SA/L03

18/05/23

Centroid of a triangle

Centroid of a rectangle

Centroid of a Arc of a Circle

$$\bar{x} = \frac{\int x\,dl}{L} = \dots = \frac{2r^2 \sin\beta}{2r\beta}$$

$$= r\times\frac{\sin\beta}{\beta}$$

$$\bar{y} = h/2$$

Centroid of Sector of a Circle

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \dots = \frac{2r}{3}\cdot\frac{\sin\beta}{\beta}$$

Lines by integration sample problems:

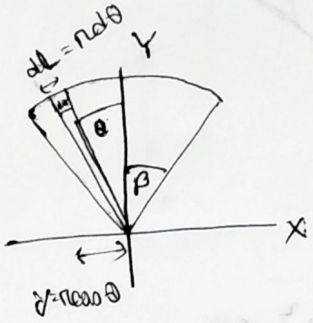
Q1

$\pi \times 0.49$

$\pi \times 0.737$

Q2

$0.637 \times \pi$



$\beta = 75°$

$$\bar{x} = \frac{\int x\,dl}{L}$$

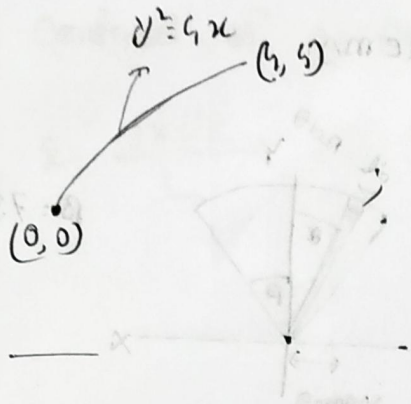
$$\int x\,dl = \int_0^{\pi/2} r\cos\theta \cdot r\,d\theta = r^2 [\sin\theta]_0^{\pi/2}$$

$$= r^2 \sin \pi/2$$

$$= r^2$$

$$L = \int dl = \int_0^{\pi/2} r\,d\theta = r\,\frac{\pi}{2}$$

$$\therefore \bar{x} = \frac{r^2}{\frac{r\pi}{2}} =$$



Find centroid

$\rightarrow y^2 = 4x$

$\Rightarrow \int 2y \frac{dy}{dx} = 4\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$

Here,

$dL^2 = dx^2 + dy^2$

$\Rightarrow dL = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= \sqrt{1+\frac{4}{y^2}}\,\frac{y}{2}\,dy$

$= \sqrt{1+\frac{4}{y^2}}\,y$

$= \sqrt{\frac{y^2+4}{y^2}}\,\frac{y}{2}\,dy$

$= \frac{1}{2}\sqrt{y^2+4}\,dy$

$L = \int_0^4 dL$

$= \int_0^4 \frac{1}{2}\sqrt{y^2+4}\,dy$

$= \frac{1}{2}\int_0^4 \sqrt{y^2+4}\,dy$

$= \cdots$

$= 9.701\ in^2$

$= 5.915\ in$

$$\int x\,dL = \int x \frac{1}{2}\sqrt{4+x^2}\,dx$$

$$= \int_0^4 \left(\frac{4}{x^2}\right)\left(\frac{1}{2}\sqrt{4+x^2}\right) dx$$

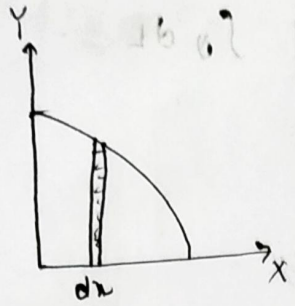
$$= \frac{1}{8}\int_0^4 x^2\sqrt{4+x^2}\,dx$$

$$= 9.701 \text{ in}^2$$

$$\bar{x} = \frac{\int x\,dL}{L} = \ldots$$

$$\bar{y} = \frac{\int y\,dL}{L}$$

$$\int y\,dL = \ldots$$



task: Quarter circle w/ angle

$$\bar{x} = \frac{\int x \, dA}{\int dA} \qquad \bar{y} = \frac{4R}{3\pi}$$

$$\bar{x} = \frac{2}{3} \frac{\sin\beta}{\beta}$$

Composite figure

Maintain +/-



$\Delta OAC = \frac{1}{2} \times 12 \times 30 \cos 60^\circ \times 30 \sin 60^\circ$

