Curve

Curve are generally used on **highway and railways** where it is necessary **to change the direction** of motion.

Types of Curves In Surveying

Two primary types: Horizontal curves and Vertical curves.

Horizontal curves are used to transition smoothly between straight sections of a road or railway or other linear infrastructure.
Vertical curves, on the other hand, are employed to provide a gradual change in elevation along the alignment.

Horizontal curves:

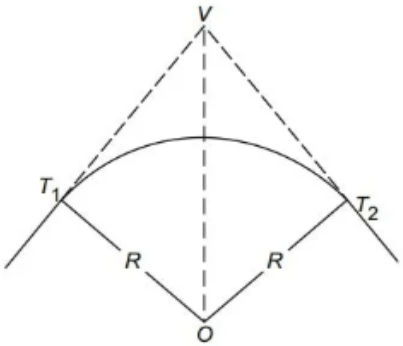
1. **Circular curve**
    i. **Simple circular curve**
    ii. **Compound curve**
    iii. **Reverse Curve**
2. **Spiral curve**
3. **Transition curve**

Vertical curves:

1. Summit curve
2. Sag curve
3. Falling curve
4. Rising curve

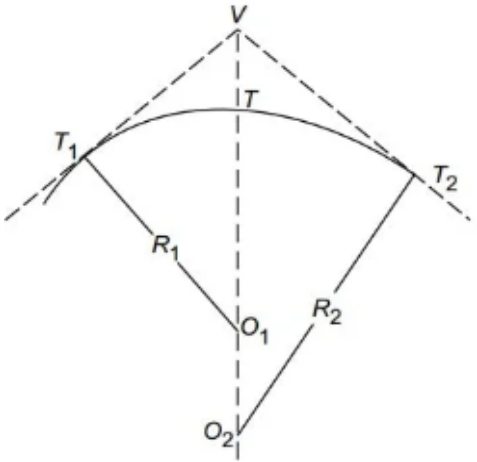
Horizontal Curve: Circular Curve

A circular curve is the most common type of horizontal curve. It follows a circular arc and is defined by its radius. It provides a constant radius throughout the curve.

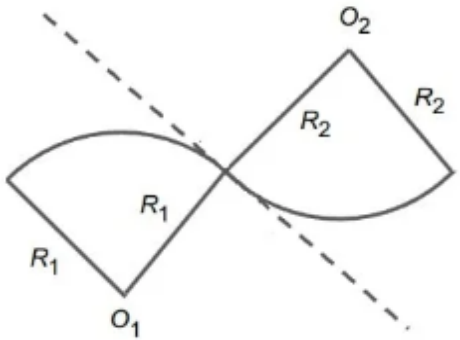


Simple Circular curve

Consists of a single arc of a circle. It is tangential to both the straight lines.



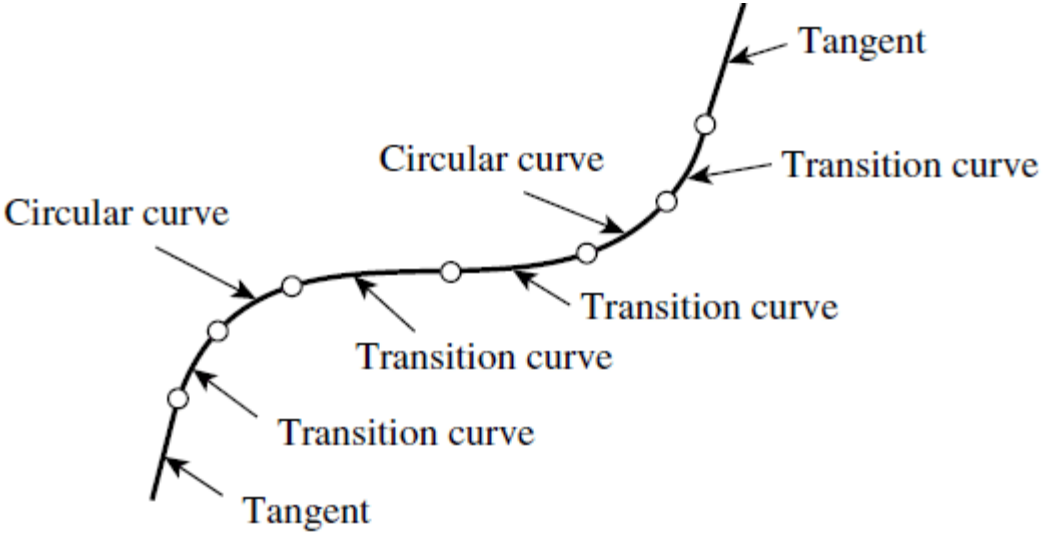
Compound curve: Consists of two or more simple arcs that turn in the same direction and join at common tangent point. Used when a more gradual transition is needed between two different alignments.

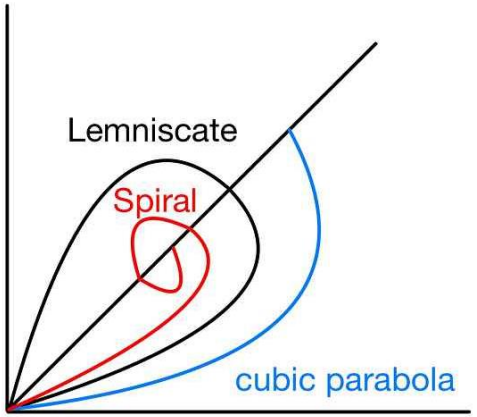


Reverse curve: Consists of two or more circular arcs of same or different radii, having their centers to the different sides of the common tangent. Used when a significant change in alignment direction is needed.

Horizontal Curve: Transition Curve

Transition Curve: A transition curve is used to provide a smooth transition between a tangent section and a circular curve.
It is often employed when a vehicle or train needs to transition from a straight path to a circular curve or vice versa.

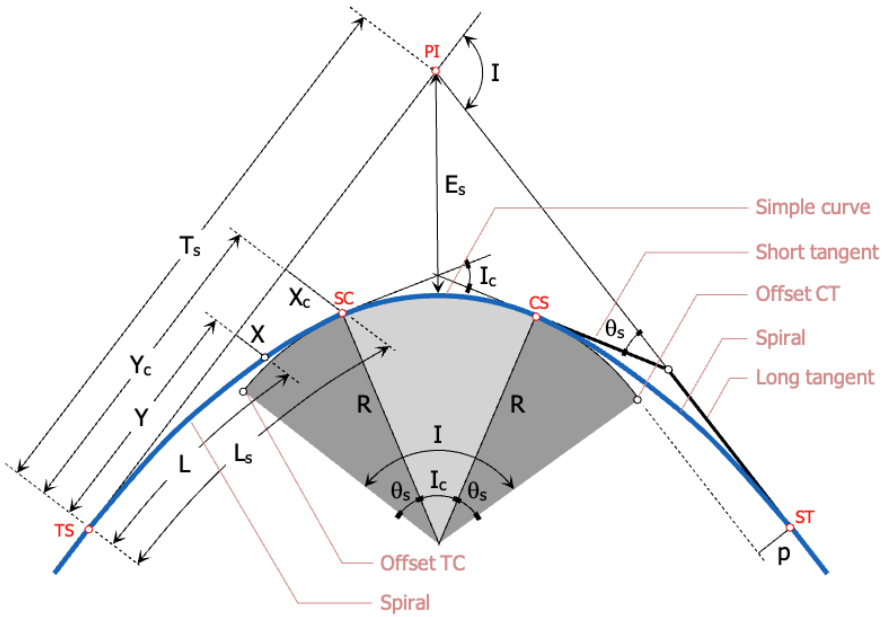




Types of Transition curve

Horizontal Curve: Spiral Curve

Spiral Curve: A spiral curve, also known as a transition curve or clothoid, is a special type of curve. It provides a smooth transition between a straight section and a circular curve.
It is not a true circular arc but follows a mathematical spiral shape.
Spiral curves are used to ensure a gradual change in curvature.
These minimize abrupt changes in centrifugal force encountered by vehicles.

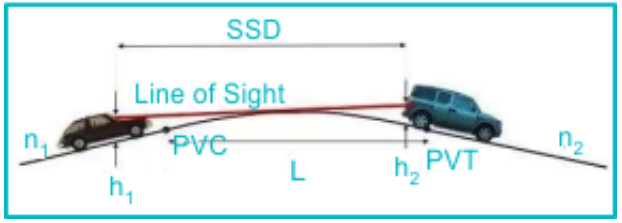


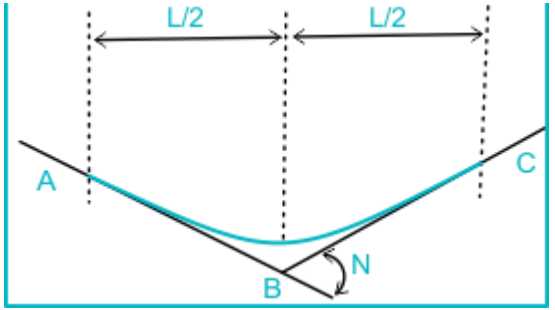
Vertical Curve

A vertical curve provides a transition between two sloped roadways, allowing a vehicle to negotiate the elevation rate change at a gradual rate rather than a sharp cut.

The design of the curve is dependent on the intended design speed for the roadway, drainage, slope, acceptable rate of change, and friction.

1. Sag curve
2. Crest or summit curve
3. Rising curve
4. Falling Curve





Importance of Curves

Curves play a vital role in surveying and civil engineering projects, particularly in the **design and construction of roads, highways, and railways**.

These ensure safe and comfortable travel by **minimizing abrupt changes in direction** along the horizontal plane **(horizontal curve) and gradient** along the vertical axis **(**vertical curve).

By understanding these curves effectively, engineers can create well-designed and efficient transportation systems.

Importance of Curves

•**Smoother Transitions/Improve safety/reduce accident:** Curves allow for smooth transitions between straight sections, reducing abrupt changes in direction and providing a more comfortable and safer path for transportation. Vertical curve provide adequate sight distance.

•**Increased network (road, railway etc) Capacity:** by allowing for smoother traffic flow and facilitating higher speeds on curved sections.

•**Improved Aesthetics:** Curves can enhance the visual appeal of roadways, railways, and other infrastructure projects.

•**Efficient Land Use:** by accommodating topographical features, minimizing the need for extensive earthwork or land acquisition.

• **Improved Drainage**

Designation of Curve

Sharpness or flatness of the curve is designated either by its **radius of curve (R)** (Great Britain) or **degree of curvature** (D) (America, Canada, India etc).

Large radius curves are relatively flat, whereas small radius curves are relatively sharp.

In most countries, two methods of defining **degree of curve** are in use:

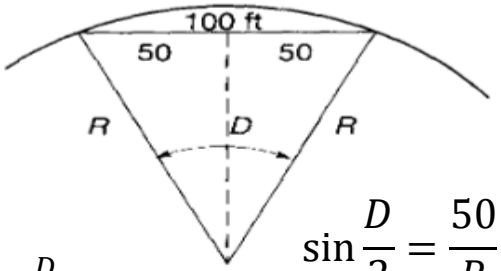
- Chord Definition
- Arc Definition

Designation of Curve

Chord Definition

Degree of curve is the central angle subtended by a chord of 100 ft (30.48 m) in length.

Used primarily for **civilian railroad construction** and **both roads and railroads for military**.



$$\sin\frac{D}{2} = \frac{50}{R}$$

If D is small, $\sin\frac{D}{2}$ **may be approx as** $= \frac{D}{2} radians = \frac{D}{2}.\frac{\pi}{180} degrees.$

$$So, \frac{D}{2}.\frac{\pi}{180} = \frac{50}{R}$$

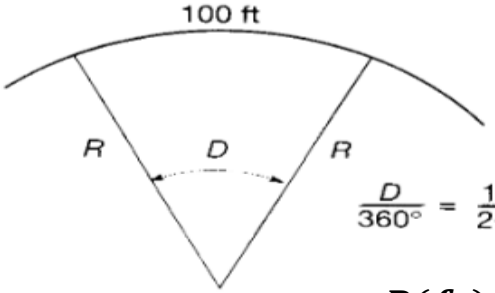
$$or, R = \frac{50 \times 2 \times 180}{\pi \times D}$$

$$\therefore \boldsymbol{R(ft) = \frac{5730}{D^\circ}}$$

Arc Definition

Degree of curve is the central angle subtended by an arc of 100 ft (30.48 m) in length.

Used primarily for highways and streets.

