EWCE 101

ANALYTICAL MECHANICS

3 Credit Hour

Department of Environmental, Water Resources, and Coastal Engineering (EWCE)

Military Institute of Science and Technology



Impulse and Momentum

Principle of Impulse and Momentum

From the theory of impulse & momentum,

$$mv_1 + \Sigma F \int_{t_1}^{t_2} dt = mv_2$$

v_1 = velocity at time t_1

v_2 = velocity at time t_2

We know,

$$\Sigma F = ma$$

$$\Rightarrow \Sigma F = m \cdot \frac{dv}{dt}$$

$$\Rightarrow \Sigma F dt = m dv$$

$$\Rightarrow \Sigma \int_{t_1}^{t_2} F dt = \int_{v_1}^{v_2} m dv$$

$$\Rightarrow \Sigma \int_{t_1}^{t_2} F dt = m \int_{v_1}^{v_2} dv$$

$$\Rightarrow \Sigma \int_{t_1}^{t_2} F dt = m(v_2 - v_1)$$

$$\Rightarrow \Sigma \int_{t_1}^{t_2} F dt = mv_2 - mv_1$$

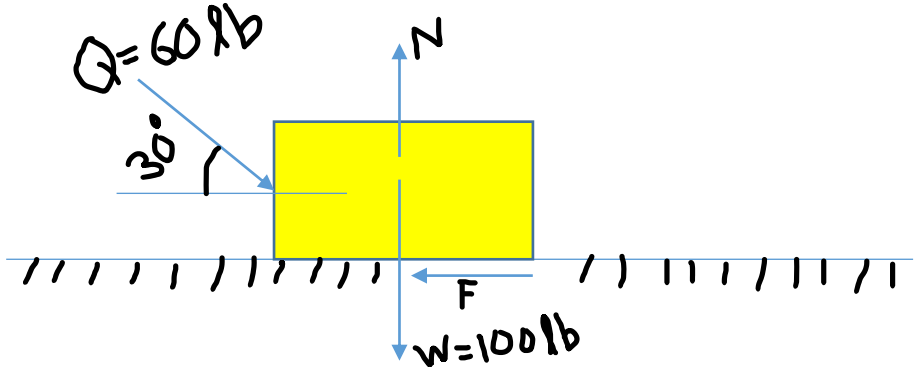
$$\therefore mv_1 + \Sigma \int_{t_1}^{t_2} F dt = mv_2$$

$$mv_1 + \Sigma F \int_{t_1}^{t_2} dt = mv_2$$

$$\Rightarrow mv_1 + \Sigma F (t_2 - t_1) = mv_2$$

$$\therefore mv_1 + \Sigma F \Delta t = mv_2$$

300 Example: A 100-lb body is on a horizontal plane, where the coefficient of kinetic friction is f=0.2. A force of Q=60 lb acts as shown in fig. (a) What is the impulse of Q in the horizontal direction during 2 sec? (b) What is the net impulse on the body during the same time.



(#)

(a) $Q_x = (60)\cos 30^\circ = 52\ lb$

$Q_x \cdot \Delta t = (52)(2) = 104\ lb\text{-sec}$

(b)

$\Sigma F_y = 0$

$\Rightarrow 60\sin 30^\circ + W - N = 0$

$\Rightarrow N = 60\sin 30^\circ + W$

$= 60\sin 30^\circ + 100 = 130$

$F = fN$

$= (0.2)\times(130)$

$= 26\ lb$

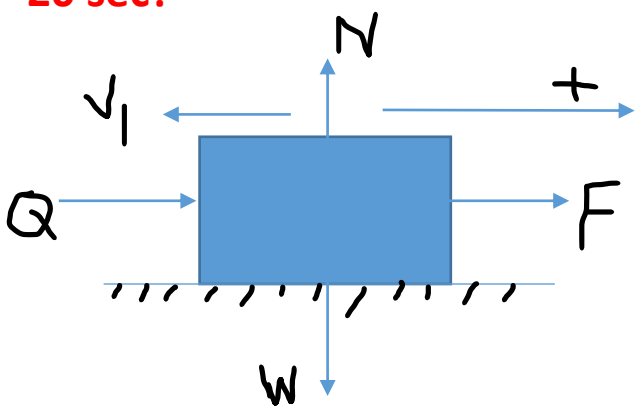
So, net impulse= $\int R\,dt = (\Sigma F_x)\Delta t$

