EWCE 101

ANALYTICAL MECHANICS

3 Credit Hour

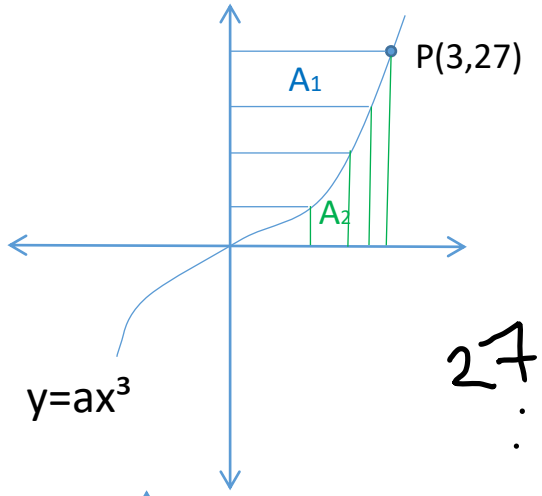
Lecturer
Department of Environmental, Water Resources, and Coastal Engineering (EWCE)

Military Institute of Science and Technology

Chapter
Moment of Inertia of Areas

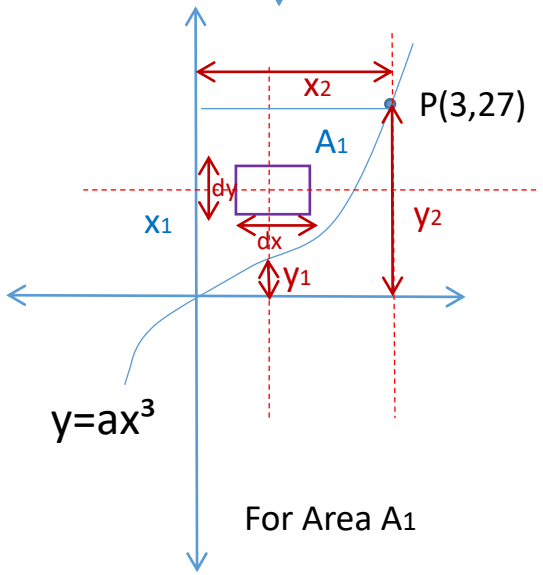
Reference Book: Analytical Mechanics by Virgil Moring Faires

Q(779): If the coordinates of P in Fig are (3 in,27 in), find the moment of inertia with respect to the y axis of area A_1 and A_2



$27 = a(3)^3$

$\therefore a = 1$



For Area A_1

$$I_{y_1} = \int x^2 \, dA$$

$$= \int_0^{27} \int_{x_1}^{x_2} x^2 \, dx \, dy$$

$$= \int_0^{27} \int_0^{y^{\frac{1}{3}}} x^2 \, dx \, dy$$

$$= \int_0^{27} \left[\frac{x^3}{3}\right]_0^{y^{\frac{1}{3}}} dy$$

$$= \int_0^{27} \frac{1}{3}\left[y^1 - 0\right] dy$$

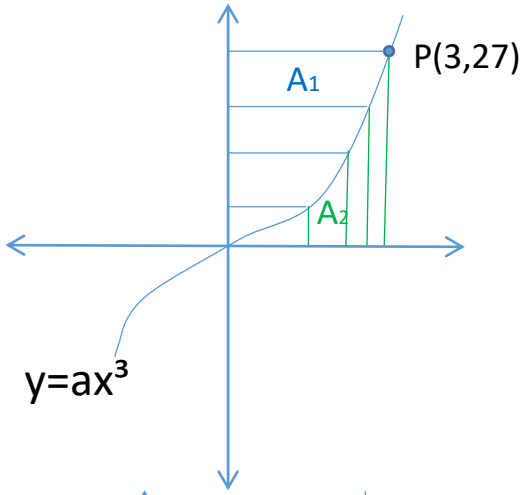
$$= \frac{1}{3}\int_0^{27} y \, dy$$

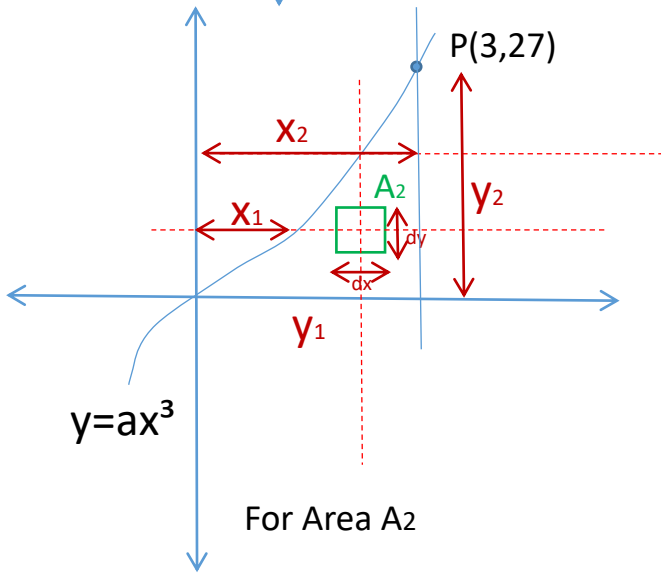
$$= \frac{1}{3}\left[\frac{y^2}{2}\right]_0^{27}$$