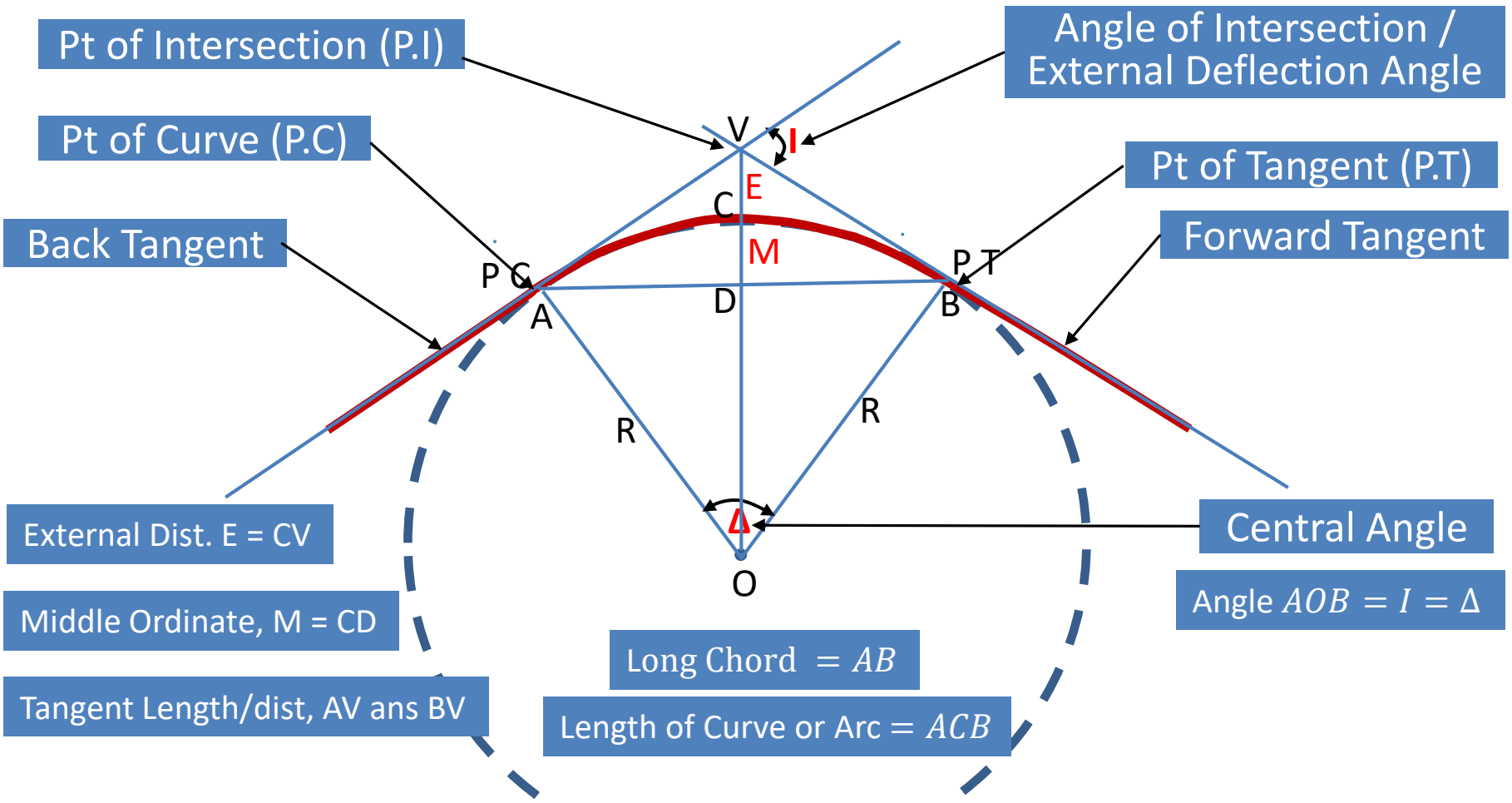


$$\frac{D}{360^\circ} = \frac{100}{2\pi R}$$

$$\therefore \boldsymbol{R(ft) = \frac{5730}{D^\circ}}$$

Metric Degree of Curve: For 20 m arc, $\mathbf{R(m) = \frac{1146}{D^\circ}}$**, and for 10 m arc,** $\mathbf{R(m) = \frac{573}{D^\circ}}$

Elements of Simple Circular Curve



Elements of Simple Circular Curve

Back Tangent/ First Tangent: Tangent previous to the curve. Straight line that connects the PC to the PI

Forward Tangent / Second Tangent: The tangent following the curve. Straight line that connects the PI to the PT

Point of Intersection (PI): at which the two tangents (back and forward tangents) to the curve intersect

Point of Curvature (PC): The beginning point of the curve. The back tangent is tangent to the curve at this point.

Point of Tangency (PT): the end point of the curve. The forward tangent is tangent to the curve at this point.

Tangent Distance (T): the distance along the tangents from the PI to the PC or PT. These distances are equal on a simple curve.

External Distance (E): the distance from the PI to the middle point of the curve. The external distance bisects the interior angle at the PI.

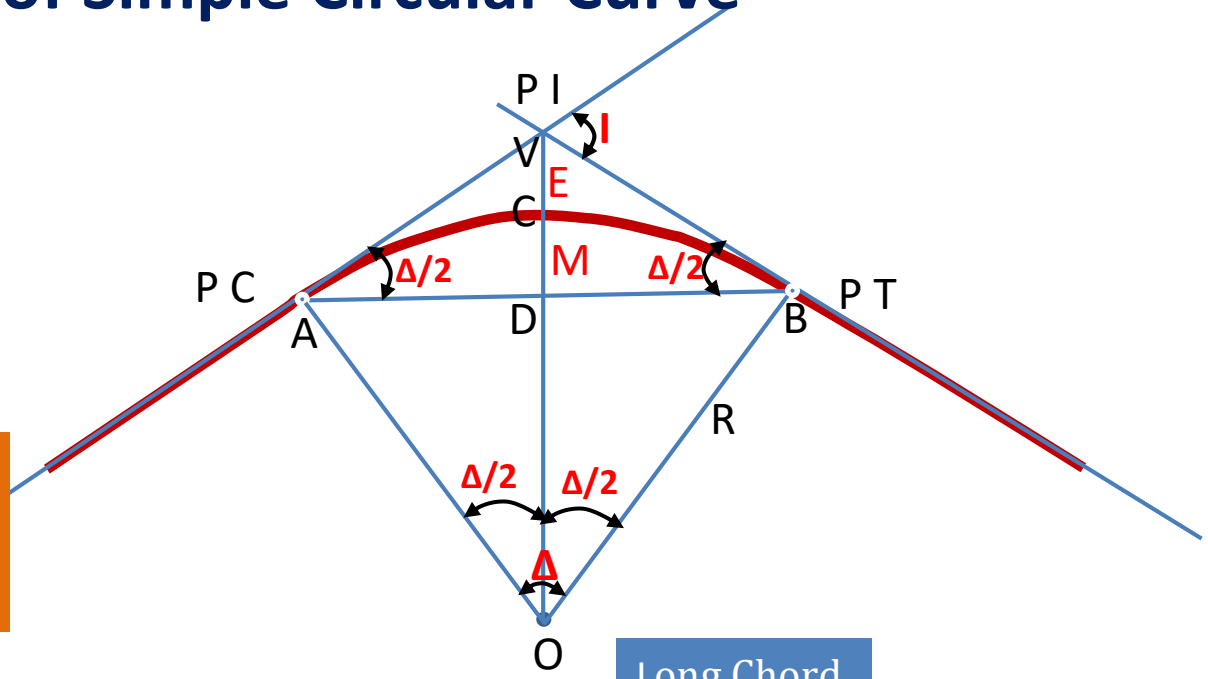
Middle Ordinate (M): the distance from the midpoint of the curve to the midpoint of the long chord. The extension of the middle ordinate bisects the central angle.

The Central Angle (Δ): the angle formed by two radii drawn from the center of the circle (O) to the PC or PT. The central angle is equal in value to the intersecting angle ($\Delta = I$).

Intersection Angle (I): The angle between produced back tangent and forward tangent at PI. Also called external deflection angle between two tangents.

Deflection Angle (Δ): The angle at PC between the back tangent and the chord from that point to PC.

Elements of Simple Circular Curve



External/Apex Dist.

E = CV = VO – CO = $R.\sec(\frac{\Delta}{2}) - R$

= $R\{\sec(\frac{\Delta}{2}) - 1\}$

Middle Ordinate

M = CD = CO – DO = $R - \sqrt{R^2 - (\frac{L}{2})^2}$

$= R - R.\cos(\frac{\Delta}{2})$ = $R\{1 - \cos(\frac{\Delta}{2})\}$

Tangent Length/dist

T = AV = BV = $R.\tan(\frac{\Delta}{2})$

Long Chord

Line joining the PC and the PT,

AB = $2.R.\sin(\frac{\Delta}{2})$

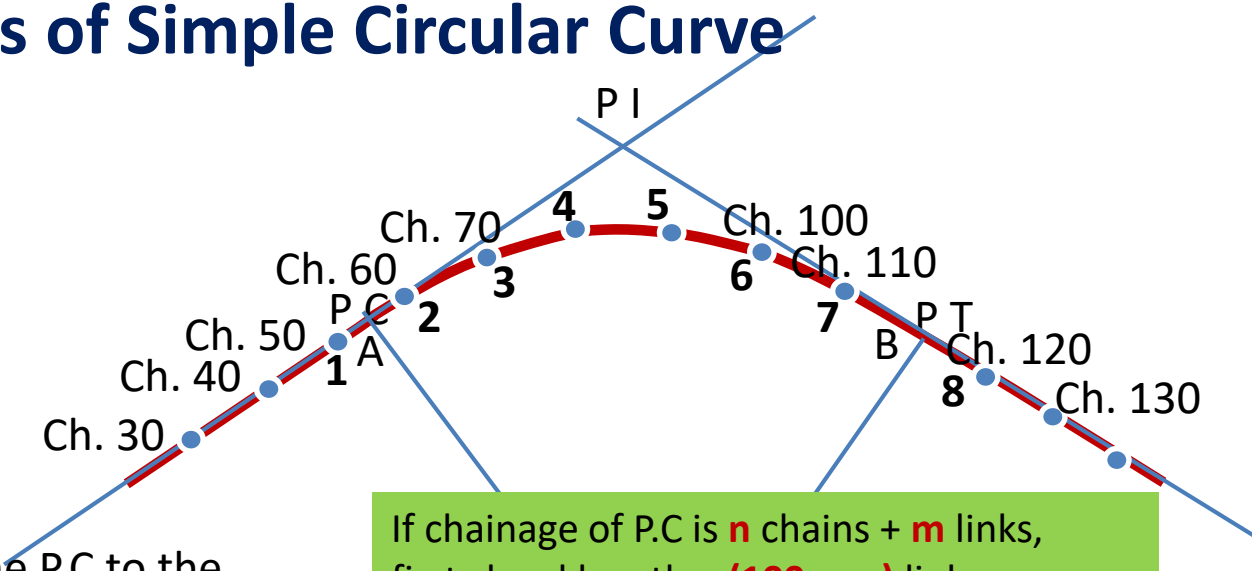
Length of Curve or Arc

Dist. from PC to PT measured along the curve, ACB = $R\Delta = \pi R\Delta/180$

Elements of Simple Circular Curve

Peg interval – distance between subsequent pegs

Full chord or normal chord -100 ft in English unit, 20m in metric unit



Sub chord - first chord joining the P.C to the first peg and the last chord joining the last peg to the P.T. All intermediate chord will be Normal Chord.

Full station - station having chainage in the multiple of chain length.
Usually Tangent points will not be full stations.

If chainage of P.C is **n** chains + **m** links,
first chord length = **(100 – m)** links.
And if chainage of P.T is **n** chains + **m** links,
last chord length = **m** links.

Length of the chord should be so selected that there is no appreciable difference between the length of the chord and arc.

If $l = \frac{1}{10}R$, error 8 mm in 20m. If $l = \frac{1}{20}R$, error is only 2 mm in 20 m.

Setting out of a Simple Circular Curve

Location of Tangents, P.I and deflection angle (Δ)

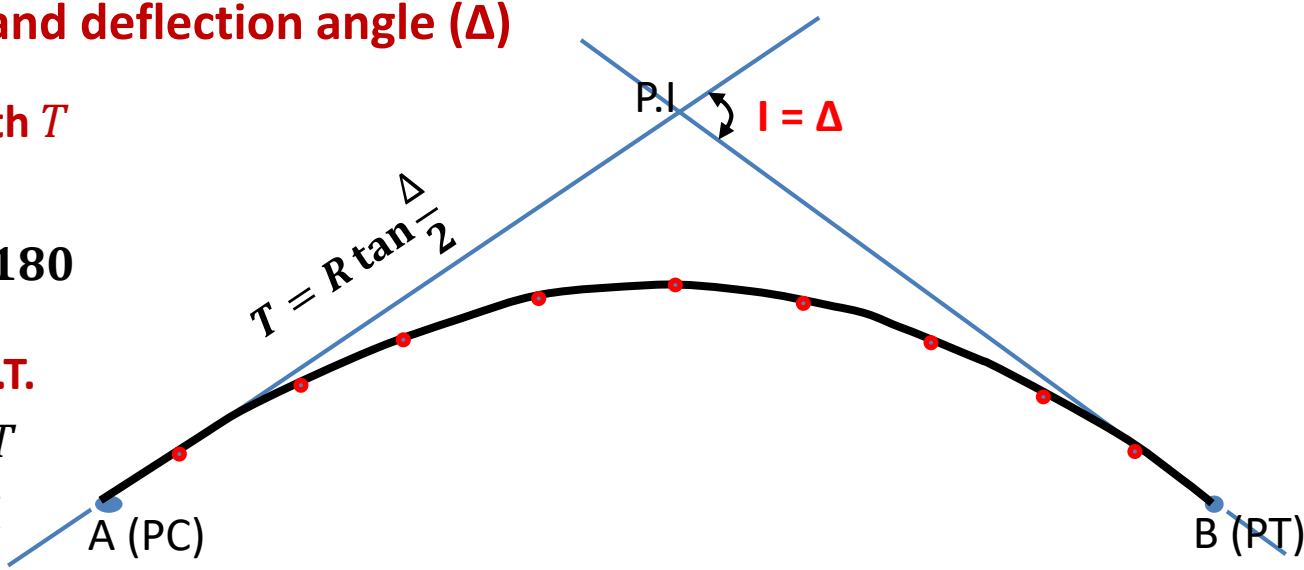
Determine the tangent length T and length of curve l

$$T = R\tan\frac{\Delta}{2} \qquad l = \pi R\Delta/180$$

Determine the ch. P.C. and P.T.

