN is therefore

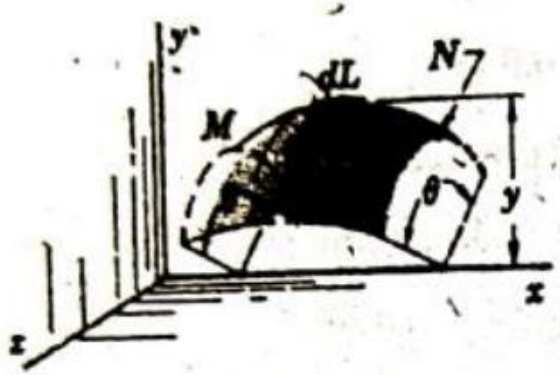


Fig. 346. Generating an Area.

$$A = \int \theta y\, dL = \theta \int y\, dL$$

for a particular value of θ. But since from equation (i), § 112, $\int y\, dL = L\bar{y}$, we have the area of the surface generated by MN as

$$A = \theta \int y\, dL = \theta \bar{y} L,$$

where $\theta\bar{y}$ is the distance traveled by the centroid of the curve MN.

(6) ***Theorem II.*** If a volume of revolution is generated by a plane area revolving about any non-intersecting axis in the plane of the area, the generated volume *is equal to* the magnitude of the area *times* the distance traveled by the centroid of the area while the volume is being generated.

PROOF. The plane area MN is revolved about the x axis, Fig. 347(a), through an angle of θ radians ($\leqq 2\pi$ radians). An element of this area dA moves through a distance of θy and generates a differential volume of $\theta y\, dA$. Thus the total volume is

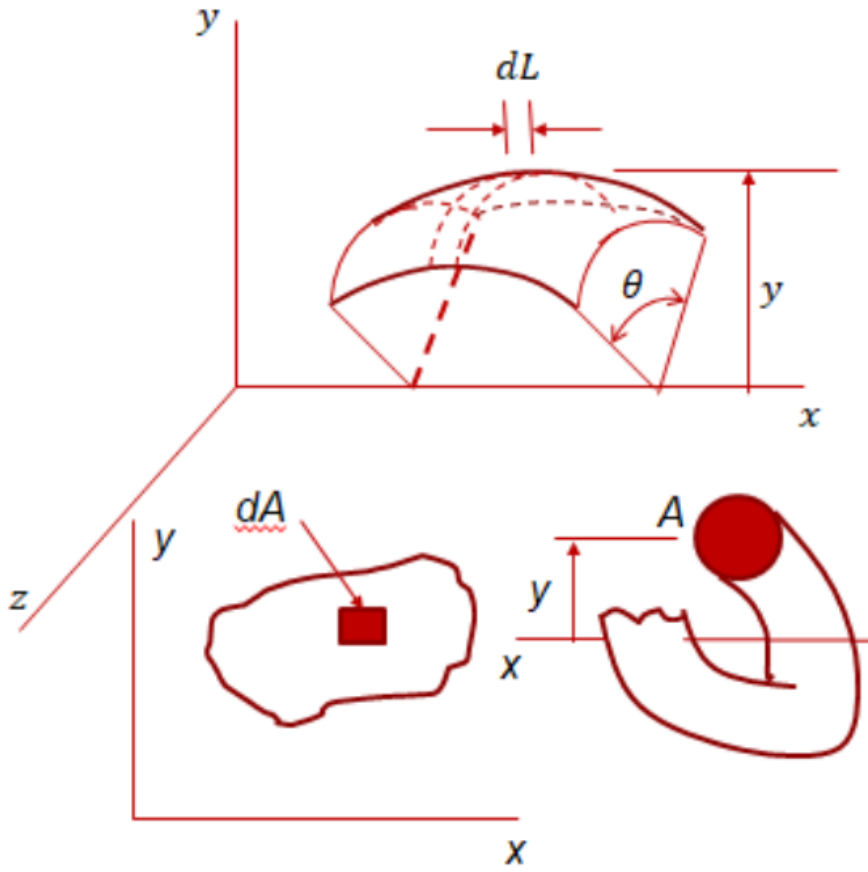
$$V = \int \theta y\, dA = \theta \int y\, dA$$

for a particular value of θ. However, from equation (h), § 111, we note that $\int y\, dA = A\bar{y}$. Therefore, the volume generated by revolving MN is

$$V = \theta \int y\, dA = \theta \bar{y} A,$$

where $\theta\bar{y}$ is the distance traveled by the centroid of the area.