$$= \frac{1}{3}\left[\frac{(27)^2}{2} - \frac{0}{2}\right]$$

$$= 121.5 \text{ in}^4$$

Q(779): If the coordinates of P in Fig are (3 in,27 in), find the moment of inertia with respect to the y axis of area A_1 and A_2





For Area A_2

$$I_{y_2} = \int x^2 \, dA$$

$$= \int_0^{27} \int_{x_1}^{x_2} x^2 \, dx \, dy$$

$$= \int_0^{27} \int_{y^{\frac{1}{3}}}^{3} x^2 \, dx \, dy$$

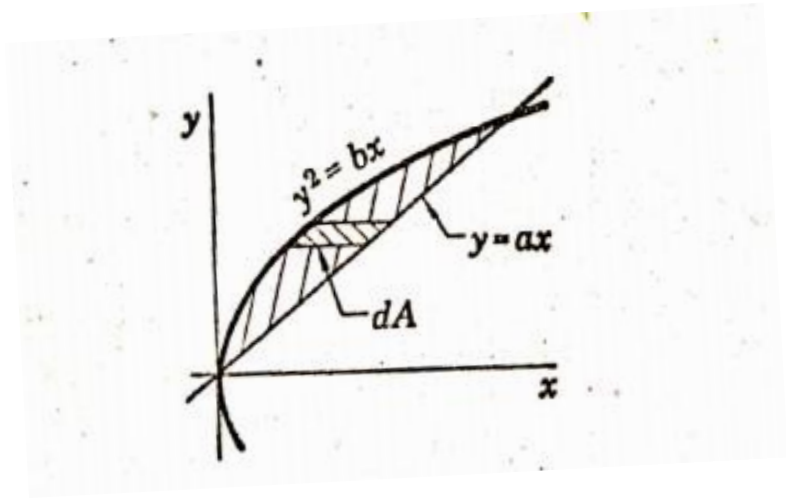
$$= \int_0^{27} \left[\frac{x^3}{3} \right]_{y^{\frac{1}{3}}}^{3} dy$$

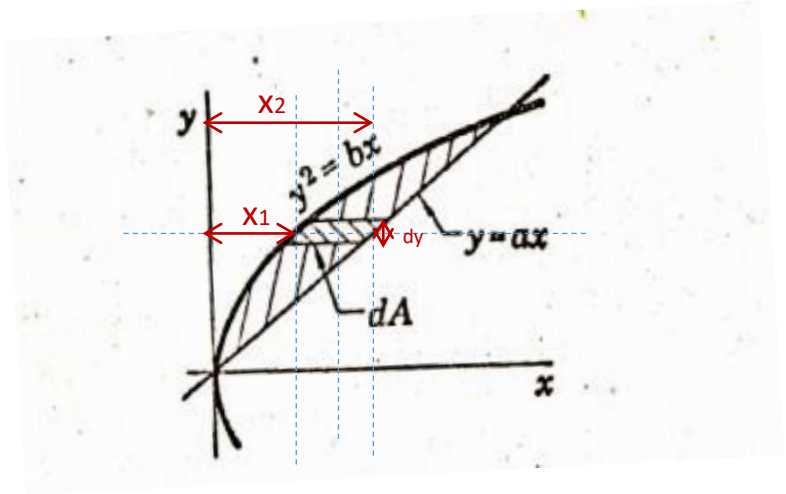
$$= \int_0^{27} \frac{1}{3} \cdot \left[3^3 - y \right] dy$$

$$= \frac{1}{3} \left[3^3 . y - \frac{y^2}{2} \right]_0^{27}$$

$$= 121.5 \text{ in}^4$$

Q(781): Find Ix for the shaded area in Fig between the curve $y^2=bx$ and the line $y=ax$ using the element dA shown .