Ch. of P.C. (A) = Ch. of P.I. – T

Ch. of P.T (B) = Ch. of P.C. + l



Set the pegs on the curves. Pegs should be on regular interval.

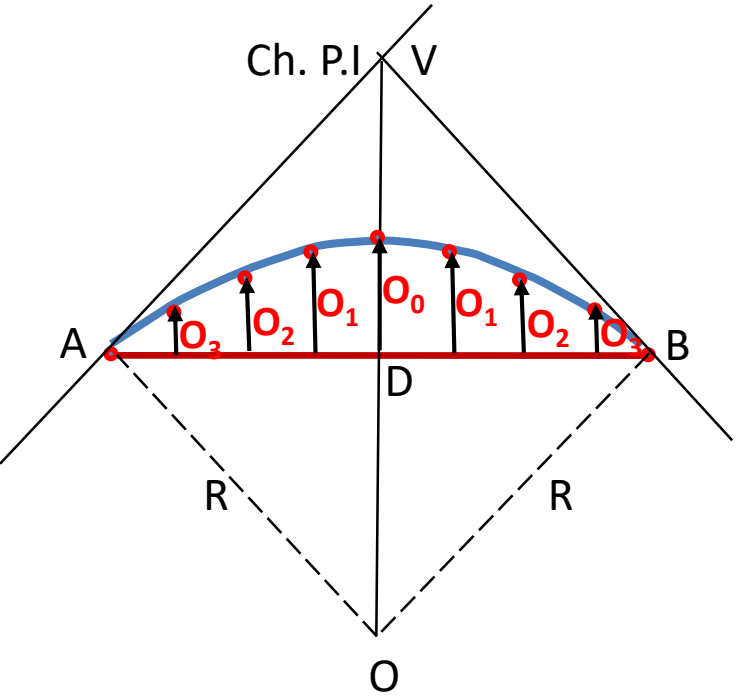
Connect the pegs and develop the curve

Method of Setting out of a Simple Circular Curve

1. **Linear method:** Only chain or tape used. Used when high degree of accuracy is not required and when the curve is short.
   a. By ordinate or offset from long chord
   b. By successive bisection of arcs or chords
   c. By offsets (radial and perpendicular)from tangents
   d. By offsets from chord produced or by deflection distances

2. **Angular Method:** Theodolite is used with or w/o chain or tape
   a. By Rankine's method of tangential / deflection angles
   b. By Two theodolite method
   c. By Tachemetric method

Setting out by Ordinate from Long Chord

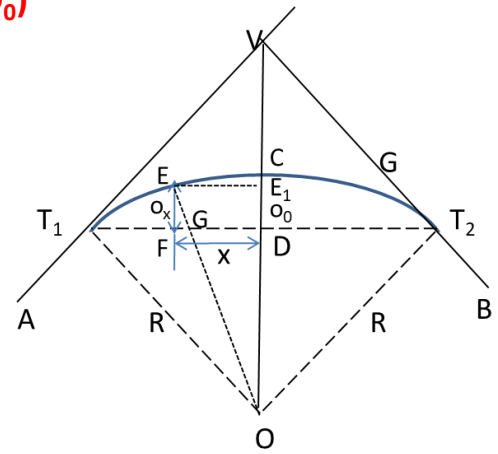


Locate mid ordinate O_0= CD = $R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$

In order to calculate ordinate O_x to any point E (at a dist. x from the mid point of the chord), draw the line EE_1, parallel to the long chord T_1T_2. Join EO to cut the long chord at G

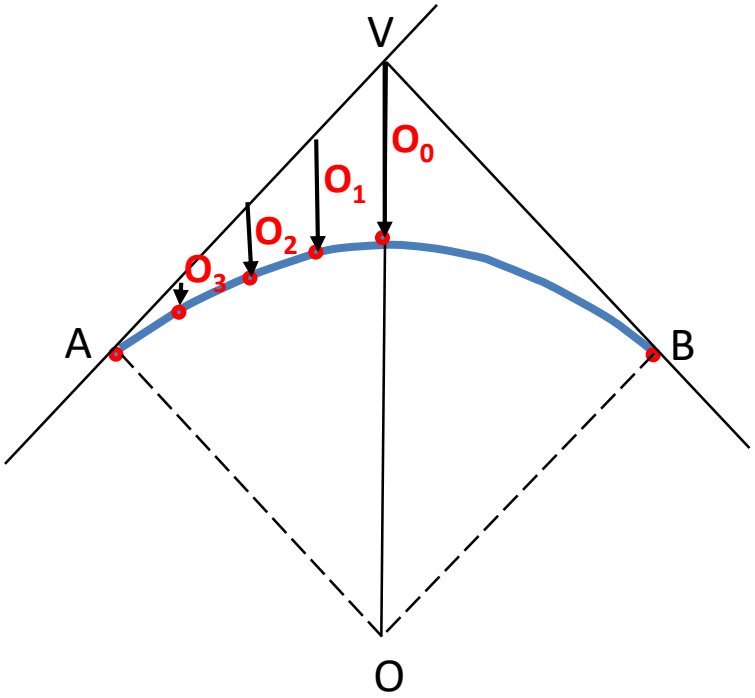
Then, O_x= EF = E_1D = E_1O - DO = $\sqrt{EO^2 - EE_1^2}$ – (CO-CD)

= $\sqrt{R^2 - X^2}$ - ($R - O_0$)

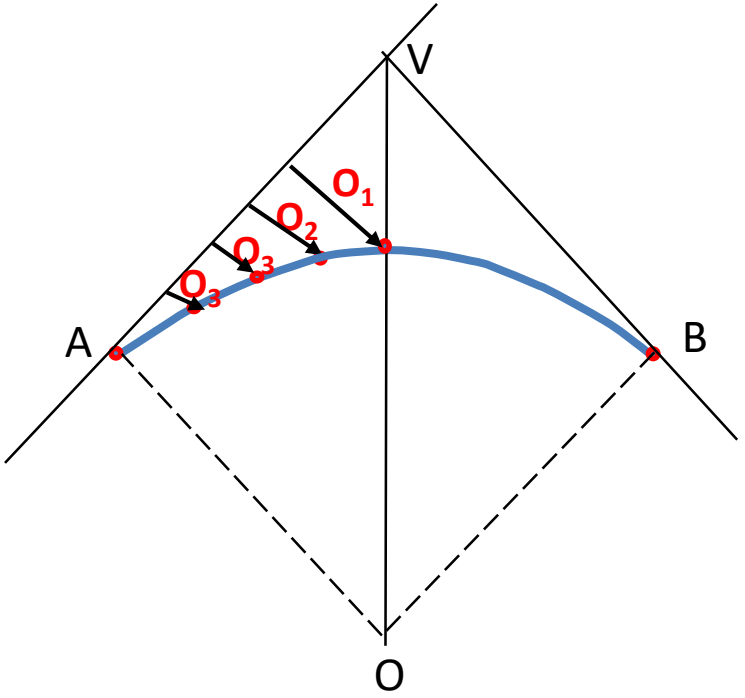


Setting out by Offsets from the Tangents

Radial Offset



Perpendicular Offset



Setting out by Offsets from the Tangents

Radial Offset

Exact Formula:

$$O_x = \sqrt{(R^2+x^2)} - R$$

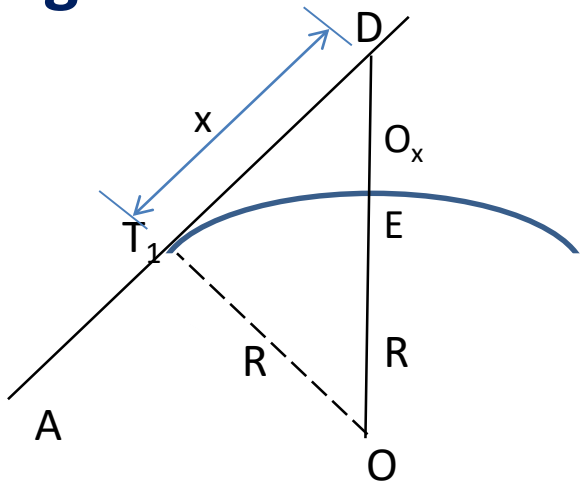
Approx.: $O_x = \frac{x^2}{2R}$

From triangle T_1DO

$DO^2 = T_1D^2+T_1O^2$

or, $(DE + EO)^2 = T_1D^2+T_1O^2$

or, $(O_x + R)^2 = R^2+x^2$



When R is large compared to L, approx. formulae may be used.

Perpendicular Offset

Exact Formula:

$$O_x = R - \sqrt{(R^2-x^2)}$$

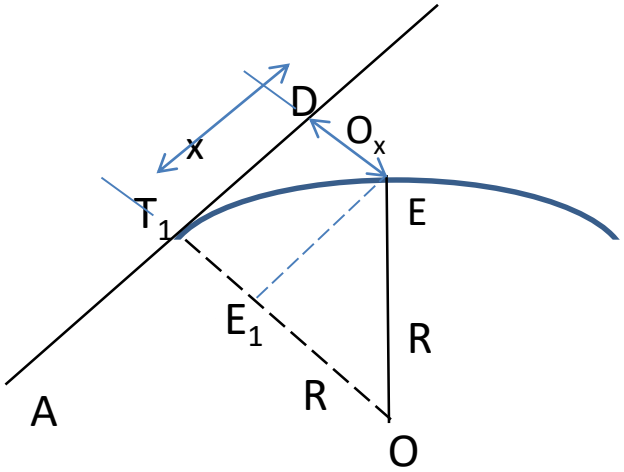
Approx.: $O_x = \frac{x^2}{2R}$

From triangle EE_1O

$E_1O^2 = EO^2 - EE_1^2$

or, $(T_1O - T_1E_1)^2 = EO^2 - EE_1^2$

or, $(R - O_x)^2 = R^2 - x^2$



Setting out by successive bisection of arcs or chords

Add PC (T_1) and PT (T_2).
And Divide the chord say at D.

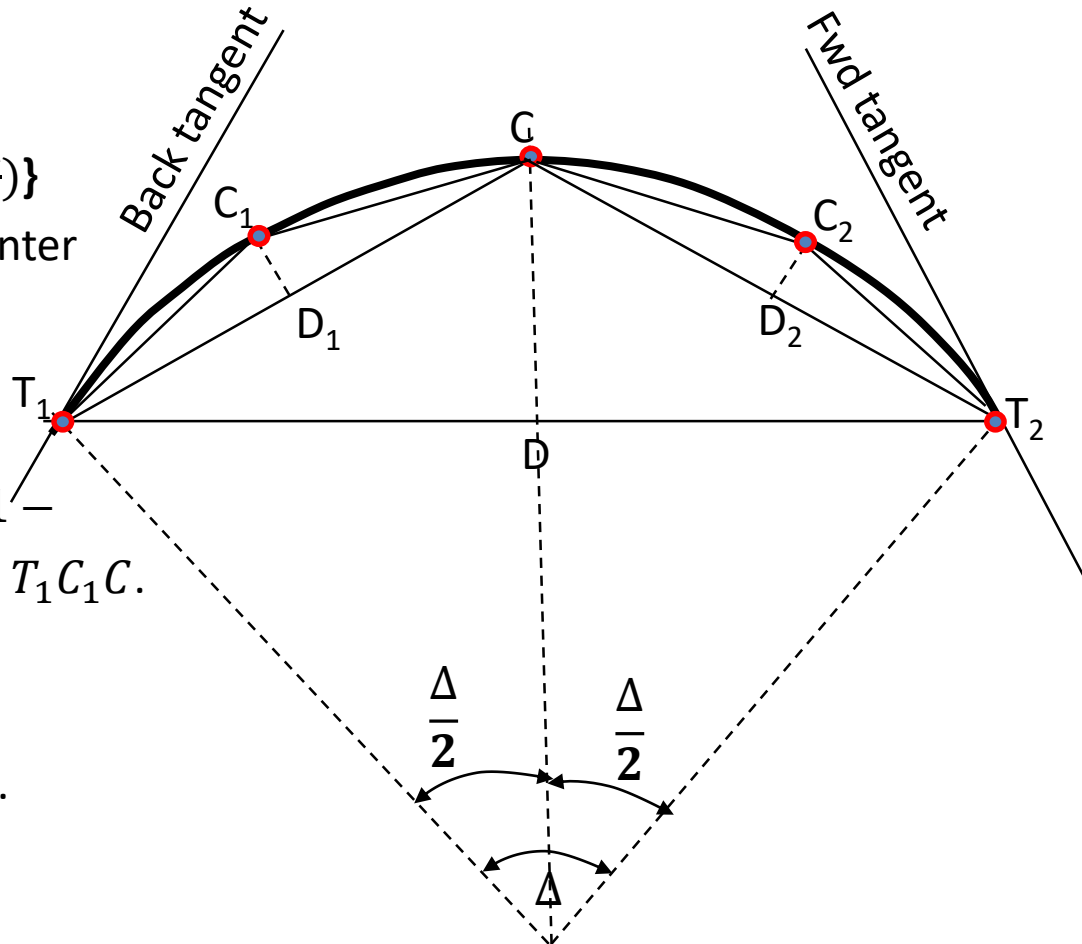
Mark C from **mid ord. CD =** $\boldsymbol{R\{1-\cos(\frac{\Delta}{2})\}}$ when Δ is the subtended angle at the center of the arc T_1CT_2.

Add T_1 and C. And divide D_1.