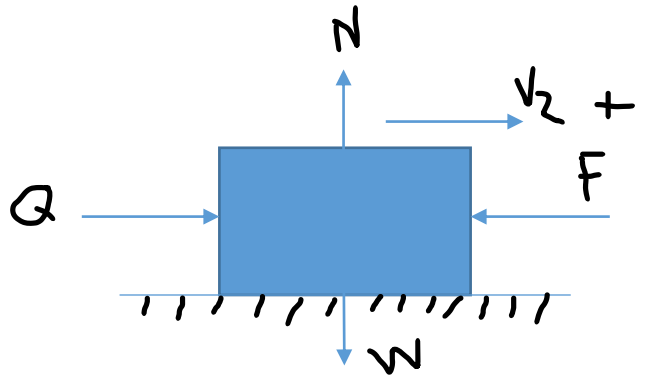
$= (Q\cos 30^\circ - F)\Delta t$

$= [(60\times\cos 30^\circ) - 26](2)$

$= 51.9\ lb\text{-sec}$

302 Example: A 200-lb body is moving toward the left with a velocity v1=50 fps at the instant that a force Q=50 lb acting toward the right is applied. There is a constant resistance to motion of F=40 lb. What is the body's velocity after 20 sec?





Let the rightward sense be positive. Using

$R\Delta t = \Delta m\bar{v}$,

fig 1) $(Q+F)\Delta t = \frac{W}{g}(V_2 - V_1)$

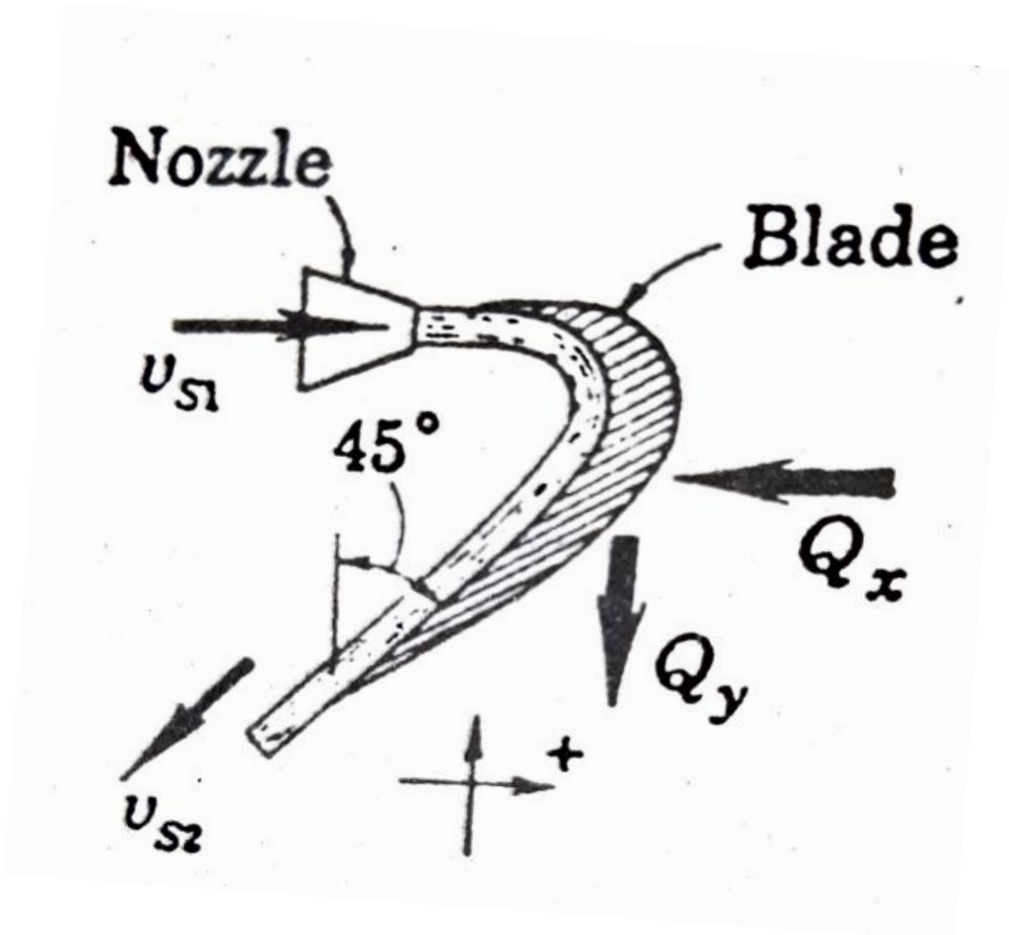
$\Rightarrow (50+40)\Delta t = \frac{200}{32.2}[0-(-50)]$