$= 389.71 \text{ in}^2$

$\bar{x} = \frac{x_1 A_1 - x_2 A_2}{A_1 - A_2}$

$= \dots$

$A_1 = \pi r_1^2 \times \frac{60^\circ}{360^\circ}$

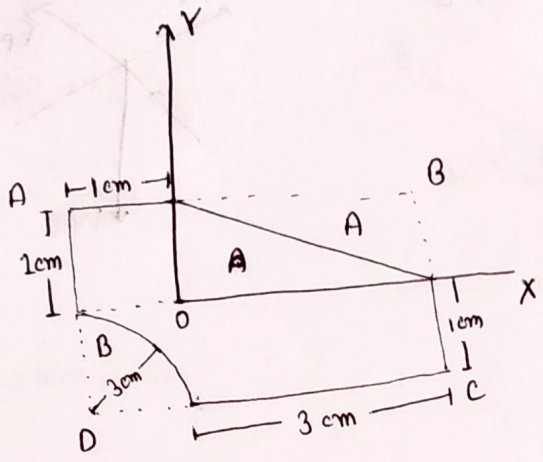
$= \dots$

$x_1 = \frac{2}{3} r_1 \frac{\sin\beta}{\beta}$

$= \dots$

Assignment

given origin is I.

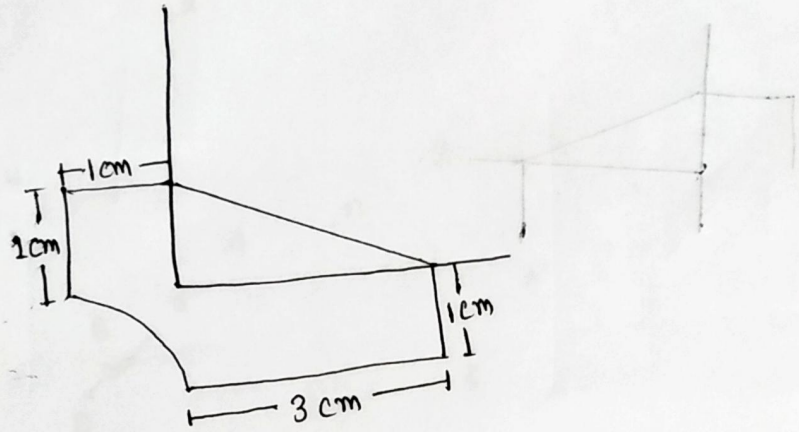


Centroid of the shape ABCD:

$$\bar{x} = \frac{(3+1)\text{ cm}}{2} = 2\text{ cm}$$

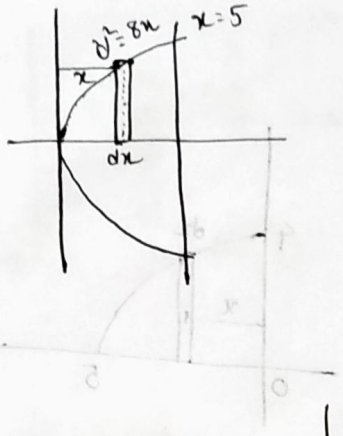
$$\bar{y} = \frac{(1+1)\text{ cm}}{2} = 1\text{ cm}$$

Centroid of A:


1cm
2cm
1cm
3 cm

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$y = \pm\sqrt{40}$

$$\bar{x} = \frac{\int_0^5 x\,dA}{\int_0^5 dA}$$

$$= \int_0^5 3 \text{ in}$$

$x =$

$dA = dx\sqrt{8x}$

$$\int dA = \int_0^5 \sqrt{8x}\,dx = 21.08 \text{ in}^2$$

$$\int_0^5 x\,dA = \int_0^5 x\sqrt{8x}\,dx$$

$$= 63.25 \text{ in}^3$$

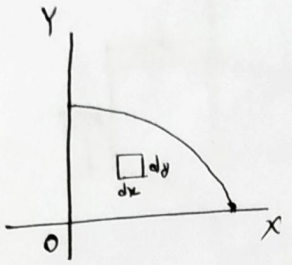
$$\bar{y} = \frac{\int y\,dA}{\int dA} = 0 \text{ in}$$

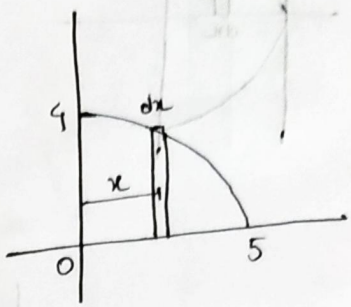
$$= \frac{\int_0^5 \sqrt{8x}\sqrt{8x}\,dx}{\int_0^5 \sqrt{8x}\,dx}$$

$$= \frac{100}{21.08}$$

$16\left(1-\frac{x^2}{25}\right)$

$\frac{x^2}{25}+\frac{y^2}{16}=1 \Rightarrow y=$





$\int y\,dA$

$= \int \left(4\sqrt{1-x^2/25}\right)^2 dx$

$= 16\int \left(1-\frac{x^2}{25}\right) dx$

$=$

$\bar{x} = \frac{\int x\,dA}{\int dA}$

$= \frac{\int_0^5 x \cdot 4\sqrt{1-\frac{x^2}{25}}\,dx}{\int_0^5 4\sqrt{1-\frac{x^2}{25}}\,dx}$