Pappus and Guldinus Theorem



Theorem I:

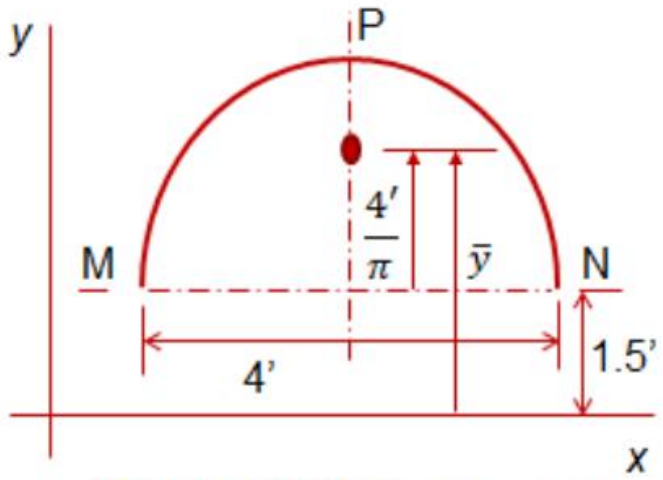
$$A = \int \theta y dL = \theta \int y dL = \theta \bar{y} L$$

Theorem II:

$$V = \int \theta y dA = \theta \int y dA = \theta \bar{y} A$$

Q: Determine the area generated by revolving the semicircular arc MPN of the following figure through 270 degree about the x axis.

Determine the volume generated by revolving a semicircular area through 360 degree about its diameter.



Given: MPN is an area.

Given: MPN is an arc.

$\theta = 270^0$ → Transfer it to radian

$\bar{y} = 1.5 + \frac{r \sin\beta}{\beta} = 1.5 + \frac{4}{\pi}$

$\frac{r\sin\beta}{\beta} = \frac{2 \times \sin 90^\circ}{90 \times \frac{\pi}{180}} = \frac{4}{\pi}$

$L = \pi r = 2\pi$

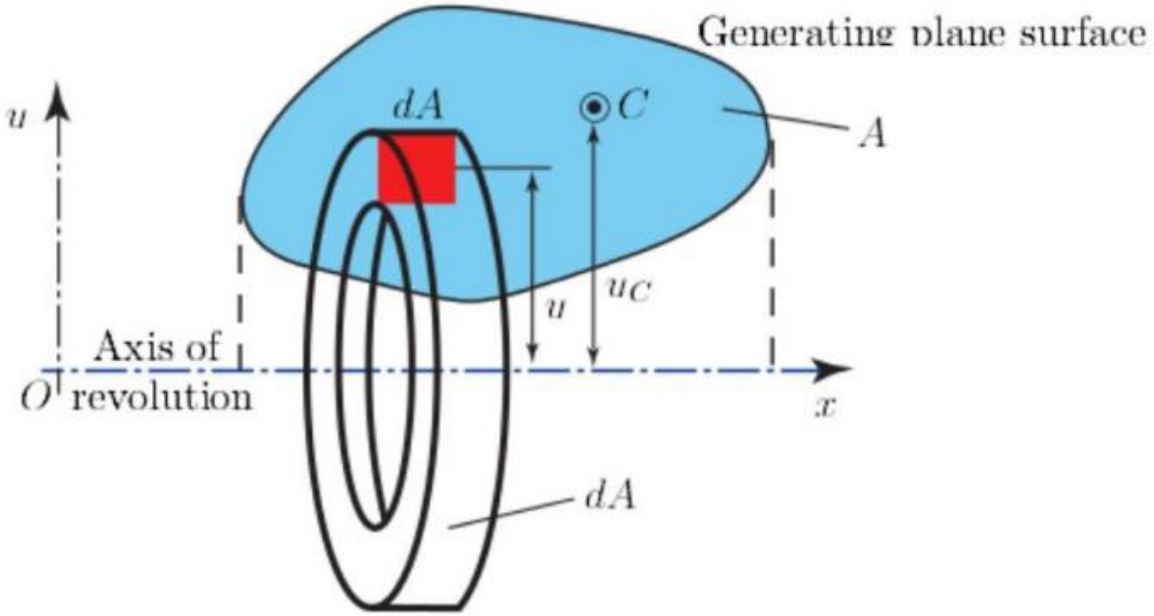
$A = \theta \bar{y} L = 82$ sq. ft.