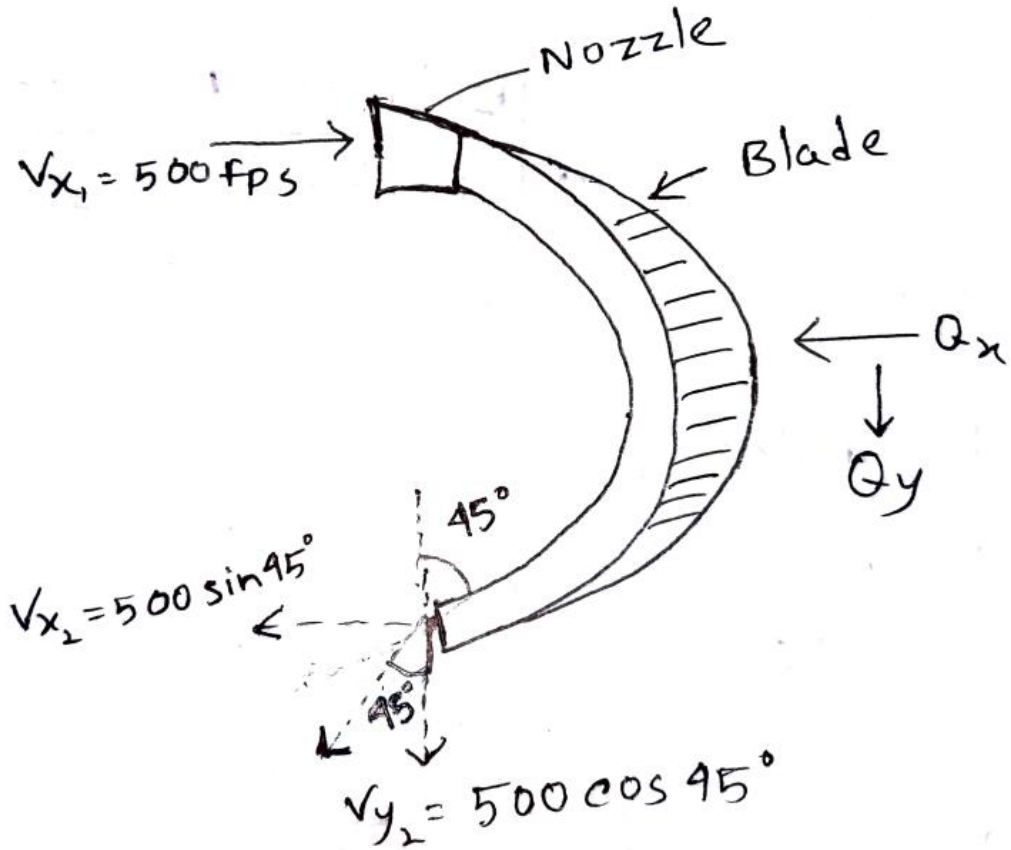
$\therefore \Delta t = 3.45 \text{ sec}$

For fig 2, $(50-40)(20-3.45) = \frac{200}{32.2}(V_2 - V_1)$ ($V_1 \to 0$)

$\therefore V_2 = 26.6 \text{ fps}$

303 Example: A jet of stream , flowing at the rate of W=1.5 lb per sec, issues from a nozzle with a velocity of vo1 = 500 fps, It enters a fixed blade and is turned through an angle of 135 degree before it is discharged. What are the horizontal and vertical components of the force exerted upon the fixed blade? Assume that the passage of stream across the blade is frictionless.