Then, mark C_1 **from mid ord.** $\boldsymbol{C_1D_1 = R\{1-\cos(\frac{\Delta}{4})\}}$. $\frac{\Delta}{2}$ *is* the subtended angle by arc T_1C_1C.

Similarly mark C_2 on other half.

Thus, we get the points, C_1, C_2 and so on.



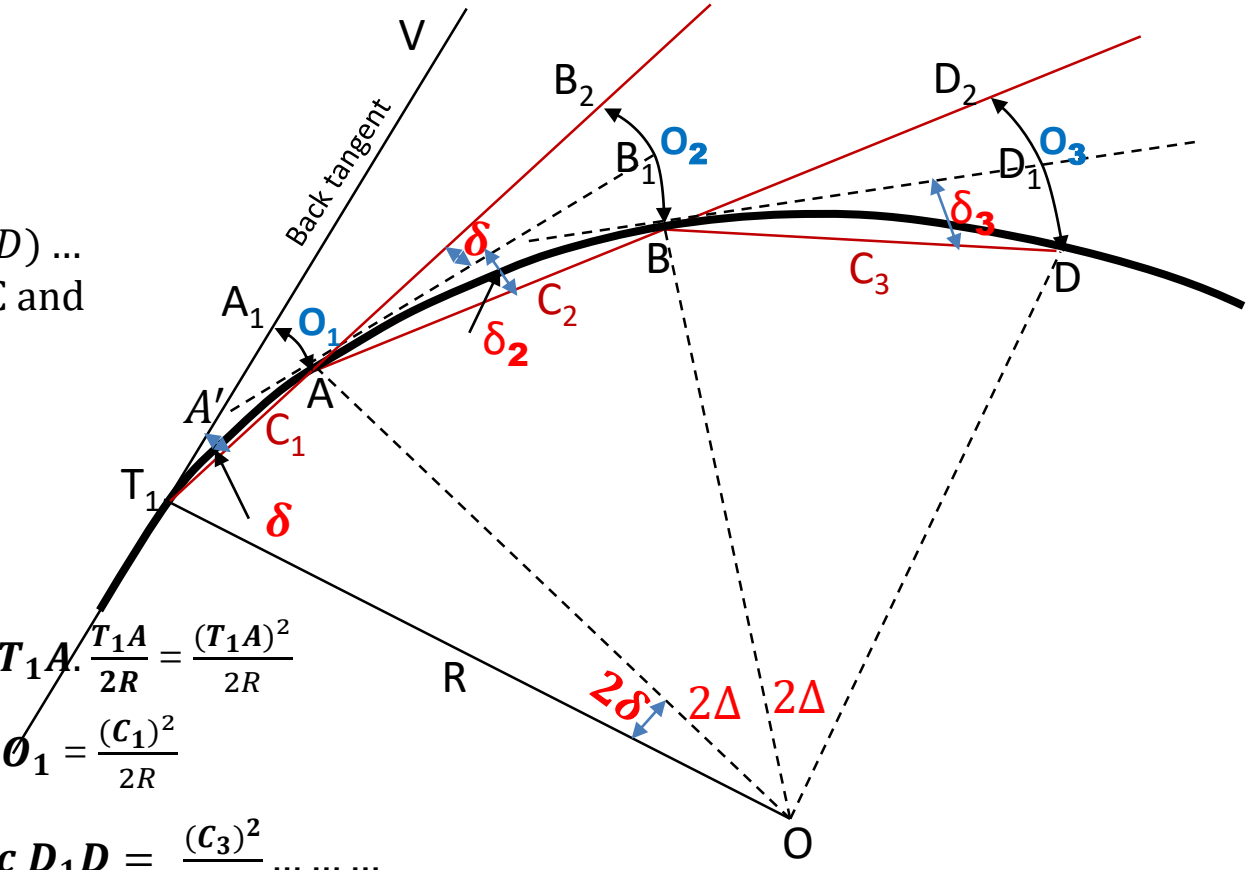
Setting out by Offsets from the Chord Produced

$T_1A = first\ subchord = C_1$
$AB(= C_2), BD(= C_3)\ are$
$2nd, 3rd\ normal\ chords$

$O_1(= A_1\ A), O_2(= B_2B), O_3(= D_1D)$ … are offsets from the tangent at PC and chords produced.

$\angle T_1OA = 2\angle A_1T_1A = 2\delta$

$T_1A = R.2\delta \qquad \therefore \delta = \frac{T_1A}{2R}$



$\boldsymbol{Now, Arc\ O_1 = A_1A = T_1A.\delta = T_1A.\frac{T_1A}{2R} = \frac{(T_1A)^2}{2R}}$

$\boldsymbol{Takig\ arc\ T_1A = chord\ T_1A,\ O_1 = \frac{(C_1)^2}{2R}}$

$\boldsymbol{Similarly,}$ arc $\boldsymbol{B_1B = \frac{(C_2)^2}{2R},\ arc\ D_1D = \frac{(C_3)^2}{2R}}$ … … …

Setting out by Offsets from the Chord Produced

$\angle B_1AB_2 = \angle A'AT_1 = \delta,$

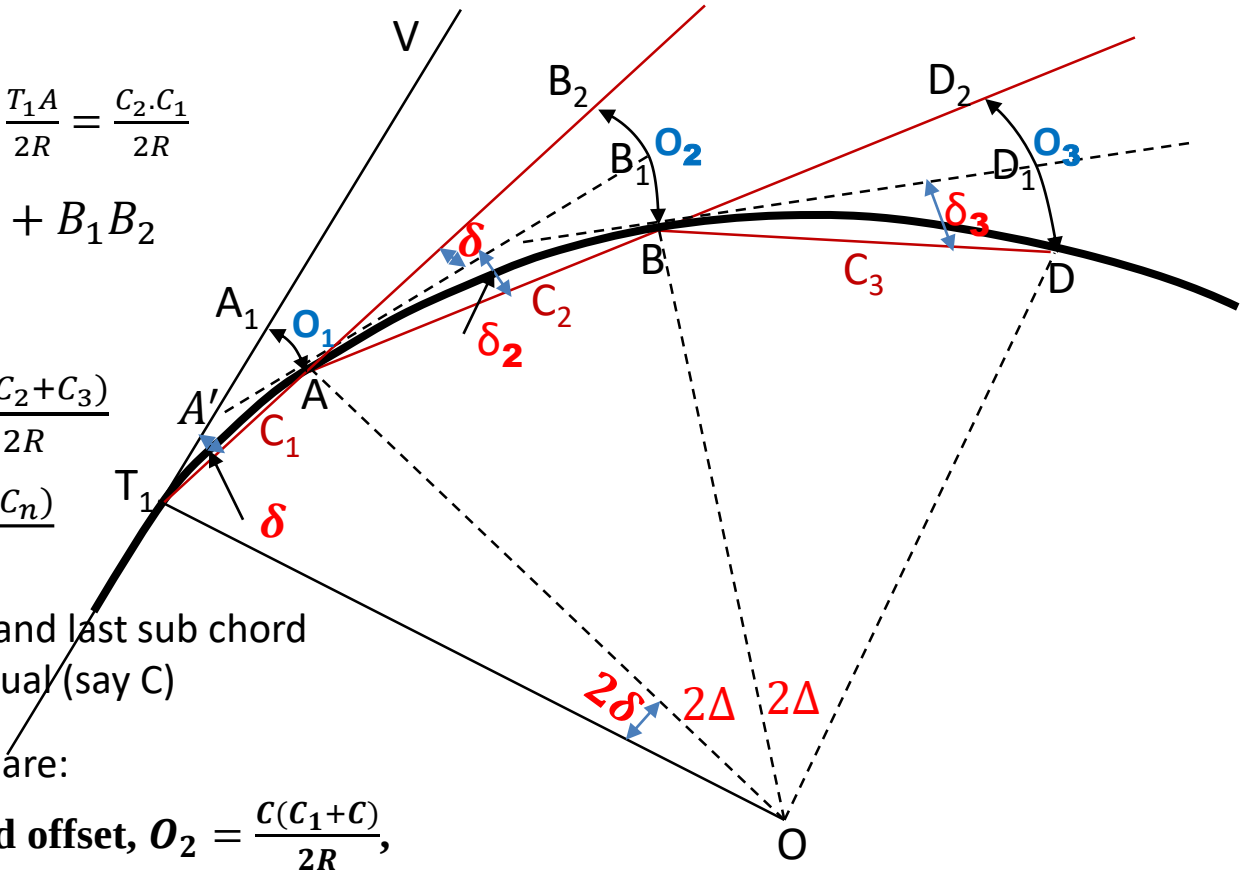
$\therefore$ arc $B_1B_2 = AB_2.\,\delta = C_2.\,\delta = C_2.\frac{T_1A}{2R} = \frac{C_2.C_1}{2R}$

Now, arc $BB_2(i.e.O_2) = BB_1 + B_1B_2$

$= \frac{(C_2)^2}{2R} + \frac{C_2.C_1}{2R} = \frac{C_2(C_1+C_2)}{2R}$

Similarly, arc $DD_2(i.e.O_3) = \frac{C_3(C_2+C_3)}{2R}$

And the last arc, $O_n = \frac{C_n(C_{n-1}+C_n)}{2R}$



But, except first sub chord (say C_1) and last sub chord (C_n), all intermediate chords are equal (say C)

Therefore formulae for the offsets are:

$\boldsymbol{First\ offset, O_1 = \frac{(C_1)^2}{2R}}$, **Second offset,** $\boldsymbol{O_2 = \frac{C(C_1+C)}{2R}}$,

Intermediate offsets, $\boldsymbol{O_3 = O_4 = \cdots \ldots\ldots O_{n-1} = \frac{C(C+C)}{2R} = \frac{C^2}{R}}$, **and last offset,** $\boldsymbol{O_n = \frac{C_n(C+C_n)}{2R}}$

Setting out by Offsets from the Chord Produced

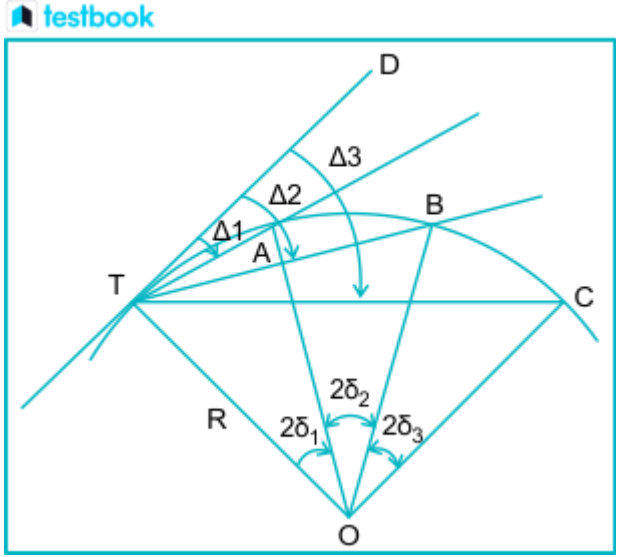
- Locate the tangent points T_1 and T_2. Fond out their chainages. Calculate length of first sub chord and decide upon other chord length.
- Hold the zero of the tape at T_1 and spread along the tangent T_1V, thus mark A_1 so that $T_1A_1 = c\ (first\ sub\ chord)$.
- Calculate O_1, O_2, O_3…….etc.
- With T_1 as center and T_1A_1 as radius, swing the tape and fix A so that $A_1A = O_1$
- Extend the tape along T_1A, mark AB_2 so that $AB_2 = \mathrm{C}$ (normal chord length). With A as center and AB_2 as radius, swing the tape and fix B so that $B_2B = O_2$.
- Spread the tape along AB and repeat the above steps and continue.
- Join the points T_1, A, B, D, …….T_2 to the curve.

Setting out by Rankine's method or Tangential method or Deflection angle method)

Rankine Method / Deflection Angles

The Rankine Method or Tangential Method, is an angular method of curve setting which utilizes a combination of measurements with a tape (or chain) and angles with a theodolite to calculate the positions of points along the curve.

