Chapter: 1

Notes: Integration is (1) process of summation

(2) antiderivative

Antiderivative

Let us consider the functions: $F(x) = x^2 - 2x + 1$ and

$$f(x) = 2x - 2$$

We see that $\frac{dF(x)}{dx} = \frac{d}{dx}(x^2 - 2x + 1) = 2x - 2$

And $\int f(x)dx = \int(2x - 2)dx = x^2 - 2x$ [Where is 1 ???]

Integrate the Following Expressions:

1 $\frac{1}{\sin(x-a)\sin(x-b)}$; $Hints$: $\frac{1}{\sin(b-a)}[\frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)}]$[5/37]

2 $cosec(x - a)cosec(x - b)$; Hints: same as 1

3. $\frac{1}{\cos(x-a)\cos(x-b)}$; $Hints$: $\frac{1}{\sin(b-a)}[\frac{\sin\{(x-a)-(x-b)\}}{\cos(x-a)\cos(x-b)}]$

4. $sec(x - a)sec(x - b)$; Hints: same as 3 [5/39]

5. $\frac{1}{\cos(x-5)\cos(x-7)}$;

6. $\frac{cos\,2x-cos\,2\alpha}{cosx-cos\alpha} = \frac{(2\cos^2 x-1)-(2\cos^2\alpha-1)}{cosx-cos\alpha} = 2\frac{(cosx+cos\alpha)(cosx-cos\alpha)}{cosx-cos\alpha}$

7. $\frac{2-sin\,2x}{1-cos\,2x}$; $Hints$: $[\frac{2-2\sin x\cos x}{2\sin^2 x} = cosec^2x - \frac{cosx}{sinx}]$:

Ans. $-cotx - logsinx$

8. $\frac{1}{1+tanx} = \frac{cosx}{cosx+sinx} = \frac{1}{2}\frac{cosx+sinx+cosx-sinx}{cosx+sinx}$

$$= \frac{1}{2}\left(1 + \frac{cosx - sinx}{cosx + sinx}\right) Bipul\ 459$$

9. $\frac{sin\,2x}{acos^2x+bsin^2x}$, [hints: $acos^2x + bsin^2x = z$, $(b - a)sin2xdx = dz$]

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$$\int \frac{dx}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(b-a)} \int \frac{Sin(b-a)}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{Sin\{(x-a)-(x-b)\}}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{Sin(x-a)cos(x-b) - cos(x-a)Sin(x-b)}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int [\left\{\frac{Sin(x-a)cos(x-b)}{\cos(x-a)\cos(x-b)}\right\} - \left\{\frac{cos(x-a)Sin(x-b)}{\cos(x-a)\cos(x-b)}\right\}] dx$$

$$= \frac{1}{\sin(b-a)} \int [\left\{\frac{Sin(x-a)}{\cos(x-a)}\right\} - \left\{\frac{Sin(x-b)}{\cos(x-b)}\right\}] dx$$

$$= \frac{1}{\sin(b-a)} \int [tan(x-a) - tan(b-a)] dx$$

$$= \frac{1}{\sin(b-a)} [logsec(x-a) - logsec(b-a)] + c$$

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Chapter-2

1. (i) $\int cos2\cot^{-1}\sqrt{\frac{1-x}{1+x}}$, (ii) $\int cos2\cot^{-1}\sqrt{\frac{1+x}{1-x}}$, $[x = cos\theta]$

(iii) $\int cos2\tan^{-1}\sqrt{\frac{1-x}{1+x}}$, (iv) $\int cos2\tan^{-1}\sqrt{\frac{1+x}{1-x}}$

2. $\int \frac{dx}{\sqrt{sinx}\sqrt{\cos^7 x}}$; $[\frac{1}{\sqrt{\frac{sinx}{cosx}}\sqrt{cosx}\sqrt{\cos^7 x}} = \frac{1}{\sqrt{\frac{sinx}{cosx}}\sqrt{\cos^8 x}}$

$= \frac{1}{\sqrt{tanx}\cos^4 x} = \frac{\sec^2 x \sec^2 x}{\sqrt{tanx}} = \frac{\sec^2 x\,(1+\tan^2 x)}{\sqrt{tanx}}; let\ tanx = z]$