$R_x = -Q_x$, $\Delta t = 1\,sec$, $w = 1.5\,lb/sec$

Here,

$$R_x \Delta t = m(V_{x_2} - V_{x_1})$$

$$\Rightarrow -Q_x \times 1 = \frac{1.5}{32.2}(-500\sin 45^\circ - 500)$$

$$\therefore Q_x = 39.76\,lb$$

Again, $R_y = -Q_y$, $V_{y_1} = 0$, $V_{y_2} = 500\cos 45$

$$R_y \Delta t = m(V_{y_2} - V_{y_1})$$

$$\Rightarrow -Q_y \times 1 = \frac{1.5}{32.2}(-500\cos 45^\circ - 0)$$

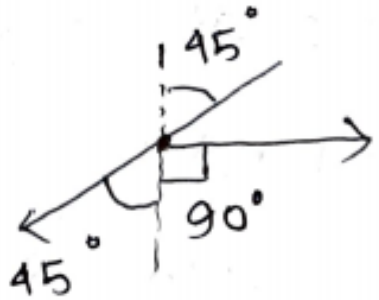
$$\therefore Q_y = 16.46\,lb$$

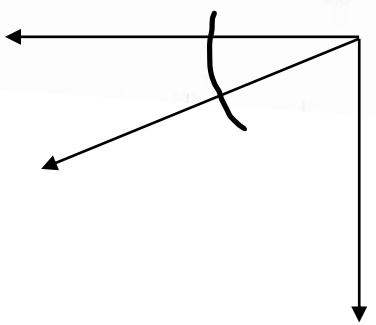
$$\therefore R = \sqrt{Q_x^2 + Q_y^2} = \sqrt{(39.76)^2 + (16.46)^2}$$

$$= 43\,lb$$

$$\theta = \tan^{-1}\frac{Q_y}{Q_x} = \tan^{-1}\frac{16.5}{39.8}$$

$= 22.5^\circ$ (with the horizontal, pointing downward toward the left)





Example: A box of sand A in Figure 688(a), weighing 10 lb, is moving towards the left on a horizontal plane, where f=0.3, with a velocity of 5 fps when it is stuck by a bullet B weighing 0.2 lb, which is moving towards the right with a velocity of 2000 fps. The bullet embeds itself in the box. (a) What is the resulting velocity of the box? (b) As measured from the instant of impact, when does the box come to rest? How far does it move from the point of impact?

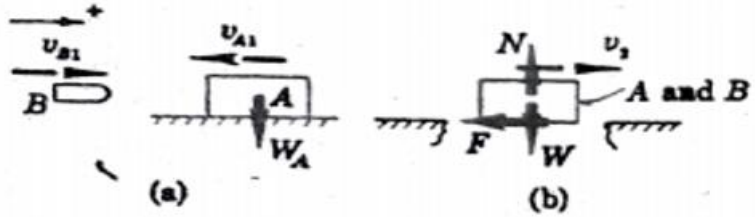


Fig. 688. It is assumed that the impact is consummated in an instant, so that the effect of friction during impact is negligible.

(a) Let the positive sense toward the right, $V_{B_1} = +2000$ fps & $V_{A_1} = -5$ fps. From the principle of the conservation of momentum,

$$m_A V_{A_1} + m_B V_{B_1} = (m_A + m_B) V_2$$

$$\Rightarrow \{10 \times (-5)\} + (0.2 \times 2000) = (10 + 0.2) V_2$$

$\therefore V_2 = +34.3$ fps; the positive sign indicating that the motion is toward the right.

(b) $N = W = 10.2$ lb

$\therefore F = fN = (0.3 \times 10.2) = 3.06$ lb

$R\Delta t = \Delta m v$

$\Rightarrow (-3.06)\Delta t = \frac{10.2}{32.2}(0 - 34.3)$ (W → 10.2; g → 32.2; final velocity (v') → 0; v → 34.3)