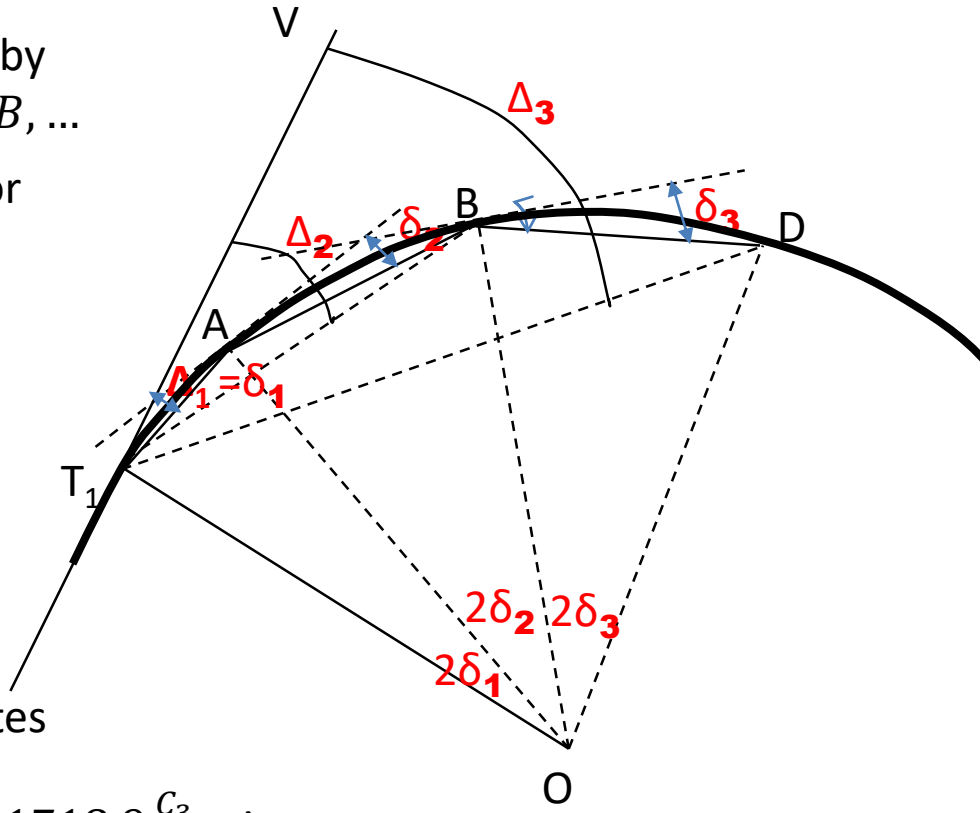
For this particular method, **the chord length should not exceed R/20.**



Setting out by Rankine's method or Tangential method or Deflection angle method)

$\delta_1, \delta_2, \delta_3$, ……are the tangential angles made by chords T_1A, AB, BC, …….at points T_1 , A, B, …

$\Delta_1, \Delta_2, \Delta_3$……are the total tangential angles or deflection angles made by chords T_1A, T_1B, T_1C, …….with the back tangent T_1V



$$\frac{\angle T_1OA}{C_1} = \frac{180°}{\pi R}, \qquad \therefore \angle T_1OA = \frac{180°C_1}{\pi R}$$

Again, $\angle T_1OA = 2\angle VT_1A = 2\delta_1$

$$\therefore 2\delta_1 = \frac{180°C_1}{\pi R}$$

$$or, \delta_1 = \frac{90°C_1}{\pi R} = \frac{90\times60C_1}{\pi R} = 1718.9\frac{C_1}{R} \text{ minutes}$$

$$Similarly, \delta_2 = 1718.9\frac{C_2}{R} \text{ minutes}, \quad \delta_3 = 1718.9\frac{C_3}{R} \text{ minutes}$$

Setting out by Rankine's method or Tangential method or Deflection angle method)

$In\ general, \delta = 1718.9\frac{C}{R}$ minutes

Again, $\angle AOB = 2\delta_2, \therefore\ \angle AT_1B = \delta_2$

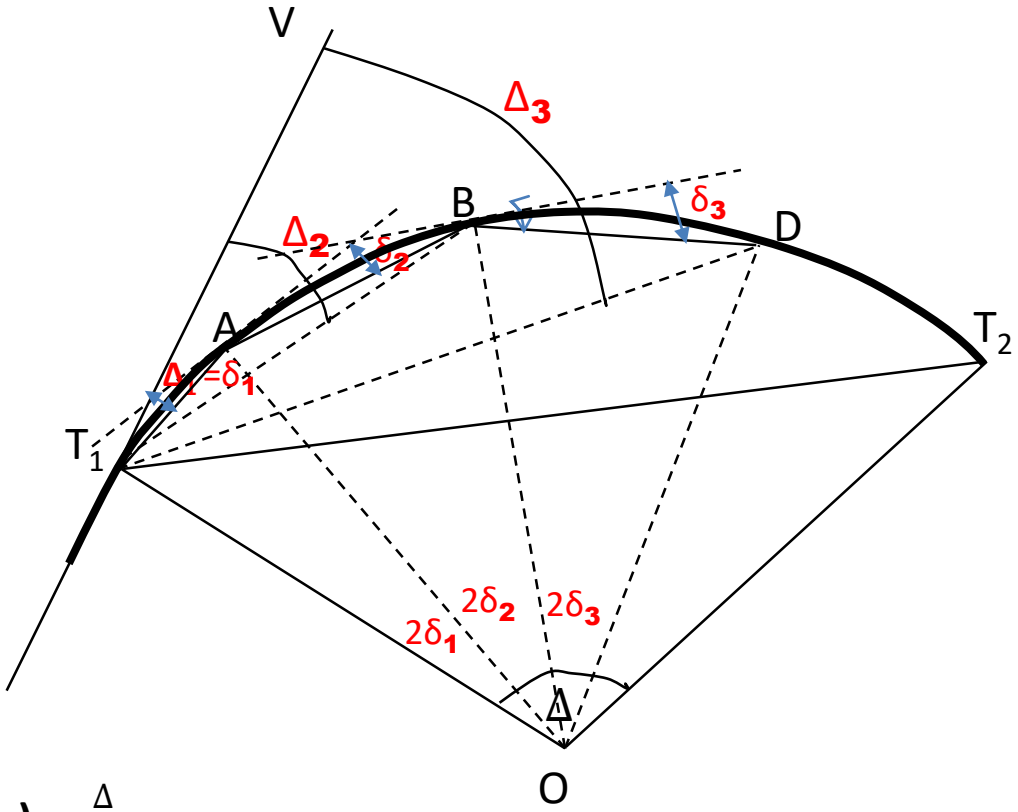
Now, $\Delta_2 = \angle VT_1B = \angle VT_1A + \angle AT_1B$
$= \Delta_1 + \delta_2 = \delta_1 + \delta_2$

Similarly, $\Delta_3 = \Delta_2 + \delta_3 = \delta_1 + \delta_2 + \delta_3$

$\Delta_4 = \Delta_3 + \delta_4 = \delta_1 + \delta_2 + \delta_3 + \delta_4$

$\Delta_n = \Delta_{n-1} + \delta_n$

Hence, deflection angle of any chord = deflection angle of the previous chord + tangential angle of that chord.

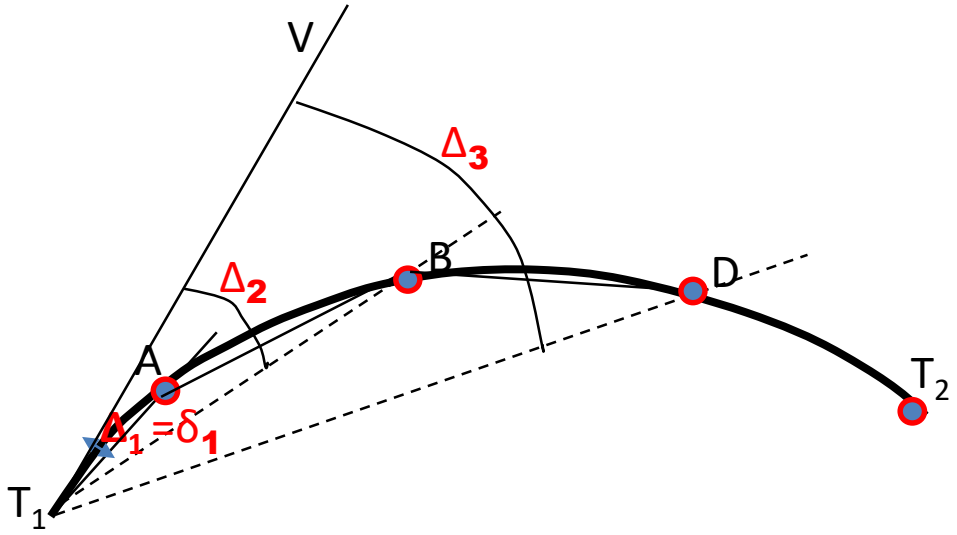


Check: Deflection angle of long chord

$\angle VT_1T_2 =$ ½(angle of intersection) $= \frac{\Delta}{2}$

Setting out by Rankine's method or Tangential method or Deflection angle method)

- Set the Theodolite at T_1. Focus towards $PI\ i.e. V$, thus align with back tangent
- Release the Vernier and set angle $\Delta_1 = \delta_1$. Thus line of sight is directed to A.
- Mark the point A by swing the tape around T_1 so that $T_1A = c$ (first sub chord).
- Set the second deflection angle Δ_2 on the vernier. Focus along T_1B.
- Now swing the tape around A and mark the point B so that $AB = C$ (normal chord)
- Set the Δ_3 on the vernier. Focus along T_1D.
- Now swing the tape around B and mark D when $BD = C$
- Repeat the above steps until the point T_2 is reached.
- Join the points $T_1, A, B, C, \ldots\ldots T_2$ to the curve.



Example

Prob: For a circular curve having radius R 250m,500m,and 1200m, what is their degree of curve for 20 m and 10 m arc/chord?

For 20 m

$$D = \frac{1146}{\mathrm{R}} = \frac{1146}{250} = 4.6\,m$$

$$D = \frac{1146}{\mathrm{R}} = \frac{1146}{500} = 2.3\,m$$

For 10 m

$$D = \frac{573}{\mathrm{R}} = \frac{573}{250} = 9.2\,m$$

$$D = \frac{573}{\mathrm{R}} = \frac{573}{500} = 4.6\,m$$

Example

Prob: Calculate the ordinates from chord at 25 ft intervals for a circular curve having a long chord of 200 ft and a versed sine (Mid ordinate) of 10 ft.

Sol: Given, $O_0 = 10\ ft.$ We know, $O_0 = R - \sqrt{R^2 - (\frac{L}{2})^2}$

or, $10 = R - \sqrt{R^2 - (100)^2}$ or, $R^2 - (100)^2 = (R - 10)^2$ or, $\mathrm{R} = 495\ ft$

Half of the long chord = 100 ft, So ordinates in each side of mid ordinate will be at 25 ft, 50 ft, 75 ft and 100 ft.

$$o_x = \sqrt{(R^2 - x^2)} - (R - O_0)$$

At $x = 25\ ft,$ $o_1 = \sqrt{(495^2 - 25^2)} - (495 - 10) = 9.4\ ft$

At $x = 50\ ft,$ $o_2 = \sqrt{(495^2 - 50^2)} - (495 - 10) = 7.5\ ft$

At $x = 75\ ft,$ $o_2 = \sqrt{(495^2 - 75^2)} - (495 - 10) = 4.3\ ft$

At $x = 75\ ft,$ $o_2 = \sqrt{(495^2 - 100^2)} - (495 - 10) = 0\ ft$

Example

Prob: determine the offset to be set out at half chain interval along the tangent to locate a 16 –chain curve, the length of each chain being 20m.

Sol:

Radial Offset: $o_x = \sqrt{(R^2 + x^2)} - R$

Or, Perpendicular Offset: $o_x = R - \sqrt{(R^2 - x^2)}$

Or, Approximate Offset: $o_x = \frac{x^2}{2R}$

Note: x is measured from the P.C

Example

Prob: Two tangents **intersect at chainage 59+60**, the **deflection angle being 50°30'**. Calculate the necessary data for setting out a curve of **15 chains radius** to connect the two tangents if it is intended to set out the curve **by offset from chords**. Take **peg interval equal to 100 links**, length of the **chain 20m (100 links).**

Determine Tangent Length (T)

Determine Length of curve (l)

Obtain P.C (Chainage of P.I – Tangent length T)

Obtain P.T (Chainage of P.C + length of Curve l)

Determine Chainage of each peg: Offset from Chord produced:

$$o_n = \frac{C_n\,(C_{n-1} + C_n)}{2R}$$

This can also be determined from long chord by knowing the long chord

Solution

Determine Tangent Length (T)

$$T = R\tan\frac{\Delta}{2} = 15 \times \tan\frac{50°30'}{2} = 7.074\ chains \times 20 = 141.48\ m$$

Determine Length of curve (l)

$$l = \frac{\pi R\Delta}{180°} = \frac{\pi \times 15 \times 50°30'}{180°} = 13.221\ chains = 264.42\ m$$

Obtain ch. of P.C = Ch. of P.I – Tangent length T

$$= 59\ chain + 60\ links - 7.074\ chains = 1192\ m\ - 141.48\ m = 1050.52\ m$$

Obtain ch. of P.T = Ch. of P.C + length of Curve l)

$$= 1050.52 + 264.42 = 1314.94\ m$$

Solution

The ch of each peg will be multiple of one chain=20 m = 100 links

Length of first sub chord = 1050.52 m to next 20 m round figure i.e. 1060 m

$= 1060.00 - 1050.52 = 9.48\ \text{m}$

Length of last sub chord $= 1314.94 - 1300.00 = 14.94\ \text{m}$

Except two sub chords, rest length of the curve $= 1300 - 1060 = 240\ \text{m}$

So, number of full chords $= \frac{240}{20} = 12$

$First\ offset, O_1 = \frac{(C_1)^2}{2R} = \frac{(9.48)^2}{2\times 300} = 0.15\ m$

Second offset, $O_2 = \frac{C(C_1+C)}{2R} = \frac{20(9.48+20)}{2\times 300} = 0.98\ m,$

Intermediate offsets, $O_3 = O_4 \ldots\ldots = O_{13} = \frac{C^2}{R} = \frac{(20)^2}{300} = 1.33\ m$

Last offset, $O_n = \frac{C_n(C+C_n)}{2R}$ $\quad O_{14} = \frac{C_{14}(C+C_{14})}{2R} = \frac{14.94(20+14.94)}{2\times 300} = 0.87\ m$

Example

Prob: Solve the previous problem by Rankine's method of Tangential angle. The theodolite has a least count of 20". Tabulate the actual theodolite readings of deflection angles to be set out.

Determine all data as previous.

Calculate deflection angles

$\delta = 1718.9 \frac{c}{R}$min.

$\Delta_1 = \delta_1, \Delta_2 = \Delta_1 + \delta_2, \Delta_3 = \Delta_2 + \delta_3$, and so on.