3. $cos^3 x\sqrt{sinx} = \sqrt{sinx}\,cos^2 x cosx = \sqrt{sinx}\,(1 - sin^2 x)cosx$

[Let $sinx = z$]

4. $\frac{\sqrt{x}}{\sqrt{a^3 - x^3}}$ [2D/28/14(i),$x = a(sin\theta)^{2/3}$], 5. $\sqrt{1 + secx} = \sqrt{\frac{1+cosx}{cos\,x}} = \sqrt{\frac{2\cos^2\frac{x}{2}}{1-2\sin^2\frac{x}{2}}} =$

$\frac{\sqrt{2}cos\frac{x}{2}}{\sqrt{1-2\sin^2\frac{x}{2}}}$; [2D/28/15,$\sqrt{2}sin\frac{x}{2} = z$]

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Chapter: 2A

Integrate the Following Expressions:

1. $\frac{1}{(a^2-x^2)^{\frac{3}{2}}}$; $[Hints: Let\ x = asin\theta\ ]$[14/6]

2. $\frac{1}{sinx+cosx}$; $[Hints: \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}(sinx+cosx)} = \frac{\frac{1}{\sqrt{2}}}{sinxcos\frac{\pi}{4}+cosxsin\frac{\pi}{4}}$

$\frac{1}{\sqrt{2}}\text{cosec}(x+\frac{\pi}{4})]$; then apply $\int cosecxdx = -\log(cosecx+cotx)$

3. $\frac{cosx}{a+b^2\sin^2 x}[bsinx = z, bcosxdx = dz\ , cosxdx = \frac{dz}{b}]$; 10/8

4. $\frac{cosx}{(a+bsinx)^2}[a+bsinx = z, bcosxdx = dz\ , cosxdx = \frac{dz}{b}]$; 11/10

5. $\frac{\cos^3 x}{\sqrt[3]{sinx}}[hints\ \frac{\text{cosxcos}^2 x}{\sqrt[3]{sinx}} = \frac{\text{cosx}\,(1-\sin^2 x)}{\sqrt[3]{sinx}}, let\ sinx = z]$; 10/9

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6. $\frac{cotx}{logsinx}\left[logsinx = z, cotxdx = dz\ \right]$; 12/13

7. $\frac{\tan^2 x sec^2 x}{1+\tan^6 x}\left[\tan^3 x = z, 3\tan^2 x sec^2 x\, dx = dz\ \right]$; 13/17

8. $\frac{x^2 \tan^{-1} x^3}{1+x^6}\left[\tan^{-1} x^3 = z, \frac{3x^2 dx}{1+x^6} = dz\ \right]$; 18/29

9. $\frac{e^x(1+x)}{\cos^2(xe^x)}\left[xe^x = z, e^x(1+x)dx = dz\ \right]$; 20/33

10. $\frac{1}{\sqrt{x}+x}\left[\sqrt{x} = z, \frac{dx}{2\sqrt{x}} = dz, dx = 2\sqrt{x}dz = 2zdz\ \right]$; 21/36

11. $\frac{sin2x}{(a+bcosx)^2} = \frac{2sinxcosx}{(a+bcosx)^2}$, $a + bcosx = z, sol: \frac{25}{43}$

12. $\frac{1}{acosx + bsinx}$, $a = csin\alpha, b = ccos\alpha$

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Chapter 2(B):

Integrate the Following Expressions:

1. $e^x cosx$; [*Hints*: *uv method*][D:16/4]

2. $\tan^{-1}\frac{3x-x^3}{1-3x^2}$; [*Hints*: $3\tan^{-1}x$, *then uv*][17/20]

3. $\sin^{-1}\frac{2x}{1+x^2}$; [*Hints*: $2\tan^{-1}x$, *then uv*][17/22(i)]

4. $\cos^{-1}\frac{1-t^2}{1+t^2}$; [*Hints*: $2\tan^{-1}t$, *then uv*][17/23]

5. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$; $[x = cos\theta\ ]$[D:17/22]

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6. $\sin^{-1}\sqrt{\frac{x}{a+x}}$; $[x = a\tan^2\theta, tan\theta = \sqrt{\frac{x}{a}}\ then\ uv\ ]$[D:17/20(i)]