$$I_x = \int y^2 dA$$

$$= \int y^2 \cdot x\, dy$$

$$= \int_0^{b/a} y^2 (x_2 - x_1)\, dy$$

$$= \int_0^{\frac{b}{a}} y^2 \left(\frac{y}{a} - \frac{y^2}{b}\right) dy$$

$$= \int_0^{\frac{b}{a}} \left(\frac{y^3}{a} - \frac{y^4}{b}\right) dy$$

$$= \left[\frac{y^4}{4a} - \frac{y^5}{5b}\right]_0^{\frac{b}{a}}$$

$$= \frac{b^4}{4a^5} - \frac{b^5}{5ba^5}$$

$$= \frac{b^4}{20a^5} \quad \text{Unit}^4$$

Here, $y^2 = bx$ & $y = ax$

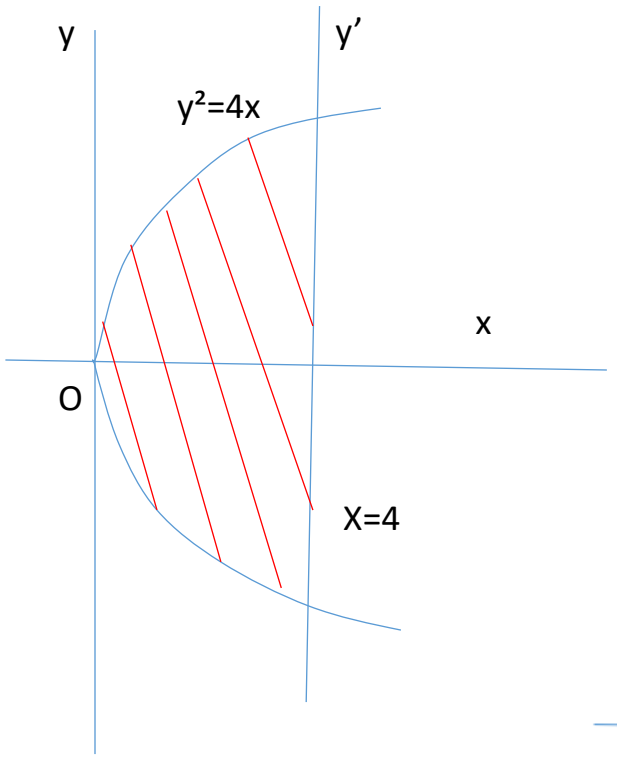
$a^2x^2 = bx$

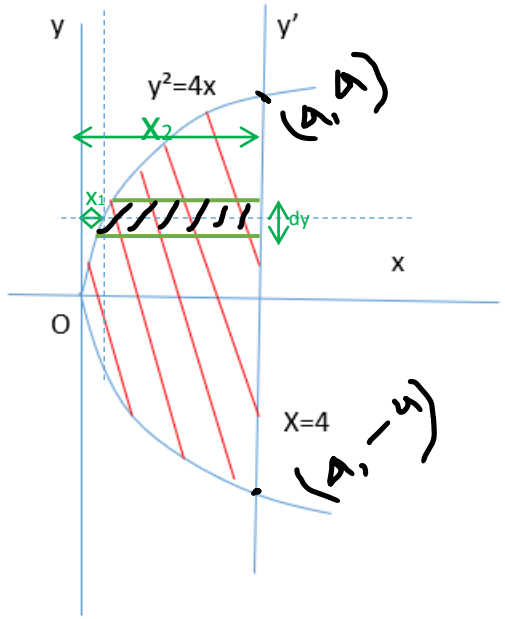
$\Rightarrow a^2x = b$

$\therefore x = \frac{b}{a^2}$

& $y = a \cdot \frac{b}{a^2} = \frac{b}{a}$

Q(786): For the shaded area in the figure, by direct integration method find (a)Ix, (b) Iy and (c) Iy'.The linear unit is in inch.





$y^2 = 4.4$

$\therefore y = \pm 4$

(a) $I_x = \int y^2 dA$

$= \int_{-4}^{4} y^2 (x_2 - x_1)\, dy$

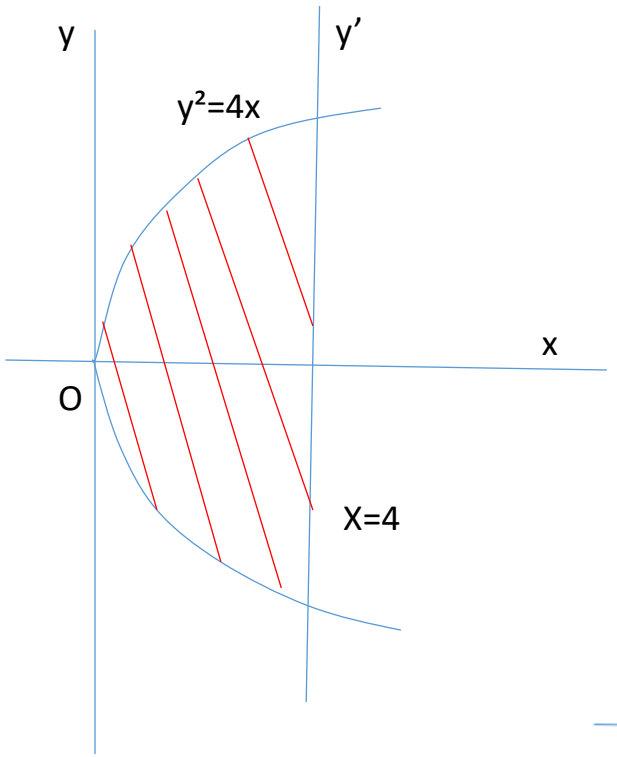
$= 2\int_{0}^{4} y^2 \left(4 - \frac{y^2}{4}\right) dy$