Check: $\Delta_n = \frac{\Delta}{2}$

Set pegs at each deflection angles fixed by Theodolite

Setting out by Rankine's method)

$$\delta = 1718.9\frac{c}{R}\text{min} = 1718.9.\frac{9.48}{300} = 54.32' = 54'19".$$

$$\delta_2 = \delta_3 = \cdots \ldots \ldots \ldots = \delta_{13} = 1718.9\frac{20}{R}\text{min} = 114.593' = 1°54'35.6".$$

$$\delta_{14} = 1718.9\frac{14.94}{R}\text{min} = 85.592' = 1°25'35".$$

	Deflection angle			Actual Theodolite Readings			Remarks
	°	'	"	°	'	"	
$\Delta_1 = \delta_1$	00	54	19.0	00	54	20	
$\Delta_2 = \Delta_1 + \delta_2$	02	48	54.6	02	49	00	
$\Delta_3 = \Delta_2 + \delta_3$	04	43	30.2	04	43	40	
$\Delta_4 = \Delta_3 + \delta_4$	06	38	5.8	06	38	00	
$\Delta_5 = \Delta_4 + \delta_5$	08	32	41.4	08	32	40	

Setting out by Rankine's method)

$\delta_2 = \cdots \ldots . = \delta_{13} = 1°54'35.6".$ $\delta_{14} = 1°25'35".$

	Deflection angle			Actual Theodolite Readings			Remarks
	°	′	″	°	′	″	
$\Delta_6 = \Delta_5 + \delta_6$	10	27	17.0	10	27	20	
$\Delta_7 = \Delta_5 + \delta_7$	12	21	52.6	12	22	00	
$\Delta_8 = \Delta_7 + \delta_8$	14	16	28.2	14	16	20	
$\Delta_9 = \Delta_8 + \delta_9$	16	11	3.8	16	11	00	
$\Delta_{10} = \Delta_9 + \delta_{10}$	18	05	39.4	18	05	40	
$\Delta_{11} = \Delta_{10} + \delta_{11}$	20	00	15	20	00	20	
$\Delta_{12} = \Delta_{11} + \delta_{12}$	21	54	50.6	21	55	00	
$\Delta_{13} = \Delta_{12} + \delta_{13}$	23	49	26.2	23	49	20	
$\Delta_{14} = \Delta_{13} + \delta_{14}$	25	15	1.2	25	15	00	

Setting out by Rankine's method)

	Deflection angle			Actual Theodolite Readings			Remarks
	°	′	″	°	′	″	
$\Delta_{11} = \Delta_{10} + \delta_{11}$	20	00	15	20	00	20	
$\Delta_{12} = \Delta_{11} + \delta_{12}$	21	54	50.6	21	55	00	
$\Delta_{13} = \Delta_{12} + \delta_{13}$	23	49	26.2	23	49	20	
$\Delta_{14} = \Delta_{13} + \delta_{14}$	25	15	1.2	25	15	00	

Check: $\Delta_{14}(25°\mathbf{15'}) = \frac{\mathbf{1}}{\mathbf{2}}\Delta\,(50°\mathbf{30'}),$

Problems Faced in setting out a Simple circular Curve

- The point of intersection (P.I) may be inaccessible.
- The point of curve (P.C) may be inaccessible
- The point of tangent (P.T) may be inaccessible
- Both tangent points (P.C and P.T) may be inaccessible.
- When P.C and P.I are inaccessible
- It may not be possible to set out the full curve from one point.
- There may be an obstacle across the curve.

Problems Faced in setting out a Simple circular Curve

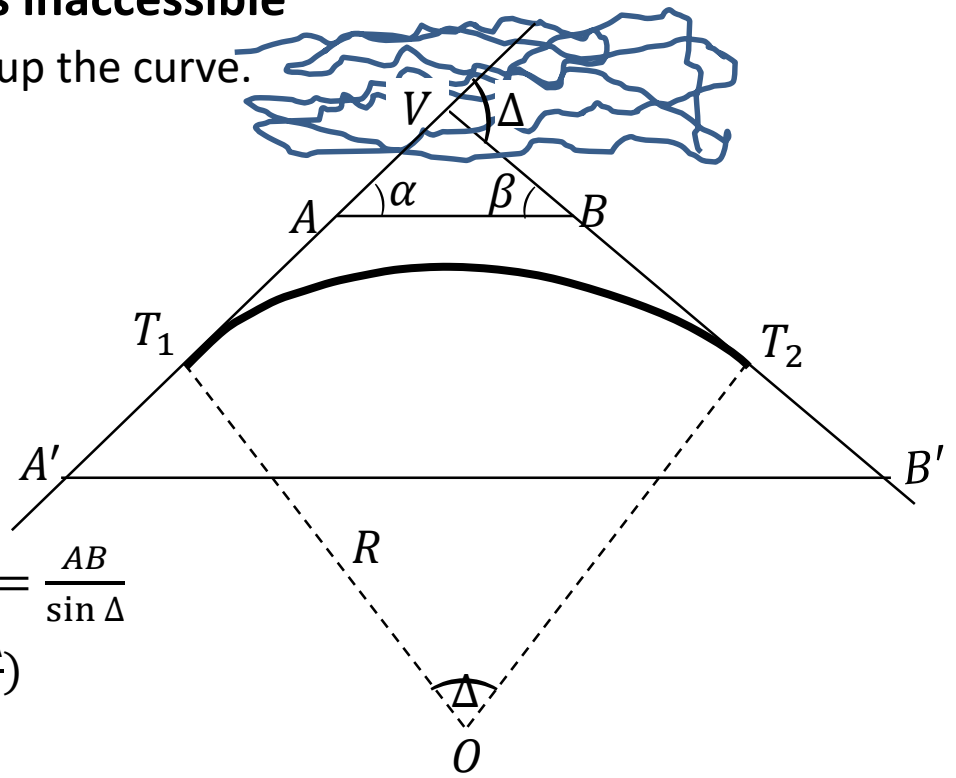
When the P.I is inaccessible

Need to know P.I, its chainage, Δ and R to set up the curve.

Select two intervisible point A and B on the tangents on fairly level ground.

Place the theodolite at A, align with AT_1, then transit the machine (180 turn) and measure the angle .

Similarly, measure β. $Get, \Delta = \alpha + \beta$



Find distances $BV\ and\ AV$, from $\frac{BV}{\sin\alpha} = \frac{AV}{\sin\beta} = \frac{AB}{\sin\Delta}$

Obtain tangent lengths, $VT_1 = VT_2 = R\tan(\frac{\Delta}{2})$

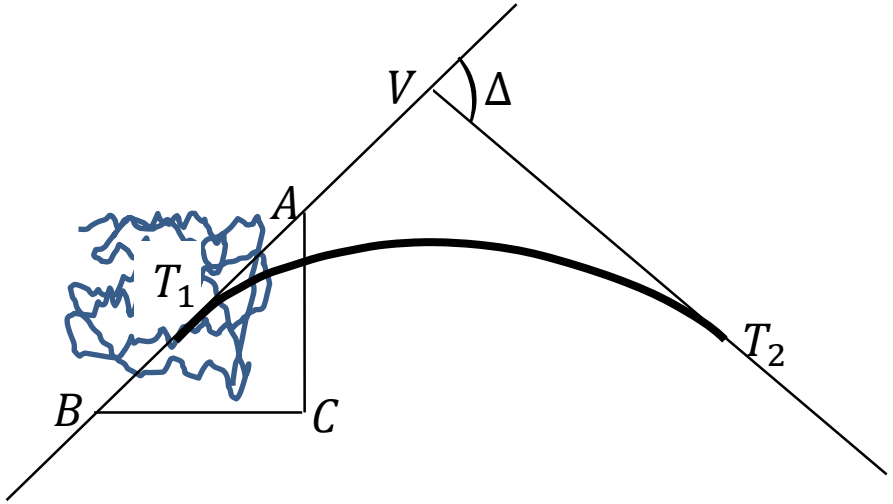
Now get the tangent points $T_1 and\ T_2$.

$$AT_1 = VT_1 - AV \qquad BT_2 = VT_2 - BV$$

If the ground toward P.I is unsuitable to measure, then take the points on other side of P.I

Problems Faced in setting out a Simple circular Curve

When the P.C is inaccessible



Chainage of P.I (i.e. V) is known.

Obtain $VT_1 = R\tan(\frac{\Delta}{2})$

Measure distances AV.

$AT_1 = VT_1 - AV$

Measure distances AB.

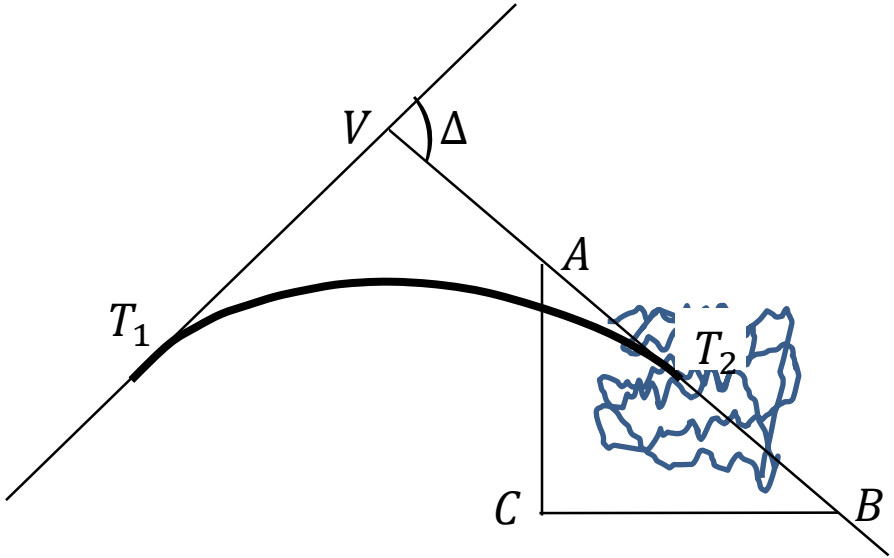
Ch. of T_1 = Ch. of B + AB - AT_1

Ch. of point T_2 = Ch. of $T_1 + length\ of\ curve(l)$

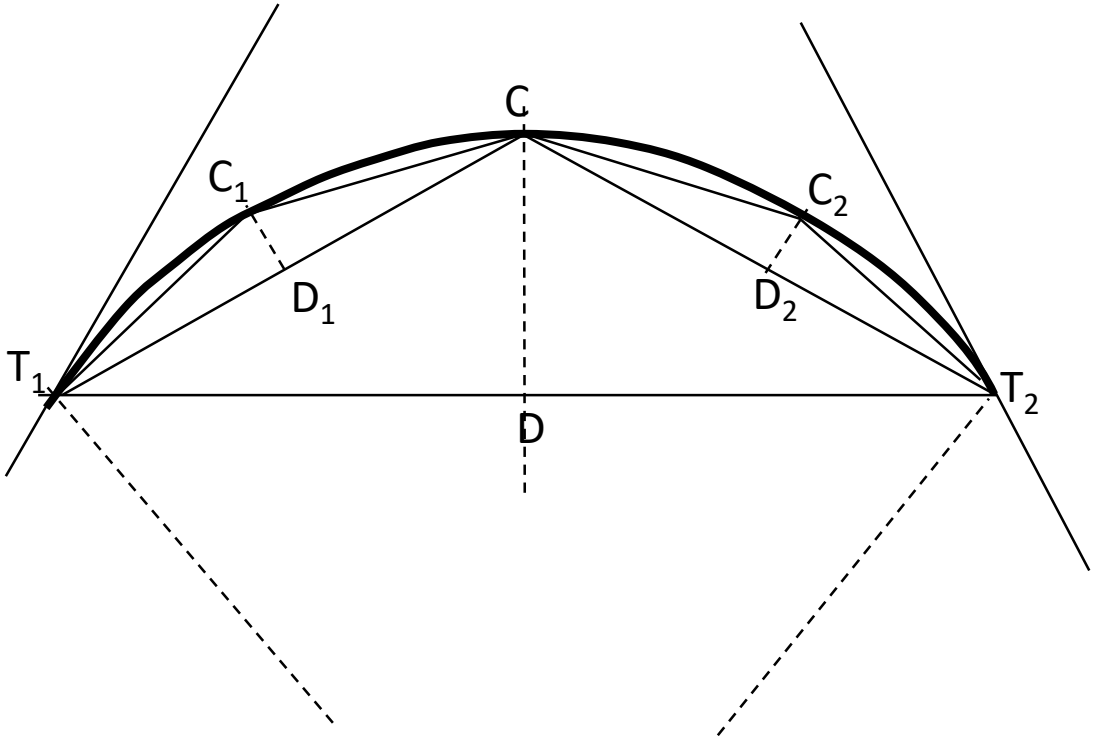
Now set the curve from reverse direction

Problems Faced in setting out a Simple circular Curve

When the P.T is inaccessible



Problems Faced in setting out a Simple circular Curve

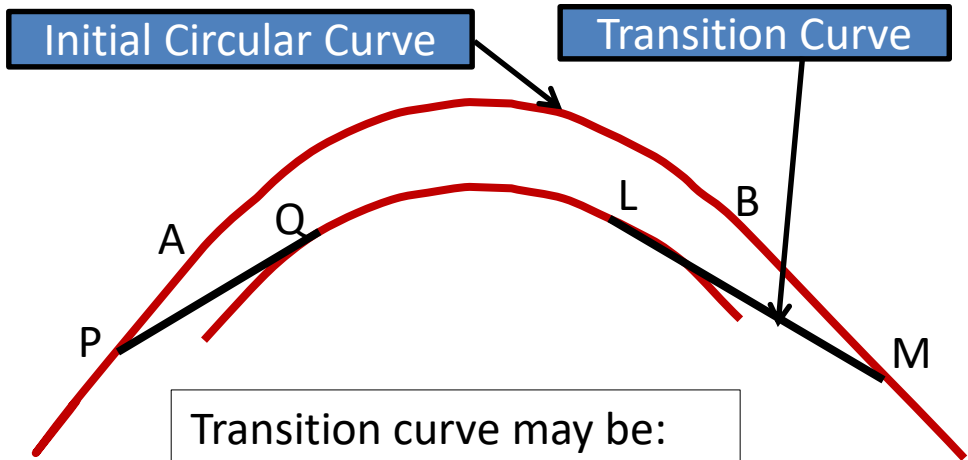


Transition Curve

A transition or easement curve is a curve of varying radius introduced between a straight or a circular curve or between two branches of compound curve or reverse curve