7. $\frac{xsin^{-1}x}{\sqrt{1-x^2}}$; $[Hints: \sin^{-1}x = z\ , x = sinz\ then\ uv\ ]$[D:17/18]

8. $\frac{\log(logx)}{x}$; $[Hints: log\ x = z\ , then\ uv\ ]$[D:17/25]

9. $xtan^{-1}x$; $[Hints: uv\ then\ s\ ]$[D:37/14]

10. $\cot^{-1}x$; $[Hints: uv\ then\ s\ ]$[D:37/15]

11. $\frac{xsin^{-1}x}{\sqrt{1-x^2}}$ $[\sin^{-1}x = z;]$[D:39/18]

12. $\tan^{-1}(\frac{1+cos\theta}{sin\theta})$ [D:43/24]

Chapter: 2C

Pattern 1. $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

2. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}Log\frac{x-a}{x+a}$

3. $\int \frac{1}{a^2-x^2}dx = -\int \frac{1}{x^2-a^2}dx = \frac{-1}{2a}Log\frac{x-a}{x+a}$

4. $\int \frac{dx}{ax^2+bx+c}$ convert ax^2+bx+c into a complete square then treat it as 1. Or 2.

5. $\int \frac{(px+q)dx}{ax^2+bx+c} = \int \frac{\frac{p}{2a}(2ax+b)-\frac{pb}{2a}+q}{ax^2+bx+c}dx$

$= \int \frac{\frac{p}{2a}(2ax+b)}{ax^2+bx+c}dx + \frac{2qa-pb}{2a}\int \frac{dx}{ax^2+bx+c}$ then treat the 2nd integral as 4.

Examples: 1. $\frac{a^2}{x^2(x^2+a^2)}$ $\left[= \frac{x^2+a^2}{x^2(x^2+a^2)} - \frac{x^2}{x^2(x^2+a^2)} = \frac{1}{x^2} - \frac{1}{x^2+a^2}\right]$

2. $\frac{1}{x^2(x^2+3)}\left[= \frac{x^2+3}{x^2(x^2+3)} - \frac{x^2}{x^2(x^2+3)} = \frac{1}{x^2} - \frac{1}{x^2+3}\right]$

3. $\int \frac{x^2dx}{x^2-a^2} = \int \frac{(x^2-a^2)dx}{x^2-a^2} + a^2\int \frac{dx}{x^2-a^2}$ [55/19],

4. $\int \frac{dx}{x^2+7x-18}$, 5. $\int \frac{dx}{x^2+5x+10}$, 6. $\int \frac{e^xdx}{e^{2x}+2e^x+5}$, 7. $\int \frac{e^xdx}{e^{2x}-2e^x+5}$,

8. $\int \frac{e^xdx}{e^{2x}-2e^x-3}$, 9. $\int \frac{(3x+2)dx}{5x^2+2x+3}$, 10. $\int \frac{(7x-9)dx}{x^2-2x+35}$,

11. $\int \frac{(3x^2-1)dx}{x^2-3x+2}$, $\left[\frac{3(x^2-3x+2)}{x^2-3x+2} + \frac{9x-7}{x^2-3x+2}\right.$ and so on]

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2(c)

2(c) More Problems

1. $\frac{x^2}{x^2-a^2} = \frac{x^2-a^2}{x^2-a^2} + \frac{a^2}{x^2-a^2} = 1 + a^2\left(\frac{1}{x^2-a^2}\right)$ [S 5/19]

2. $\frac{3x^2-1}{x^2-3x+2} = \frac{3(x^2-3x+2)+9x-7}{x^2-3x+2} = 3 + \frac{\frac{9}{2}(2x-3)+\frac{13}{2}}{x^2-3x+2}$ [59/29]

3. $\frac{4+7x}{\sqrt{4-x^2}} = \frac{4}{\sqrt{2^2-x^2}} + \frac{7x}{\sqrt{2^2-x^2}} = \frac{4}{\sqrt{2^2-x^2}} - \frac{7}{2}\cdot\frac{-2x}{\sqrt{2^2-x^2}}$ Let $2^2-x^2=z$ [68/13]