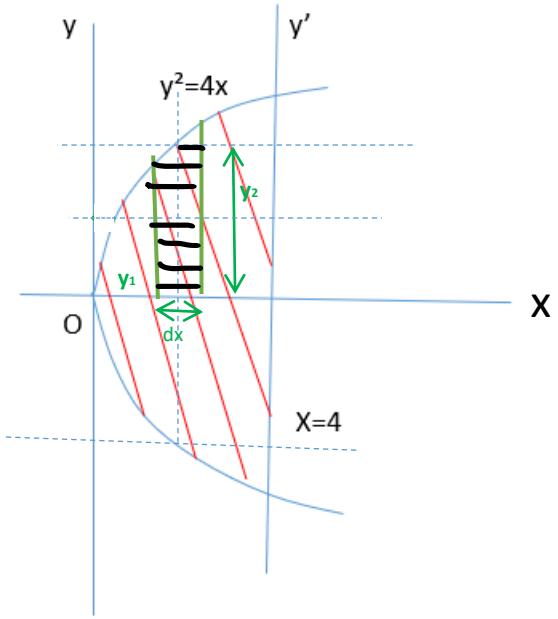
$= 2\int_{0}^{4} \left(4y^2 - \frac{y^4}{4}\right) dy$

$= 2\left[\frac{4y^3}{3} - \frac{y^5}{5\times 4}\right]_0^4$

$= 68.27 \text{ in}^4$

Q(786): For the shaded area in the figure, by direct integration method find (a)Ix, (b) Iy and (c) Iy'.The linear unit is in inch.





(b) $I_y = \int \overset{\vee}{x}\, dA$

$= \int x^{\vee} \cdot (y_2 - y_1)\, dx$

$= \int x^{\vee} \cdot (2\sqrt{x} - 0)\, dx$

$= 2\int_0^4 x^{\vee} \cdot 2\sqrt{x}\, dx$

$= 4\int_0^4 x^{\vee}\sqrt{x}\, dx$

$= 4\int_0^4 x^{\frac{5}{2}}\, dx$

$= 4 \times \left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^4$

$= 146.285\ in^4$

(c) $\bar{x} = \dfrac{\int x\,dA}{A}$

$\therefore \int x\,dA = \int_0^4 x \cdot 2y\,dx = \int_0^4 x \cdot 2 \cdot 2\sqrt{x}\,dx$

$= 4\int_0^4 x^{\frac{3}{2}}\,dx = 51.2\ in^3$

$A = \int dA = \int_0^4 2y\,dx = 2\int_0^4 2\sqrt{x}\,dx$

$= 4\int_0^4 \sqrt{x}\,dx$

$= 4\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= 4\left[\dfrac{4^{\frac{3}{2}}}{\frac{3}{2}} - 0\right] = 21.33\ in^2$