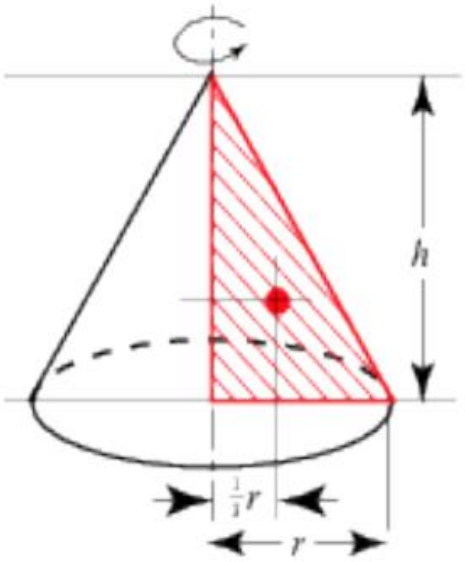
Half circle

$\theta = 360^0$ about its diameter

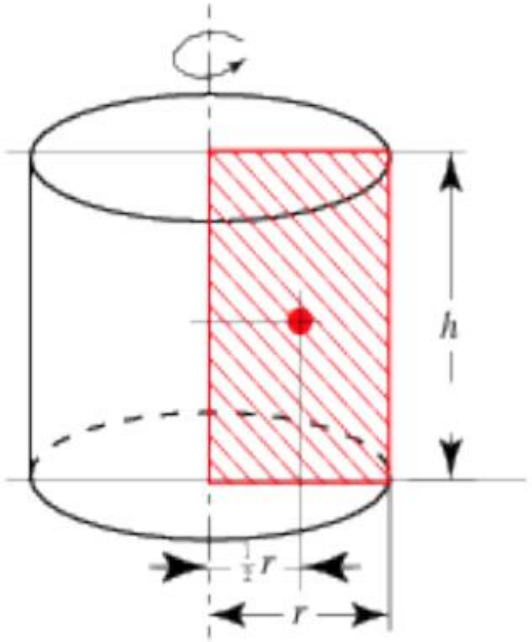
$\bar{y} = \frac{2r\sin 90^\circ}{3 \times 90 \times \frac{\pi}{180}} = \frac{4r}{3\pi}$

$V = \theta \bar{y} A = (2\pi)\left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{4}{3}\pi r^3$

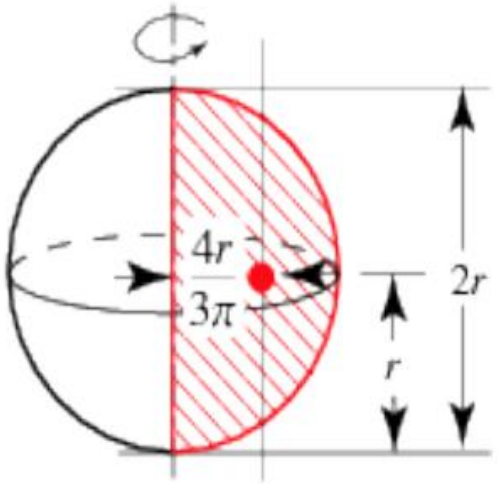




cone



cylinder



sphere

Solids By Integration

Ques: A solid of revolution is generated by the shaded area of fig 02 being revolved about the y axis. Determine the co-ordinates of the centroid of this volume.