Functions:

- To accomplish gradually the transition from the tangent to circular curve (to increase or decrease curvature gradually)
- To provide required super-elevation



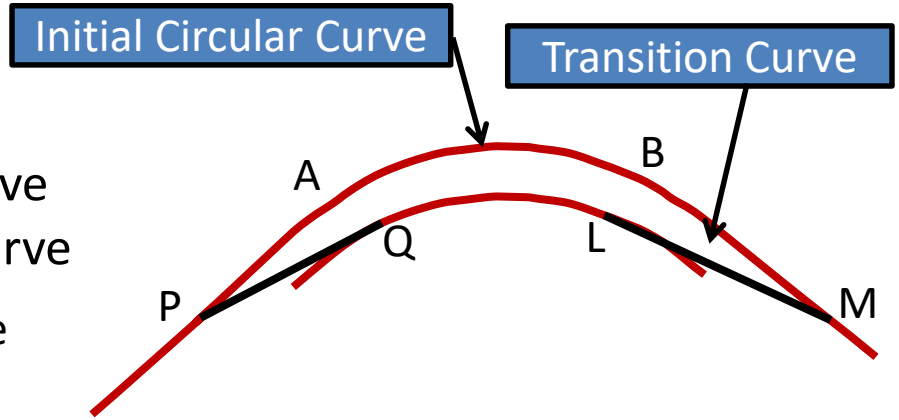
Transition curve may be:

- Clothoid or True spiral
- Cubic Spiral
- Cubic parabola

Transition Curve

Conditions to be fulfilled by a transition

1. Should be tangential to the straight
2. Should meet the circular curve tangentially
3. Curvature should be zero at origin on straight
4. Curvature at junction with circular curve should be the same that of the circular curve
5. The rate of increase of curvature along the transition should be the same as that of the increase of cant or super elevation



6. Its length should be such that full cant or super elevation is attained at the junction with the circular curve

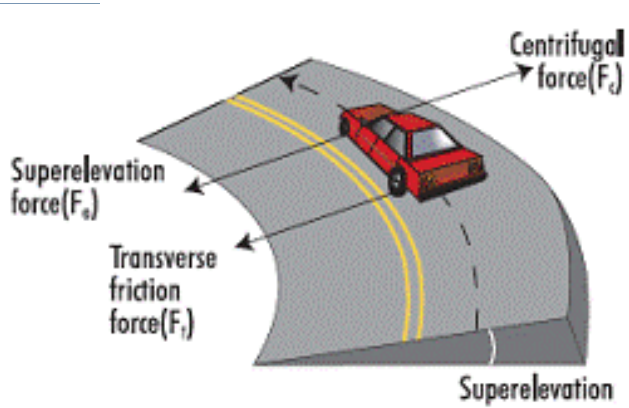
Transition Curve

Super-elevation:

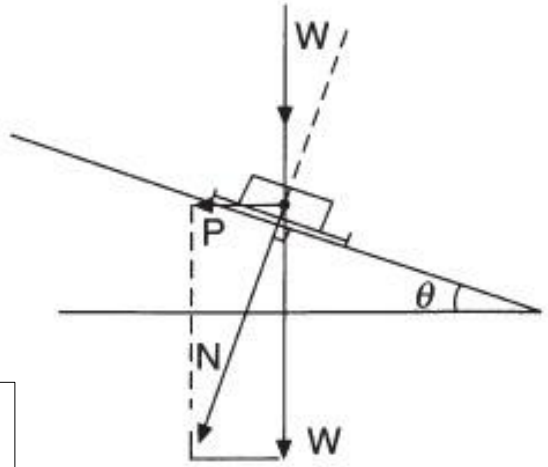
Rising of outer edge of railway or road surface above the inner edge in order to counteract or balance centrifugal force which is experienced by the train or veh when entered into a curve or turn in order to prevent derailment, overturning or side slipping of vehicles.







Transition Curve



N = sum of the forces normal to the pavement

= $P \sin\theta + W \cos\theta$

T = sum of the forces tangential to the pavement

= $P \cos\theta - W \sin\theta$

If f be the friction of the pavement, then $T = fN$

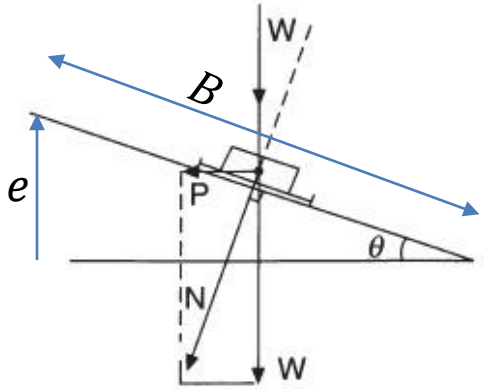
$\therefore P \cos\theta - W \sin\theta = f(P \sin\theta + W \cos\theta)$

$$\Rightarrow \quad \frac{P}{W} = \frac{\sin\theta + f\cos\theta}{\cos\theta - f\sin\theta} = \frac{\tan\theta + f}{1 - f\tan\theta}$$

Transition Curve

$$\frac{P}{W} = \frac{\tan\theta + f}{1 - f\tan\theta} = \frac{v^2}{gR}$$

Again $e = \tan\theta = \frac{P}{W} = \frac{v^2}{gR}$



A vehicle of mass m, travelling at a constant speed v, along a curve of radius R, is subjected to a radial force

$$P = m\frac{v^2}{R} = \frac{W}{g}\frac{v^2}{R}, \text{ or } \frac{P}{W} = \frac{v^2}{gR}$$

If B width of the road then, $\tan\theta = \frac{e}{B} = \frac{v^2}{gR} = \frac{1}{4}$

If G gauge of the railway, $\tan\theta = \frac{e}{G} = \frac{v^2}{gR} = \frac{1}{8}$

For road, $v = \sqrt{\frac{gR}{4}}$

For railways, $v = \sqrt{\frac{gR}{84}}$

Length of Transition Curve

Method 1: By an arbitrary gradient

If gradient is 1 in n, then L=ne

Value of n varies from 300 to 1200

$L = ne = nB.\tan\theta = B.v^2/gR$ on road

$L = ne = nG.\tan\theta = G.v^2/gR$ on railways

By an arbitrary gradient, $L = ne = B.v^2/gR$

Method 2: By the time rate

L=length of transition curve e.v/r meters

Here, super elevation e is applied at an arbitrary time rate of r units per sec.

v= speed of veh in m/sec

By the time rate, L = e.v/r

Method 3: By the rate of change of radial acceleration

In this method length of transition curve is decided on the basis of comfort of passengers. It is experienced that a rate of change of radial acceleration (α) of 1 ft/sec (0.3 m/sec) will pass unnoticed.

So $L = v^3/\alpha R$ **where v is in m/sec**. When α = 0.3 m/sec then $L = V^3/14R$ **where V is in kmph**

By the rate of change of radial acceleration, $L = v^3/\alpha R$ or $L = V^3/14R$

Prob 3.1: A transition curve is required for a circular curve of **200 m radius**, the **gauge being 1.5 m** and **max super elevation restricted to 15 cm**. The transition is to be designed for a velocity such that no lateral pressure is imposed on the rails and the rate of gain of **radial acceleration is 30 cm/sec³**. calculate the required length of the transition curve and the design speed.

$$\mathbf{e} = \tan\boldsymbol{\theta} = \frac{e}{G} = \frac{\boldsymbol{v^2}}{\boldsymbol{gR}}$$

$$\frac{15}{150} = \frac{\boldsymbol{v^2}}{\mathbf{9.81x200}}$$

Thus v = 14 m/sec

$$\text{L} = \frac{\boldsymbol{v^3}}{\boldsymbol{\alpha R}} = \frac{\mathbf{14^3}}{\mathbf{0.3x200}} = \mathbf{46\ m}$$

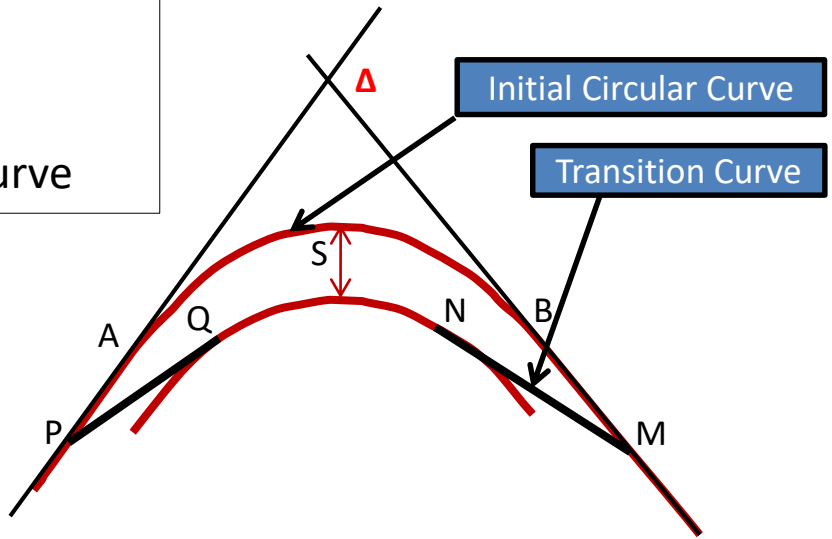
Combined Curve

Combined Curve: **two transition curves on either sides** of a circular curve.

Transition curve may be:

- Clothoid or True spiral/Ideal Transition curve
- Cubic Spiral
- Cubic parabola/ most widely used transition curve

The radius of curvature r at any point vary inversely with the distance l from the beginning of the curve.



Combined Curve

Initial circular curve $= AB$.

To ease up the curve, two transition curves are placed on either sides of the circular curve. To do so, circular curve is shifted inward by s.

New combined curve $= PM + MN + NQ$

Total length of the combined curve

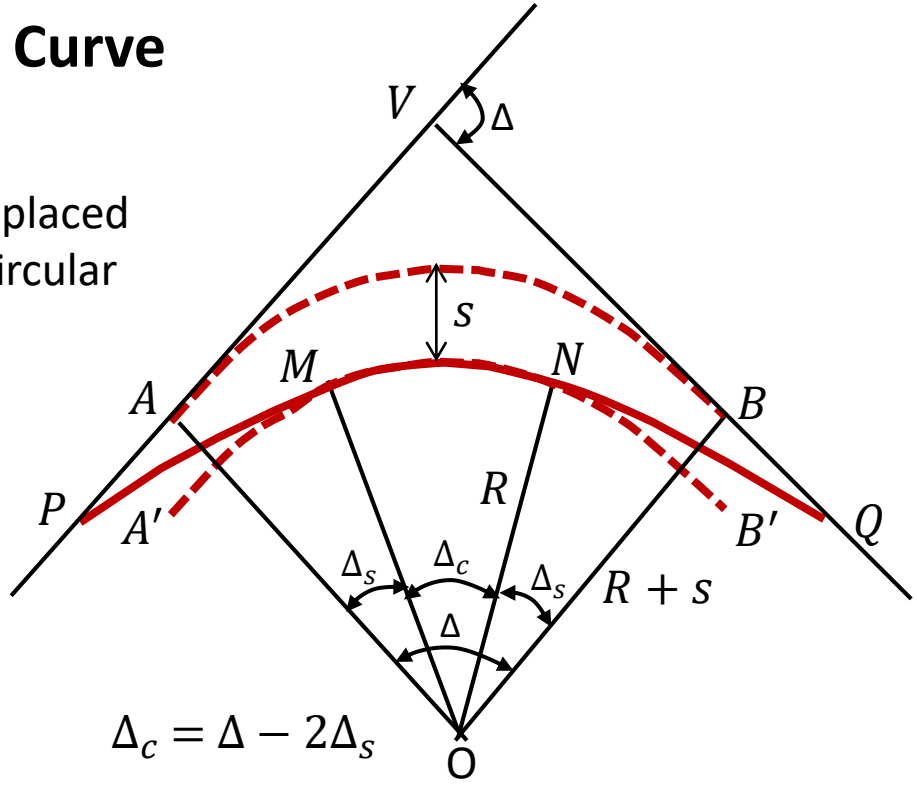
$$= L_T + L_C + L_T$$

Length of the circular curve MN,

$$L_C = \frac{\boldsymbol{\pi R \Delta_c}}{\boldsymbol{180°}} = \frac{\boldsymbol{\pi R(\Delta - 2\Delta_s)}}{\boldsymbol{180°}}$$

Total length of the combined curve

$$= L + \frac{\boldsymbol{\pi R(\Delta - 2\Delta_s)}}{\boldsymbol{180°}} + L$$



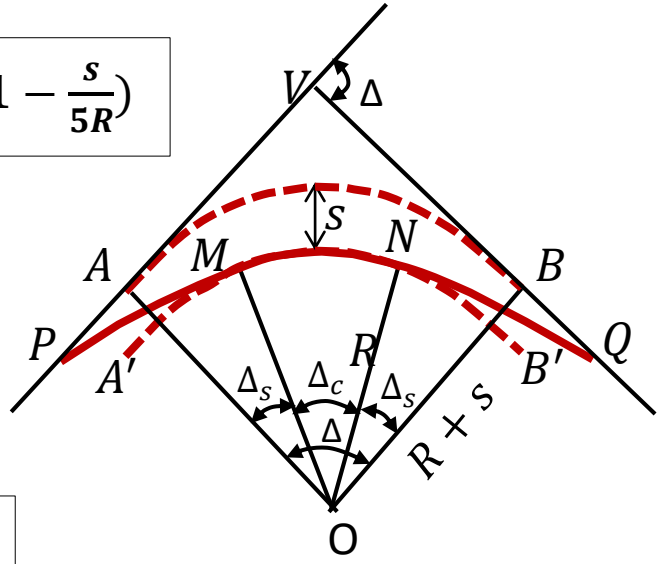
Total length can also be approximated by

$$= \frac{\boldsymbol{L}}{\boldsymbol{2}} + \frac{\boldsymbol{\pi R \Delta}}{\boldsymbol{180°}} + \frac{\boldsymbol{L}}{\boldsymbol{2}}$$

Clothoid or True spiral/Ideal Transition curve

Total tangent length, $TV = AV + AP = (\boldsymbol{R} + \boldsymbol{s})\,\boldsymbol{tan}\left(\frac{\Delta}{2}\right) + \frac{\boldsymbol{L}}{\boldsymbol{2}}\left(\boldsymbol{1} - \frac{\boldsymbol{s}}{\boldsymbol{5R}}\right)$

$y = \frac{l^3}{6RL}\left(1 - \frac{\emptyset^2}{14}\right) = \frac{l^3}{6RL}\left(1 - \frac{l^4}{56R^2L^2}\right)$,
l along the curve, y is perpendicular to tangent.