$\bar{x} = \dfrac{\int x\,dA}{A} = \dfrac{51.2}{21.33} = 2.4\ in$

$I_y = \bar{I}_y + Ad^2$

$\Rightarrow \bar{I}_y = I_y - Ad^2 = 146.285 - (21.33 \times (2.4)^2)$

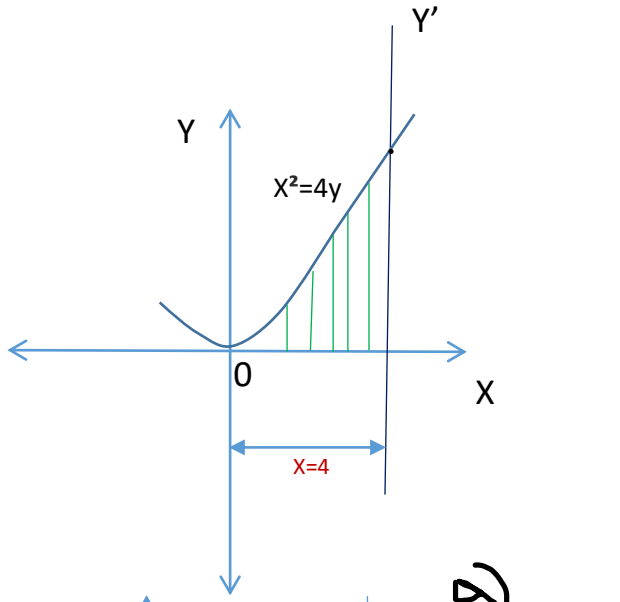
$= 23.42\ in^4$

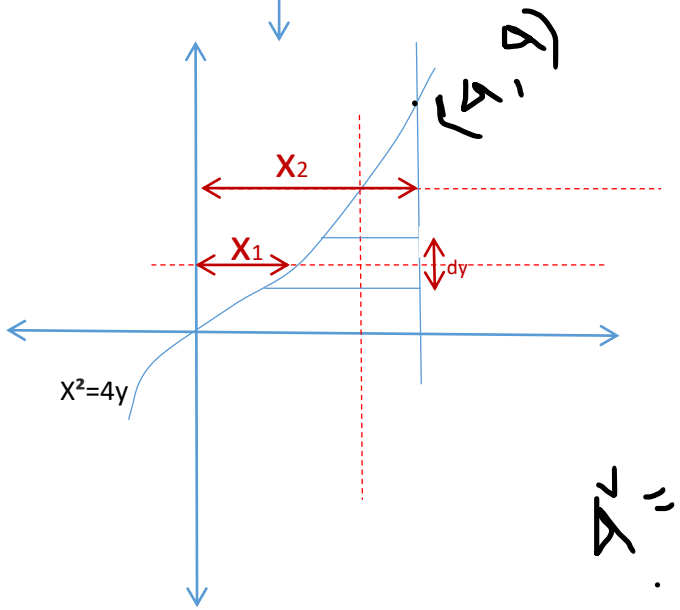
$I_{y'} = \bar{I}_y + Ad^2$

$= 23.42 + \{21.33 \times (4 - 2.4)^2\}$

$= 78.029\ in^4$

Q(788): For the shaded area in figure, find by direct integration method (a) Ix, (b) Iy and (c) Iy'. The linear units are in inches.

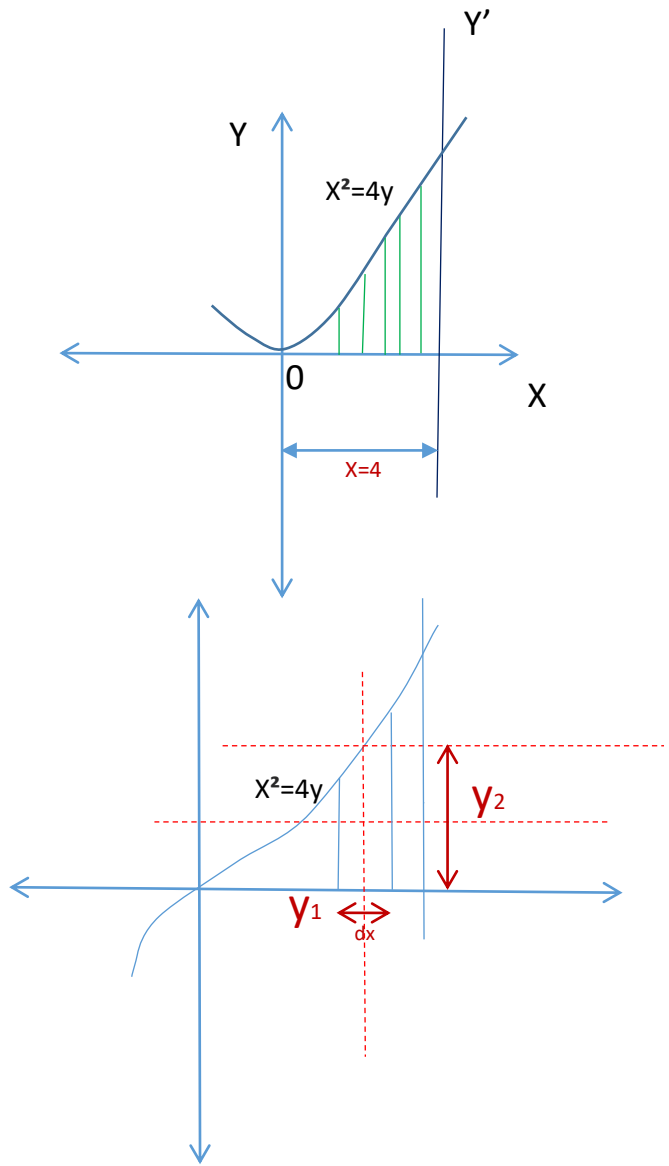




$$x^2 = 4y$$
$$\therefore y = 4$$

$$(a)\ I_x = \int y^2\, dA$$
$$= \int_0^4 y^2 \cdot (x_2 - x_1)\, dy$$
$$= \int_0^4 y^2 \cdot (4 - 2\sqrt{y})\, dy$$
$$= \int_0^4 \left(4y^2 - 2y^{\frac{5}{2}}\right) dy$$
$$= \left[4 \cdot \frac{y^3}{3} - \frac{2y^{\frac{7}{2}}}{7/2}\right]_0^4$$
$$= 12.19\ in^4$$

