BANGLADESH UNIVERSITY OF PROFESSIONALS
Military Institute of Science and Technology
B.Sc. in Environmental, Water Resources, and Coastal Engineering
Level-1, Term-I Final Examination: June 2016
Subject: MATH-107; Differential and Integral Calculus, Matrices

Time: 3.00 hours **Full Marks: 210**

INSTRUCTIONS:

a. Use SEPARATE answer scripts for each section.
b. Answer any THREE questions from each section.
c. Figures in the margin indicate full marks.
d. Assume reasonable data if necessary.
e. Symbols used have their usual meanings.

SECTION-A

Question – 1

a. Investigate the continuity and differentiability of the following function at the point $x = 0$, $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. 10

b. Find $\frac{dy}{dx}$ of the following functions: 16

i) $x + y = \tan(xy)$,

ii) $y = x^3\sqrt{\frac{x^2+4}{x^2+3}}$.

c. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. 09

Question – 2

a. If $y = \sin\left(m\sin^{-1}x\right)$ then show that 16

i) $\left(1-x^2\right)y_2 = xy_1 - m^2y$,

ii) $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2-m^2\right)y_n = 0$.

b. State Mean-value theorem. Verify the mean-value theorem for the function $f(x) = 2x - 3x^2 + x^3$ in the interval $(0,1]$. 10

c. If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$. 09

Question – 3

a. Determine the maximum and minimum values of $x + \sin 2x$ for $0 < x < 2\pi$. 10

b. If v is a function of x and y, and $x = r\cos\theta$ and $y = r\sin\theta$ then prove that, $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}$. 15

c. Prove that the angle of intersection of two curves $x^2 - y^2 = 2a^2$ and $x^2 + y^2 = 4a^2$ is $\frac{\pi}{3}$. 10

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Question – 4 Evaluate

a. $\int \frac{(x+1)\sqrt{x+2}}{\sqrt{x-2}}dx$

b. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}dx$

c. $\int \frac{1}{\sqrt{1-\sin 2x}}dx$

d. $\int \frac{11\cos x-16\sin x}{2\cos x+5\sin x}dx$

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SECTION-B

Question 5

a. Find the reduction formula for $\int \cos^m x \cos nx dx$.

b. Evaluate $\lim_{n\to\infty}\left[\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\frac{n}{(n+3)^2}+...+\frac{n}{(n+n)^2}\right]$.

c. Find $\int_1^3\int_{\pi/6}^{y^2} 2y\cos x dx dy$.

Question 6

a. Define Beta and Gamma functions. Prove that $\int_0^{\frac{\pi}{2}} \sin^p\theta\cos^q\theta d\theta = \frac{\left\lfloor\frac{p+1}{2}\right\rfloor\left\lfloor\frac{q+1}{2}\right\rfloor}{2\left\lfloor\frac{p+q+2}{2}\right\rfloor}$. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta d\theta$.

b. Find the area bounded by the curve $r=2(1+\cos\theta)$.

c. Find the volume of the solid generated by the revolution of the curve $y=x^3$ and x-axis between $x=0$ and $x=2$.

Question 7

a. Define matrix. Find the inverse of the matrix $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix}$ by matrix operation.

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b. Solve the system of linear equations 15

$$2x-3y+4z=1$$
$$3x+4y-5z=10$$
$$5x-7z+2z=3$$

by operating matrix.

Question 8

a. State Cayley-Hamilton theorem. Justify Cayley-Hamilton theorem 20
for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

b. Find the eigen-values and the corresponding eigen-vectors for the 15
matrix $A=\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.

END

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