$\therefore \Delta t = 3.55$ sec

$v' = v_2 + at$

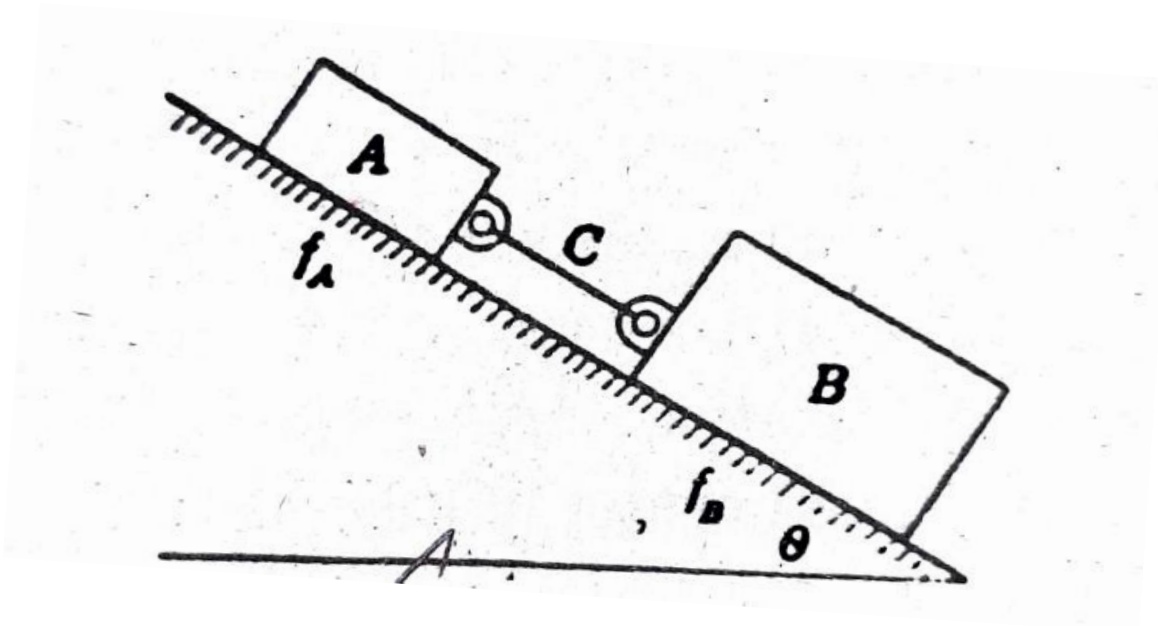
$\Rightarrow 0 = 34.3 + a \times 3.55$

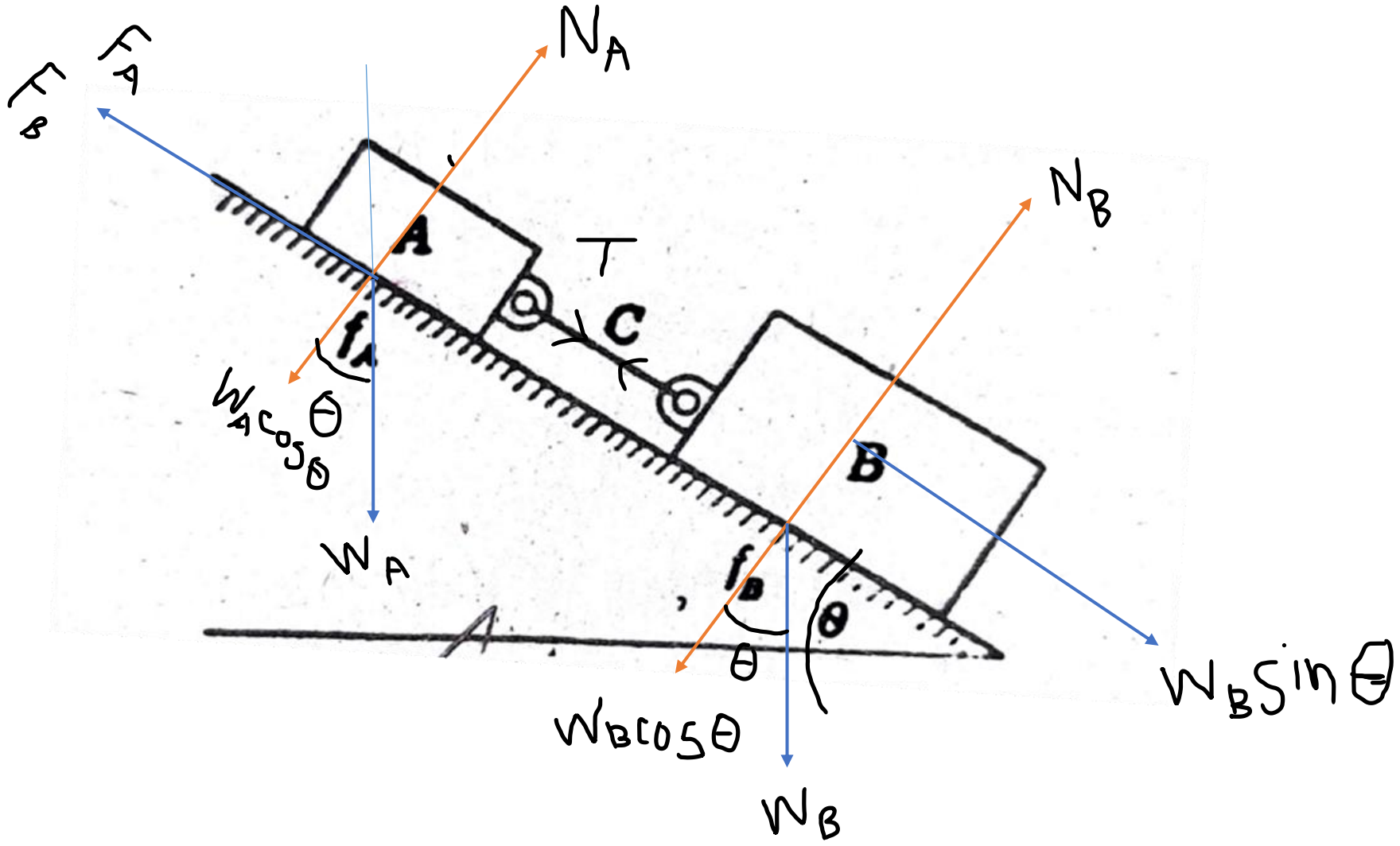
$\therefore a = \frac{-34.3}{3.55} = -9.66$ fps