Q(788): For the shaded area in figure, find by direct integration method (a) Ix, (b) Iy and (c) Iy'



$$I_y = \int x^2 dA$$

$$= \int_0^4 x^2 (y_2 - y_1) dx$$

$$= \int_0^4 x^2 \left(\frac{x^2}{4} - 0\right) dx$$

$$= \int_0^4 \frac{x^4}{4} dx$$

$$= \left[\frac{x^5}{5}\right]_0^4 = 51.2 \text{ in}^4$$

$$A = \int dA = \int_0^4 (y_2 - y_1)\, dx$$

$$= \int_0^4 \left(\frac{x^2}{4} - 0\right) dx$$

$$= \left[\frac{x^3}{3 \times 4}\right]_0^4 = \frac{16}{3}\ in^2$$

$$\bar{x} = \frac{\int x\, dA}{A}$$

$$\int x\, dA = \int_0^4 x\,(y_2 - y_1)\, dx$$

$$= \int_0^4 x\left(\frac{x^2}{4} - 0\right) dx$$

$$= \int_0^4 \frac{x^3}{4}\, dx$$

$$= \left[\frac{x^4}{4 \times 4}\right]_0^4 = 16\ in^3$$

$$\bar{x} = \frac{\int x\, dA}{A}$$

$$= \frac{16}{\frac{16}{3}}$$

$$= 3\ in$$

$$I_y = \bar{I_y} + Ad^2$$

$$\Rightarrow \bar{I_y} = I_y - Ad^2$$

$$= 51.2 - \left\{ \frac{16}{3} \times 3^2 \right\}$$

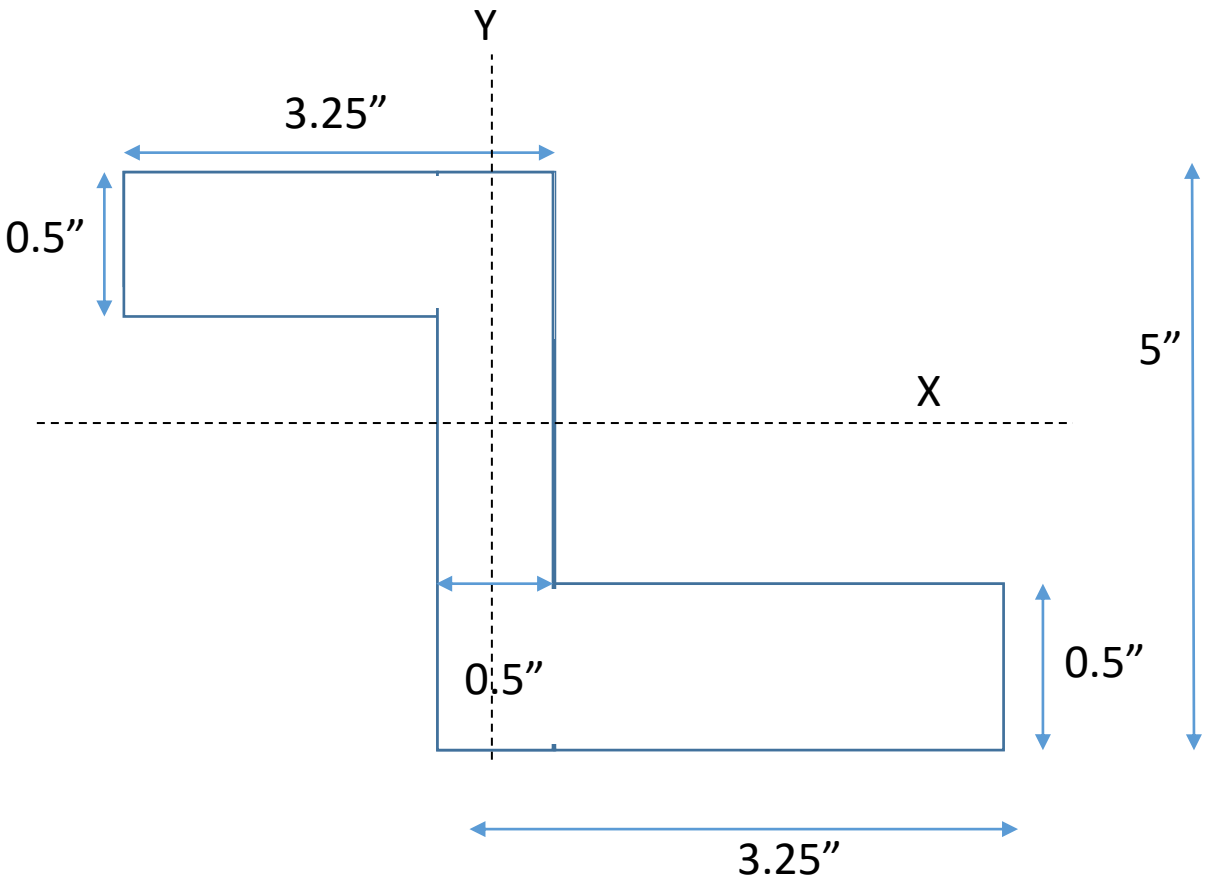
$$= 3.2 \text{ in}^4$$

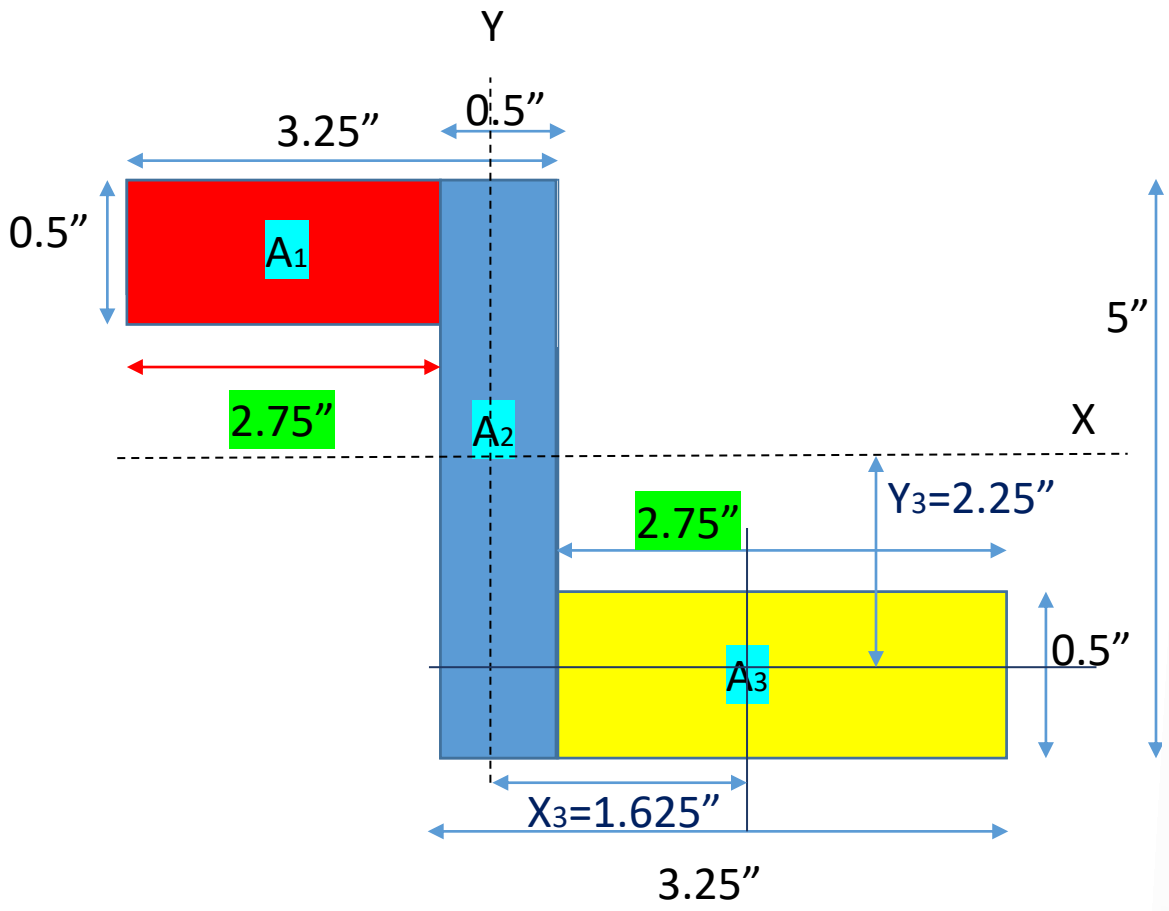
$$I_{y'} = \bar{I_y} + Ad^2$$

$$= 3.2 + \left\{ \frac{16}{3} \times (4-3)^2 \right\}$$

$$= 8.53 \text{ in}^4$$

Q 804: In the following figure, find the centroidal moment of Inertia.