$= \frac{33.33}{15.71}$

$= 2.12$ in

$\bar{y} = \frac{\int y\,dA}{\int dA}$

$= \frac{\int_0^5 \left(4\sqrt{1-\frac{x^2}{25}}\right)^2 dx}{15.71}$

$= \frac{\int_0^5 \left(2\sqrt{1-\frac{x^2}{25}}\right)\left(4\sqrt{1-\frac{x^2}{25}}\right) dx}{15.71}$

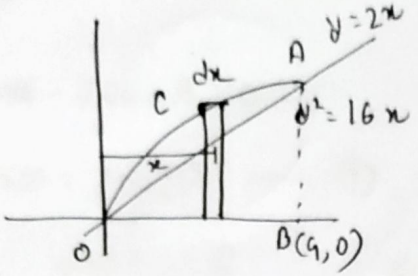
$= 16.17$ in

$\bar{y} = \frac{\int y\,dA}{\int dA}$

$= \frac{\int_0^5 y\,dA}{\int_0^5 dA} =$

$= \frac{\int_0^5 \left(4\sqrt{1-\frac{x^2}{25}}\right)^2 dx}{\int_0^5 4\sqrt{1-\frac{x^2}{25}}\,dx}$

$= \frac{53.33}{15.71} = 3.39$ in



$(2x)^2 = 16x$

$\therefore x = 4$

$A(4, 8)$

Centroid of OAB,

$\bar{y}_1 = \frac{1}{3} AB$

$= \frac{1}{3} \times 8$

$= 2.67$ in

$\bar{x}_1 = \frac{2}{3} OB = 2.67$ in

Centroid of OCAB,

$\bar{x}_2 = \frac{\int_0^4 x\, dA}{\int_0^4 dA}$

$= \frac{51.20}{21.33}$ in

$= 2.4$ in

$dA = \sqrt{16x}\, dx$

$\int_0^4 dA = = 21.33$ in^3

$\int_0^4 x\, dA = \int_0^4 x\sqrt{16x}\, dx$

$= 51.20$ in^3

$\bar{y}_2 = \frac{\int_0^4 y\, dA}{21.33} = \frac{\int_0^4 \frac{\sqrt{16x}}{2} \sqrt{16x}\, dx}{21.33}$

$= \frac{64}{21.33} = 3$ in

$\therefore \bar{x} = \frac{\bar{x}_2 \times 21.33 - \bar{x}_1 \times \frac{1}{2} 4 \times 8}{21.33 - \frac{1}{2} 4 \times 8}$

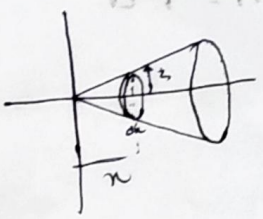
$= 1.6$ in

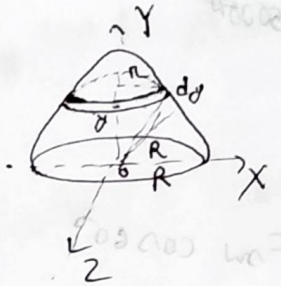
$\bar{y} = \frac{3 \times 21.33 - 2.67 \times 16}{21.33 - 16} = 3$ in

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$$\bar{x} = \frac{3}{4}h$$





$$\bar{x} = \frac{\int x\,dV}{\int dV}$$

$$= \frac{\pi R^3}{4} \times \frac{3}{2R^3\pi}$$

$$= \frac{3}{8}R$$

$dV = \pi r^2 \rho\, dx$

$= \pi(R^2 - x^2)\rho\, dx$

$\int x\,dV = \int \pi(R^2 - x^2)\rho\, dx$

$= \cdots$

$= \frac{\pi R^4}{4}$

$\int dV = V = \cdots$

$= \frac{2}{3}\times R^3\pi$

$R^2 = r^2 + x^2$

$\Rightarrow r^2 = R^2 - x^2$

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$\bar{x}_1 = \frac{3h}{4} = 22.5''$

$\bar{x}_2 = 24'' + \frac{6}{2}'' = 27''$

$V_1 = \pi h \left(\frac{D}{2}\right)^2 - 2 \times \frac{1}{2} \frac{D}{2} h = 9124.78\ in^3$

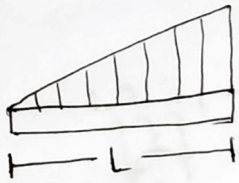
$V_2 = \pi d^2 b = 301.60\ in^3$

$\therefore \bar{x} = \frac{x_1 V_1 - x_2 V_2}{V_1 - V_2}$

$= 22.35''$

$\bar{y} = 0''$

$\bar{x} = \frac{3}{2} L$



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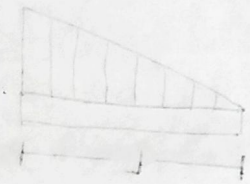
Theorem 1:

area = line height X centroid movement

= L X $\bar{x}_1$

Theorem 2:

volume = area X centroid movement



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Moment of inertia:

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

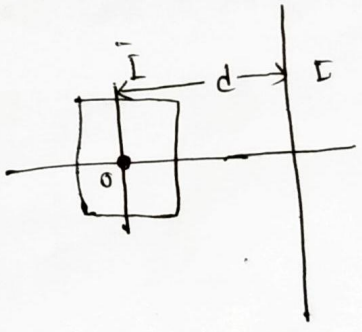
$$I_z = I_x + I_y$$

Radius of Gyration:

$$k = \sqrt{\frac{I}{A}} \quad \text{or} \quad k = \sqrt{\frac{J}{A}}$$

The parallel one:

$$I' = \bar{I} + Ad^2$$

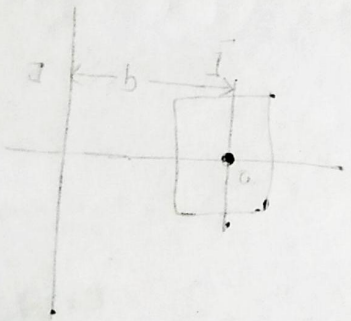


$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{bh^3}{12}}{\frac{bh}{2}}}$$

$$= \sqrt{\frac{bh^3}{12} \times \frac{2}{bh}}$$

$$= \sqrt{\frac{bh^2}{6}}$$

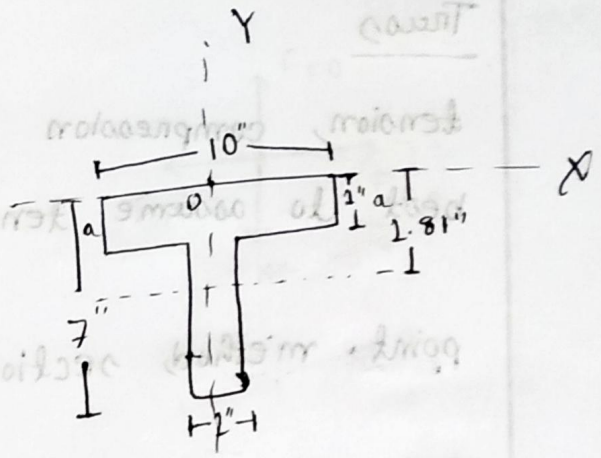
$$= \frac{h}{\sqrt{6}}$$



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$x_1 = 0'', \quad x_2 = 0''$

$y_1 = 0.5'', \quad y_2 = 1 + \frac{6}{2} = 4''$

$\therefore \bar{x}_0 = 0''$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{10 \times 0.5 + 6 \times 4}{10 + 6}$$

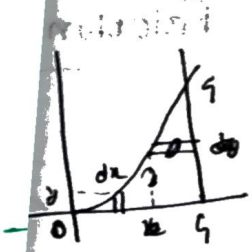
$$= 1.81''$$

$$I_x = \bar{I}_{a.a} + A_1' d^2$$

$$= \frac{bh^3}{12} + 10 \times$$

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$I_x = 12.19 \text{ in}^4$

$I_y = 51.2 \text{ in}^4$

$I_{y'} = I_y + Ad^2$

$= 51.2 + 5.33 \times (4-3)^2$

$= 56.53 \text{ in}^4$

$\bar{x} = \frac{\int_0^4 x\, dA}{\int_0^4 dA} = \frac{\int_0^4 \frac{x^3}{4} dx}{\int_0^4 \frac{x^2}{4} dx}$

$= \frac{16}{5.33}$

$= 3 \text{ in}$

$x^2 = 4y \Rightarrow y = \frac{x^2}{4}$

$x = \sqrt{4y}$

$dA = \frac{x^2}{4} dx$

$I_y = \bar{I}_y + Ad^2$

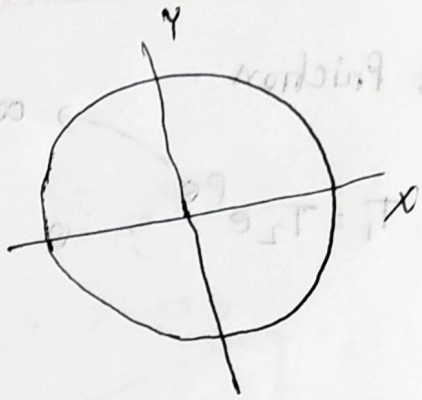
$\Rightarrow \bar{I}_y = 51.2 - 5.33 \times 3^2$

$= 3.23 \text{ in}^4$

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10/08

moment of inertia of a circle about diameter.



$$I_{xx} = I_{yy} = \frac{\pi}{64} d^4$$