Subject: Math 107, Differential, Integral Calculus and Matrix

Time: 3.00 hours **Full Marks: 210**

INSTRUCTIONS:

a. Use SEPARATE answer scripts for each section.
b. Answer any THREE questions from each section.
c. Figures in the margin indicate full marks.
d. Assume reasonable data if necessary.
e. Symbols used have their usual meanings.

SECTION-A

Question – 1

a. Discuss the limit, continuity and differentiability of the function 20

$$f(x)=\begin{cases} x^2 \text{ when } x\le 0 \\ x \text{ when } 0<x<1. \\ \frac{1}{x} \text{ when } x\ge 1 \end{cases}$$

at the point $x=1$.

b. Differentiate $y=(\sin x)^{\cos x}+(\tan x)^{\sin x}$ with respect to x. 10

c. The rate of change of radius of *a* circle is $\frac{1}{\pi}$. Find the rate of change of area of the circle at the instant when the radius is 2 cm. 5

Question – 2

a. State Leibnitz's theorem. If $y=e^{m\cos^{-1}x}$, then prove that 15

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$$

b. Verify the mean value theorem for the function $f(x)=3+2x-x^2$ in the interval (0,1). 10

c. Expand *sinx* in the power of $\left(x-\frac{\pi}{2}\right)$. 10

Question – 3

a. If $f(x,y)$, is a homogeneous function of order n, then prove that 15

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$$

b. Find the maximum and minimum values of 10
$f(x)=x^4-8x^3+22x^2-24x+5$.

c. Evaluate: $\lim_{x\to 0}(\sin x)^x$ 10

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Question – 4

a. Find the eigen values and the corresponding eigen vectors of the matrix $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$. 20

b. Use Caley-Hamilton theorem to find the inverse of the matrix $\begin{bmatrix}4 & 3 & 1\\ 2 & 1 & -2\\ 1 & 2 & 1\end{bmatrix}$. 15

SECTION-B

Question 5 Evaluate the following integrals: 11

a. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$ 12

b. $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx$ 12

c. $\int \frac{1}{(x+1)\sqrt{1+2x-x^2}}\,dx$

Question 6 17

a. Prove that, $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}=\frac{\pi}{2ab}$ 18

b. State Beta function and Gamma function.
Prove that $\int_0^1 x^6\sqrt{1-x^2}\,dx=\frac{5\pi}{256}$.

Question 7 17

a. Evaluate $\iint_D (3-x-y)\,dA$, where D is the triangle in the (x,y) plane bounded by the x-axis and the lines $y=x$ and $x=1$. 18

b. Find the area of the region between the graphs of $f(x)=3x^3-x^2-10x$ and $g(x)=-x^2+2x$.

Question 8 20

a. Solve the following system of linear equation by matrix
$x+2y+3z=-1$
$2x-y+5z=4$
$4y-z=5$

b. Define rank of a matrix. Find the rank of the matrix A, 15
$A=\begin{bmatrix}-1 & 2 & 0 & 4 & 5 & -3\\ 3 & -7 & 2 & 0 & 1 & 4\\ 2 & -5 & 2 & 4 & 6 & 1\\ 4 & -9 & 2 & -4 & -4 & 7\end{bmatrix}$

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