$S = v_2 t + \frac{1}{2} a t^2 = (34.3 \times 3.55) + \{\frac{1}{2} \times (-9.66) \times (3.55)^2\}$

$= 60.9$ ft

539 A 2000-lb. body A is connected to a 3000-lb. body B by a weightless rigid rod C, Fig. 695, and they are on a $\theta = 30°$ incline. The coefficients of friction are $f_A = 0.1$ and $f_B = 0.2$. If the bodies start from rest, how long will it take for them to acquire a speed of 30 fps? What is the force on the rod C? *Ans.* 2.58 sec., 103 lb. (C).




F_A
F_B
N_A
A
T
C
f_A
$W_A \cos\theta$
W_A
N_B
B
f_B
θ
θ
$W_B \cos\theta$
W_B
$W_B \sin\theta$

Here,

$\theta = 30^\circ$; $W_A = 2000\,lb$, $W_B = 3000\,lb$

$f_A = 0.1$, $f_B = 0.2$, $V_2 = 30\,fps$, $V_1 = 0\,fps$

We know,

$$R\,\Delta t = m(V_2 - V_1)$$

$$\Rightarrow (W_A \sin\theta + W_B \sin\theta - F_A - F_B)\,t = \left(\frac{W_A + W_B}{32.2}\right)(V_2 - V_1)$$

$$\Rightarrow \{3000\sin 30 + 2000\sin 30 - (3000\cos 30 \times 0.2) - (2000\cos 30 \times 0.1)\}\,t = \frac{5000}{32.2}(30 - 0)$$

$$\therefore t = 2.58\,s$$

Now, for body A,

$$(W_A \sin\theta + T - F_A)\,t = \frac{W_A}{g} V$$

$$\Rightarrow W_A \sin\theta + T - F_A = \frac{W_A V}{gt}$$

$$\Rightarrow T = \frac{W_A V}{gt} + F_A - W_A \sin\theta$$

$$= \frac{2000 \times 30}{32.2 \times 2.58} + 173.2 - 2000\sin 30^\circ$$

$$= -104.5\,lb$$

$F_A = f_A N_A$

$= 2000\cos 30 \times 0.1$

$= 173.2$

$N_A = W_A \cos\theta$

$$\left[\begin{array}{l} W = W_A + W_B \\ \quad = (2000 + 3000)\,lb \\ \quad = 5000\,lb \end{array}\right]$$

$F_A = f_A N_A$

$F_B = f_B N_B$

Q: Each of the cables can sustain a maximum tension of 5000 lb. If the uniform beam has a weight of 5000 lb, determine the shortest time possible to lift the beam with a speed of 10 ft/sec starting from rest.