Here, $A_1 = (3.25-0.5) \times 0.5 = 1.375 \text{ in}^2 = A_3$

$A_2 = (0.5 \times 5) = 2.25 \text{ in}^2$

$x_1 = x_3 = \frac{0.5}{2} + \frac{3.25-0.5}{2} = 1.625 \text{ in}$

$y_1 = y_3 = \frac{5}{2} - \frac{0.5}{2} = 2.25 \text{ in}$

$$I_{x_1} = I_{x_3} = \frac{(3.25-0.5)\times(0.5)^3}{12} + \{1.375\times(2.25)^2\}$$

$$= 6.98 \text{ in}^4$$

$$I_{x_2} = \frac{0.5\times 5^3}{12} + \{(0.5\times 5)\times 0^2\}$$

$$= 5.20 \text{ in}^4$$

$$\therefore \bar{I_x} = I_{x_1} + I_{x_2} + I_{x_3}$$

$$= 6.98 + 5.20 + 6.98$$

$$= 19.16 \text{ in}^4$$

$$I_{y_1} = I_{y_3} = \frac{0.5\times(3.25-0.5)^3}{12} + \{1.375\times(1.625)^2\}$$

$$= 4.497 \text{ in}^4$$

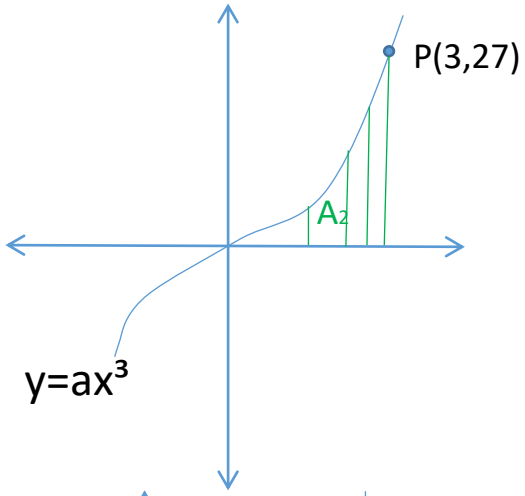
$$I_{y_2} = \frac{5\times(0.5)^3}{12} + \{(5\times 0.5)\times 0^2\}$$

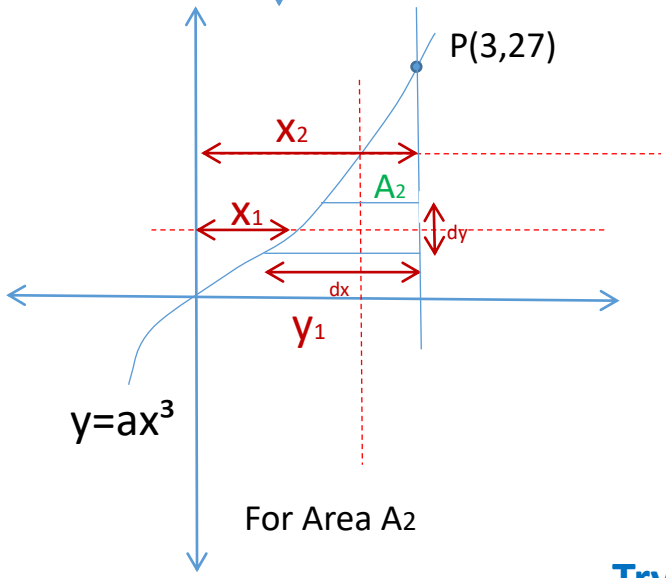
$$= 0.052 \text{ in}^4$$

$$\therefore I_y = I_{y_1} + I_{y_2} + I_{y_3} = 4.497 + 0.052 + 4.497$$

$$= 9.046 \text{ in}^4$$

Q(797): If the coordinates of P in Fig are (3 in,27 in), find the radius of gyration of area A_2 with respect to the (a) x axis (b) the centroidal x axis and (c) the line y=5. The centroid of area A_2 is given at (2.4 in, 7.71 in)





For Area A_2

(a) Here, centroid (2·4, 7·71)

$y = ax^3$ | P co-ordinate is (3, 27)

$\Rightarrow y = 1 \times x^3$ | $27 = a \times 3^3$

$\therefore y = x^3$ | $\therefore a = 1$

$$A = \int dA = \int_0^{27} \left(3 - y^{\frac{1}{3}}\right) dy$$

$$= \left[3y - \frac{y^{\frac{4}{3}}}{\frac{4}{3}}\right]_0^{27}$$

$$= 20.25 \text{ in}^2$$

$$I_x = \int y^2 dA = \int_0^{27} y^2 (x_2 - x_1)\, dy$$

$$= \int_0^{27} y^2 \left(3 - y^{\frac{1}{3}}\right) dy$$

$$= \left[3 \cdot \frac{y^3}{3} - \frac{y^{\frac{10}{3}}}{\frac{10}{3}}\right]_0^{27}$$

$$= 1968.3 \text{ in}^4$$

$$\therefore k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1968.3}{20.25}} = 9.86 \text{ in}$$

Try to solve the b and c portion by yourself, we will discuss in next class