BANGLADESH UNIVERSITY OF PROFESSIONALS
Military Institute of Science and Technology
B.Sc. in Environment, Water and Coastal Engineering
Term Final (Spring) Examination 2023: Jul-Sep 2023

Student Group: 0<Earned Credit Hours ≤ 36

Subject: Math-101, Differential and Integral Calculus

Time: 3.00 hours **Full Marks: 180**

INSTRUCTIONS:

a. Use **SEPARATE** answer scripts for each section.
b. **Question-1** in **Section-A** and **Question-5** in **Section-B** are compulsory
c. Answer any other **TWO** questions out of **THREE** from each section.
d. Figures in the margin indicate full **marks**.
e. Assume reasonable data if necessary.
f. **Symbols** and **abbreviations** used have their usual meanings.

SECTION-A

Question 1 (Compulsory)

a. Distinguish between $\lim_{x\to a} f(x)$ and $f(a)$. **06**

b. What do you mean by differentiability of a function? **14**
A function is defined as follows:

$$f(x) = \begin{cases} 3-2x; & 0 \le x < \frac{3}{2} \\ -3-2x; & x \ge \frac{3}{2} \end{cases}$$

Examine the continuity and differentiability of $f(x)$ at $x = \frac{3}{2}$.

c. Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ with respect to $\sin^{-1}\frac{2x}{1+x^2}$. **10**

Question 2

a. State Leibnitz's Theorem. If $x = \sin\left(\frac{1}{a}\log y\right)$, then prove that $(1-x^2)y_{12} - 21xy_{11} - (100+a^2)y_{10} = 0$. **12**

b. Evaluate the followings: **18**

(i) $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$.

(ii) $\lim_{x\to 0} (\sin x)^{\tan x}$

Question 3

a. State and prove Euler's theorem on homogeneous function u in x, y of degree n. **10**

b. If $u = \log(x^3+y^3+z^3-3xyz)$, show that **10**
$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.

c. Show that x^x is minimum for $x = \frac{1}{e}$. **10**

Question 4

a. State Rolle's theorem. Verify the truth of Rolle's theorem for the function $f(x) = x^2 - 3x + 2 = 0$ in the interval $(1, 2)$. **16**

b. Find the equation of the tangent at the point 't' on a curve given by $x = a\cos^3 t, \quad y = a\sin^3 t$. **14**

SECTION-B

Question 5 (Compulsory)

a. Consider the formula $\int e^x\{f(x) + f'(x)\}\,dx = e^x f(x)$ to integrate $e^x \frac{1-\sin x}{1-\cos x}$. **10**

b. Integrate the following functions: **20**

(i) $\frac{3x+2}{5x^2+2x+3}$; (ii) $\frac{x^2 \tan^{-1} x^3}{1+x^6}$.

Question 6

a. Derive a reduction formula for $\int \tan^n x\,dx$. Hence integrate $\tan^6 x$. **08**

b. Evaluate $\int_0^a \frac{a^2-x^2}{(a^2+x^2)^2}\,dx$. **10**

c. Prove that $\int_0^k f(x)\,dx = \int_0^k f(k-x)\,dx$. **12**

Hence evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\tan x}}$.

Question 7

a. State Walli's formula for $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$. Derive the formula and evaluate $\int_0^{\frac{\pi}{2}} \sin^{10} x\,dx$. **12**

b. Define Beta function. Prove that $\beta(m,n) = \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}}\,dy$; $m > 0, n > 0$. **10**

c. Write down the relation between Beta and Gamma function. Apply the relation in order to evaluate $\int_0^1 x^7(1-x)^5\,dx$. **08**

Question 8

a. Write down the formula to evaluate $\int_0^{\frac{\pi}{2}} \sin^m x\,\cos^n x\,dx$; $m > 0$, $n > 0$. Hence find the value of $\int_0^{\frac{\pi}{2}} \cos^8 x\,dx$. **10**

b. Write down the formula to calculate the arc length of polar equation. Hence find the perimeter of the cardioid $r = a(1 + \cos\theta)$. **10**

c. Calculate the volume generated by revolving the area bounded by $x^{2/3} + y^{2/3} = 4$ about x-axis. **10**