Here, $\cos\theta = \frac{4}{5}$

$\Sigma F_y = 0, \quad +P_{max} - T\cos\theta - T\cos\theta = 0$

$\Rightarrow P_{max} = 2T\cos\theta = 2 \times 5000 \times \frac{4}{5}$

$= 8000\ lb$

Now, From principle of impulse & momentum,

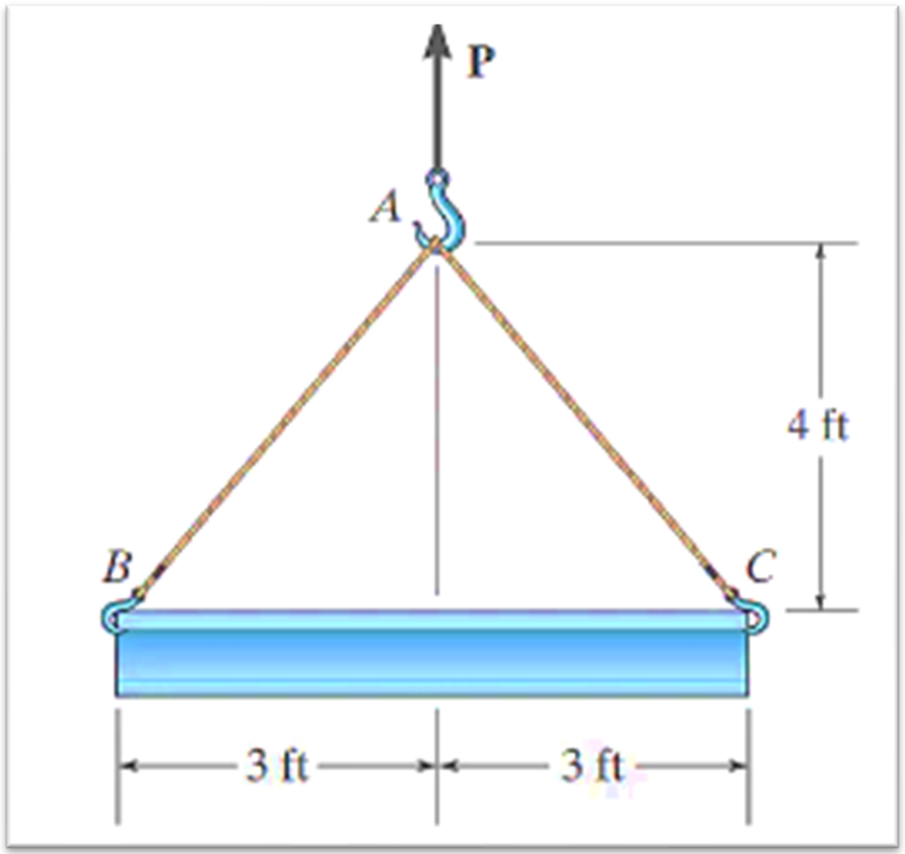
$mV_1 + \Sigma \int F\,dt = mV_2$

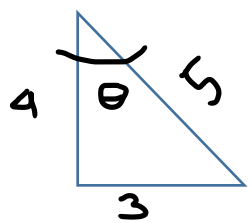
$\Rightarrow 0 + 8000\int_0^t dt + \int_0^t (-5000)\,dt = \frac{5000}{32.2}(10)$ (5000 ↝ w, 32.2 ↳ g)

$\Rightarrow 8000\,[t]_0^t - 5000\,[t]_0^t = \frac{50000}{32.2}$

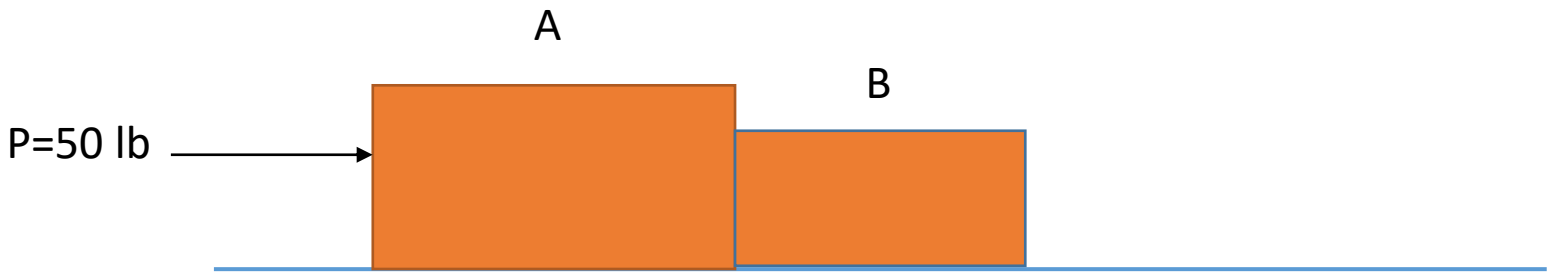
$\Rightarrow (8000t) - (5000t) = \frac{50000}{32.2}$