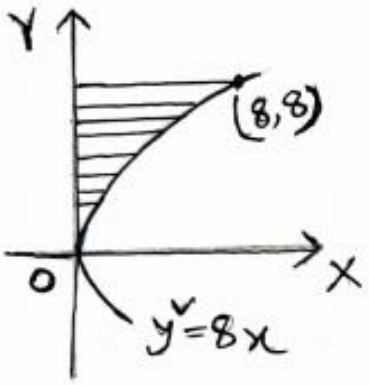
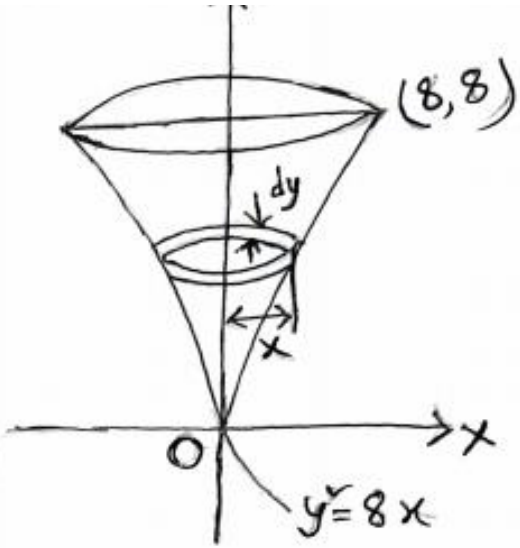
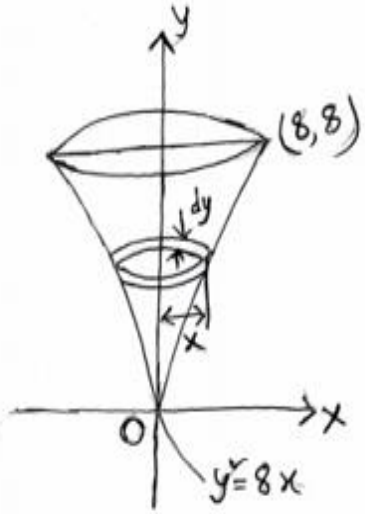


Fig: 02





Here, $y^2 = 8x$

$\Rightarrow x = \frac{y^2}{8}$

$dv = \pi x^2 dy$

$$\int dv = \int_0^8 \pi x^2 dy$$

$$= \int_0^8 \pi \left(\frac{y^2}{8}\right)^2 dy$$

$$= \int_0^8 \pi \left(\frac{y^4}{64}\right) dy$$

$$= \frac{\pi}{64} \int_0^8 y^4 dy$$

$$= \frac{\pi}{64} \left[\frac{y^5}{5}\right]_0^8$$

$$= \frac{\pi}{64} \left[\frac{8^5}{5} - 0\right]$$

$$= \frac{512}{5}\pi$$

$$\int_0^8 y\,dv = \int_0^8 y \cdot \pi x^2 dy$$

$$= \int_0^8 y \cdot \pi \left(\frac{y^2}{8}\right)^2 dy$$

$$= \frac{\pi}{64} \int_0^8 y^5 dy$$

$$= \frac{\pi}{64} \left[\frac{y^6}{6}\right]_0^8$$

$$= \frac{\pi}{64} \left[\frac{8^6}{6} - 0\right]$$

$$= \frac{2048}{3}\pi$$

$$\therefore \bar{y} = \frac{\int y\,dv}{V} = \frac{2048\pi \times 5}{3 \times 512\pi}$$

$$= 6.67$$

$\bar{x} = 0$ [$\because$ y is the axis of symmetry]