4. $\frac{\sqrt{x}}{\sqrt{a^3-x^3}}$; Let $x = a(\sin\theta)^{2/3}$, $dx = a(\sin\theta)^{-1/3}\cos\theta\, d\theta$ [68/14(i)]

5. $\frac{1}{x^{1/2}-x^{1/4}}$; Let $x^{\frac{1}{4}} = z \Rightarrow x = z^4$, $dx = 3z^3dz$, $x^{1/2} = z^2$ [50/5(i)]

6. $\frac{\cos\theta}{4-\sin^2\theta}$; let $\sin\theta = z$ [49/4]

7. $\frac{x^2-x+1}{x^2+x+1} = \frac{x^2+x+1}{x^2+x+1} - \frac{2x}{x^2+x+1} = 1 - \frac{2x+1}{x^2+x+1} + \frac{1}{x^2+x+1}$

$= 1 - \frac{2x+1}{x^2+x+1} + \frac{1}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$ [original 21/4]

Chapter :2D

1. $coshx = \frac{e^x + e^{-x}}{2}$; $sinhx = \frac{e^x - e^{-x}}{2}$; $\cosh^2 x - \sinh^2 x = 1$

2. $\int sinhx dx = coshx$; 3. $\int coshx dx = sinhx$

1. (i) $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$

   (ii) $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$

2. $\int \tan x\, dx = -\ln|\cos x| = \ln|\sec x| + c$

3. $\int \cot x\, dx = \ln|\sin x)| + c$
   $= -\ln|\operatorname{cosec} x| + c$

4. $\int \sec x\, dx = \ln|\sec x + \tan x| + c$
   $= \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

5. $\int \operatorname{cosec} x\, dx = \ln\left|\tan\frac{x}{2}\right| + c$

6. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

7. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$; $[x > a]$

8. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$; $[a > x]$

9. $\int \sqrt{a^2 - x^2}\, dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

10. $\int \sqrt{a^2 + x^2}\, dx = \frac{x\sqrt{a^2 + x^2}}{2} + \frac{a^2}{2}\ln|x + \sqrt{a^2 + x^2}| + c$

11. $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2}\ln|x + \sqrt{x^2 - a^2}| + c$

12. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln|x + \sqrt{x^2 + a^2}| + c$

13. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + c$

Examples: 1. $\frac{1}{\sqrt{16 - 9x^2}}$, 2. $\sqrt{16 + x^2}$, 3. $\frac{5x^2}{\sqrt{1 - x^6}}$ [28/3] , 4. $\frac{x}{\sqrt{1 - x^4}}$ [28/5];

5. $x^2\sqrt{x^6 - 1}$ [28/6], 6. $\frac{e^x}{\sqrt{1 + e^{2x}}}$ [28/10], 7. $\frac{4 + 7x}{\sqrt{4 - x^2}}$ [28/13],

Chapter: 2E

1. $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ ' 2. $\int \sqrt{ax^2+bx+c}\,dx$, 3. $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$,

4. $\int (px+q)\sqrt{ax^2+bx+c}\,dx$, 5. $\int \frac{dx}{(px+q)(\sqrt{ax^2+bx+c})}$, $px+q=\frac{1}{z}$,

6. $\int \frac{dx}{(ax+b)(\sqrt{cx+d})}$, $cx+d=z^2$

1. $\frac{1}{\sqrt{3x-x^2-2}}=\frac{1}{\sqrt{\frac{1}{4}-\left(x-\frac{3}{2}\right)^2}}$, 2. $\frac{1}{\sqrt{x^2-4x+3}}$, [35/7(i)],

3. $\sqrt{4-3x-2x^2}$, [35/10], 4. $\sqrt{2ax-2x^2}$, [35/15]

4. $\int \frac{(2x+3)dx}{\sqrt{x^2+x+1}}$ [own], 4. $\int (3x+5)\sqrt{2x^2+x+2}\,dx$, [own]

5. $\int \frac{dx}{(x)(\sqrt{x^2+x+9})}$, $x=\frac{1}{z}$, $dx=\frac{-dz}{z^2}$, [35/16 or,77/16]

6. $\int \frac{dx}{(x-3)(\sqrt{x+1})}$, $x+1=z^2$, [35/17 or,77/17]