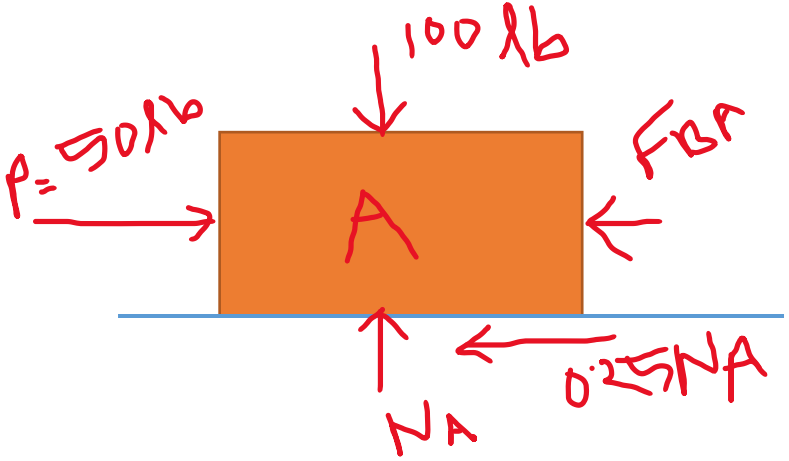
$\Rightarrow t = \frac{50000}{32.2 \times 3000} = 0.518\ sec$ //

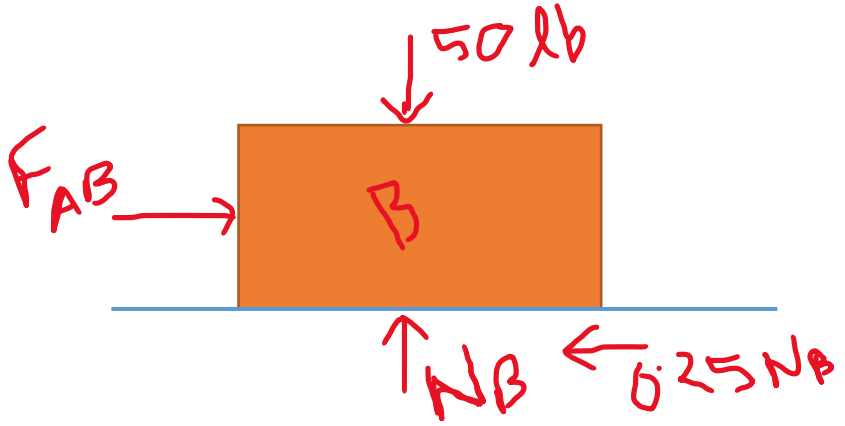




Q: Crates A and B weigh 100 lb and 50 lb, respectively. If they start from rest, determine their speed when t=5 sec. Also, find the force exerted by crate A on crate B during the motion. The coefficient of kinetic friction between the crates and the ground is μ_k=0.25







Here, $W_A = 100\ lb$; $(V_A)_1 = (V_B)_1 = 0$

$W_B = 50\ lb$ $\quad t = 5\ sec$

Here, $F_{AB} = F_{BA} = F$

$(V_B)_2 = (V_A)_2 = v = ?$

For crate A, From principle of impulse and momentum,

$$m_A(V_A)_1 + \Sigma\int_0^5 F\,dt = m_A(V_A)_2$$

$$\Rightarrow (m_A \times 0) + 50\int_0^5 dt - F\int_0^5 dt - 0.25N_A\int_0^5 dt = \frac{100}{32.2}v$$

$$\Rightarrow (50\times 5) - 5F - 0.25N_A \times 5 = 3.11v$$

$$\Rightarrow 5F + 3.11v = (0.25\times 100\times 5) - 250$$

$$\therefore 5F + 3.11v = 125 \quad \text{——(I)}$$

$\Sigma F_y = 0$,

$100 - N_A = 0$

$\therefore N_A = 100$

For crate B,

$$m_B(V_B)_1 + \Sigma\int_0^5 F\,dt = m_B(V_B)_2$$

$$\Rightarrow (m_B \times 0) + F\int_0^5 dt - 0.25N_B\int_0^5 dt = \frac{50}{32.2}v$$

$$\Rightarrow 5F - (0.25\times 50\times 5) = \frac{50}{32.2}v$$

$$\therefore 5F - 1.55v = 62.5 \quad \text{——(II)}$$

$\Sigma F_y = 0$,

$50 - N_B = 0$

$\therefore N_B = 50$

(I) − (II), $3.11v + 1.55v = 125 - 62.5$

$\Rightarrow 4.66v = 62.5$

$\therefore v = 13.41\ ft/sec$

Now, $F = \dfrac{125 - (3.11\times 13.41)}{5}$ [from eqn (I)]

$= 16.66\ lb$