Cubic spiral transition curve

Total Tangent length, $= (\boldsymbol{R} + \boldsymbol{S})\,\boldsymbol{\tan}\left(\frac{\Delta}{2}\right) + \frac{\boldsymbol{L}}{\boldsymbol{2}}\left(\boldsymbol{1} - \frac{\boldsymbol{s}}{\boldsymbol{5R}}\right)$

If Δ_s is very small, tangent length $= (\boldsymbol{R} + \boldsymbol{S})\,\boldsymbol{\tan}\left(\frac{\Delta}{2}\right) + \frac{\boldsymbol{L}}{\boldsymbol{2}}$

$y = \frac{l^3}{6RL}$, l along the curve, y is perpendicular to tangent.

If it is set by deflection angle then, $\alpha = \frac{573l^2}{RL}$ minutes

Cubic parabola/most widely used transition curve

Ttotal tangent length $= (\boldsymbol{R} + \boldsymbol{s})\,\boldsymbol{\tan}\left(\frac{\Delta}{2}\right) + \frac{\boldsymbol{L}}{\boldsymbol{2}}$

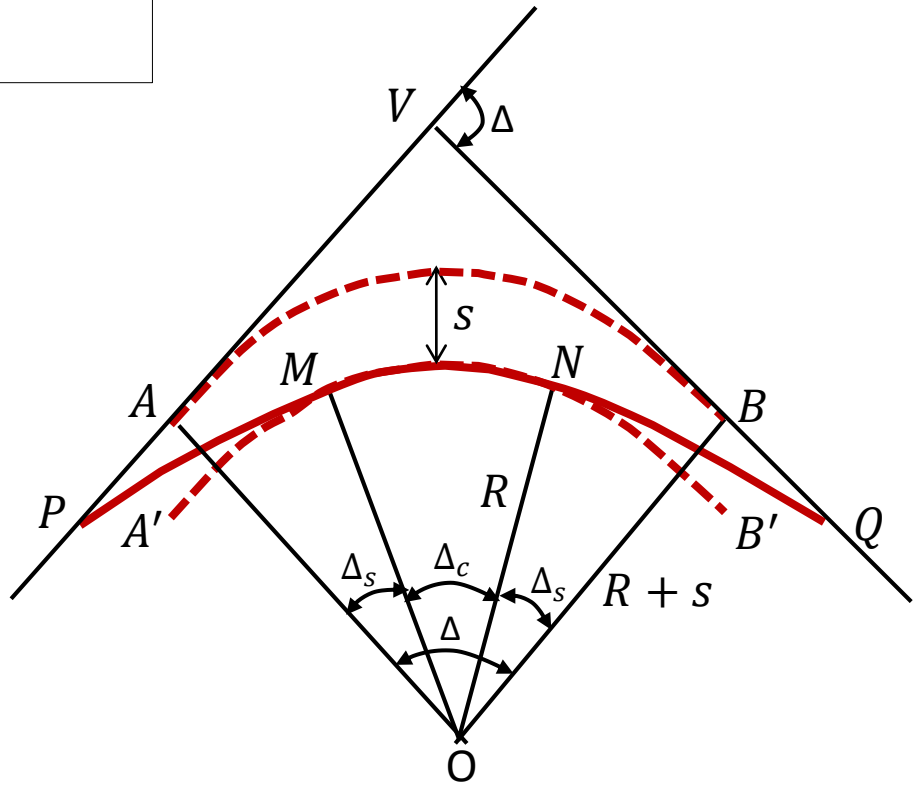
$y = \frac{x^3}{6RL}$, x is measured along the tangent,
y is perpendicular to tangent.

If it is set by deflection angle then,

$\alpha = \frac{573l^2}{RL}$ minutes

Shift of Circular curve, $\boldsymbol{s} = \frac{\boldsymbol{L^2}}{\boldsymbol{24R}}$

Spiral angle, $\boldsymbol{\Delta_s} = \frac{\boldsymbol{L}}{\boldsymbol{2R}}$



Prob 3.2: A Road bend which **deflects 80°** is to be designed for a **max speed of 100 km per hr**, a max **centrifugal ratio of ¼** and a max **rate of change of acceleration of 30 cm/sec³**. the curve consisting of a **circular arc combined with two cubic spirals**. Calculate

(a) radius of circular arc,
(b) the requisite length of transition,
(c) the total length of composite curve
(d) The chainage of the beginning and the end of transition curve and of the junctions of the transition curves with the circular arc if the **chainage of the P.I is 42862m.**

Vertical Curve

Provided where change of grade in the vertical plane to smoothen the change

A vertical curve should be designed so that

It gives **smooth riding qualities**

Should have **adequate sight distance** before the veh reaches the summit

Types

1. Sag curve
2. Crest or summit curve
3. Rising curve
4. Falling Curve

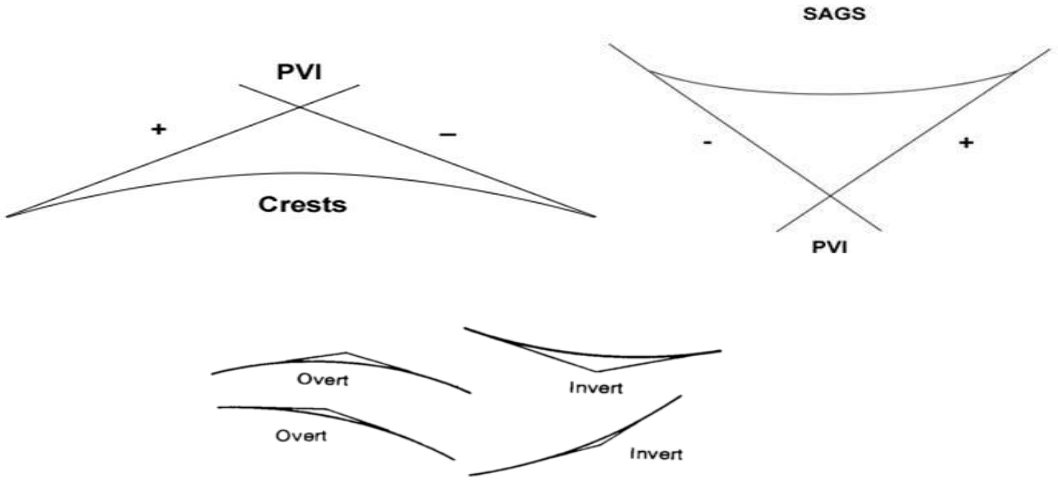


Figure 9-19. Types of vertical curves

Vertical Curve

The general equation of parabola,

$$y = ax^2 + bx + c$$

Slope of this curve, $\frac{dy}{dx} = 2ax + b$

Rate of change of slope is the rate of change of grade r, $\frac{d^2y}{dx^2} = 2a$

The grade changes uniformly throughout the curve, which is the desired condition of vertical curve

Vertical Curve

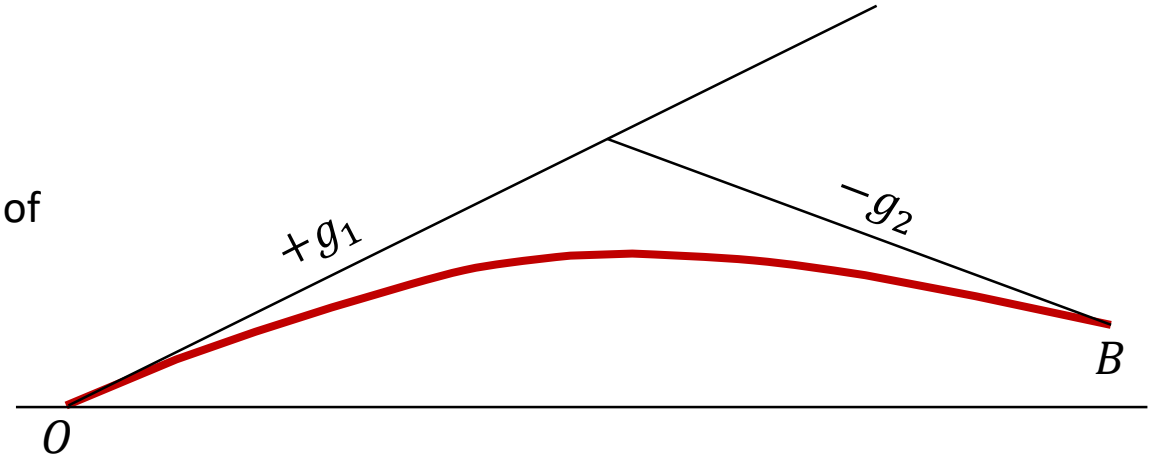
The length of the vertical curve (L)

$$= \frac{Total\ change\ of\ grade\ (with\ due\ regard\ to\ sign)}{Rate\ of\ change\ of\ grade(\%)per\ chain} = \frac{g_1 - g_2}{r}$$

$y = ax^2 + bx + c$

Slope of this curve, $\frac{dy}{dx} = 2ax + b$

Rate of change of slope is the rate of change of grade $(r) = \frac{d^2y}{dx^2} = 2a$



Vertical Curve

The first tangent point O and The second tangent point B

$\frac{dy}{dx}$ ($\frac{dy}{dx}$ means g_1) which the gradient at P.C is $= 0$,

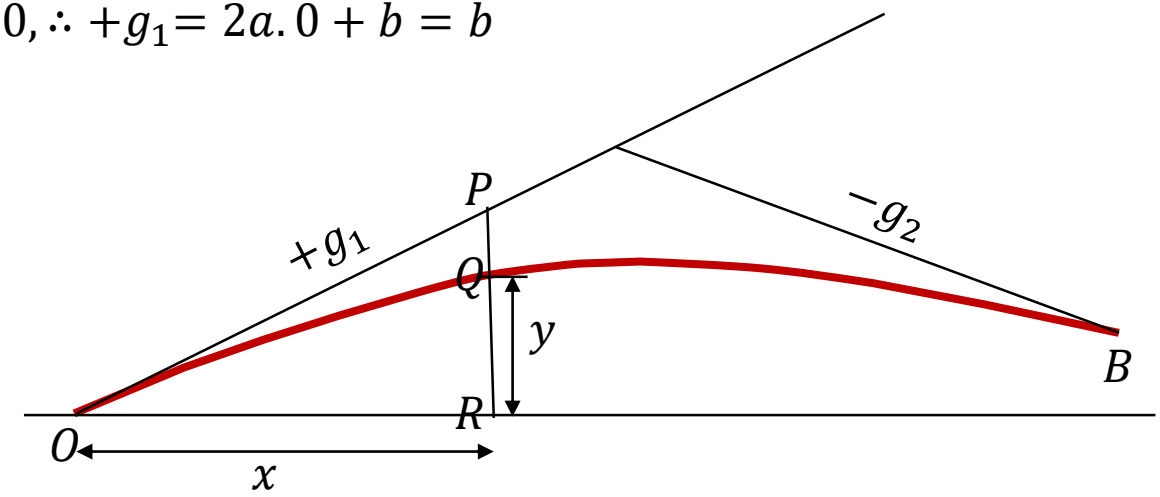
$\therefore\ 2ax + b = 0$, Again at P.C, $x = 0, \therefore + g_1 = 2a.0 + b = b$

$PQ = PR - QR$

$or, h = g_1 x - y$

$but\ -ax^2 = g_1 x - y$

$\therefore h = -ax^2 = Cx^2 = kN^2$



The difference in elevation between a vertical curve and a tangent to it varies as the square of its horizontal distance from the point of tangencc. This difference is known as tangent correction. $h = kN^2$

Vertical Curve

e_1 and $e_2$are corresponding rise anf and fall per chord kength l

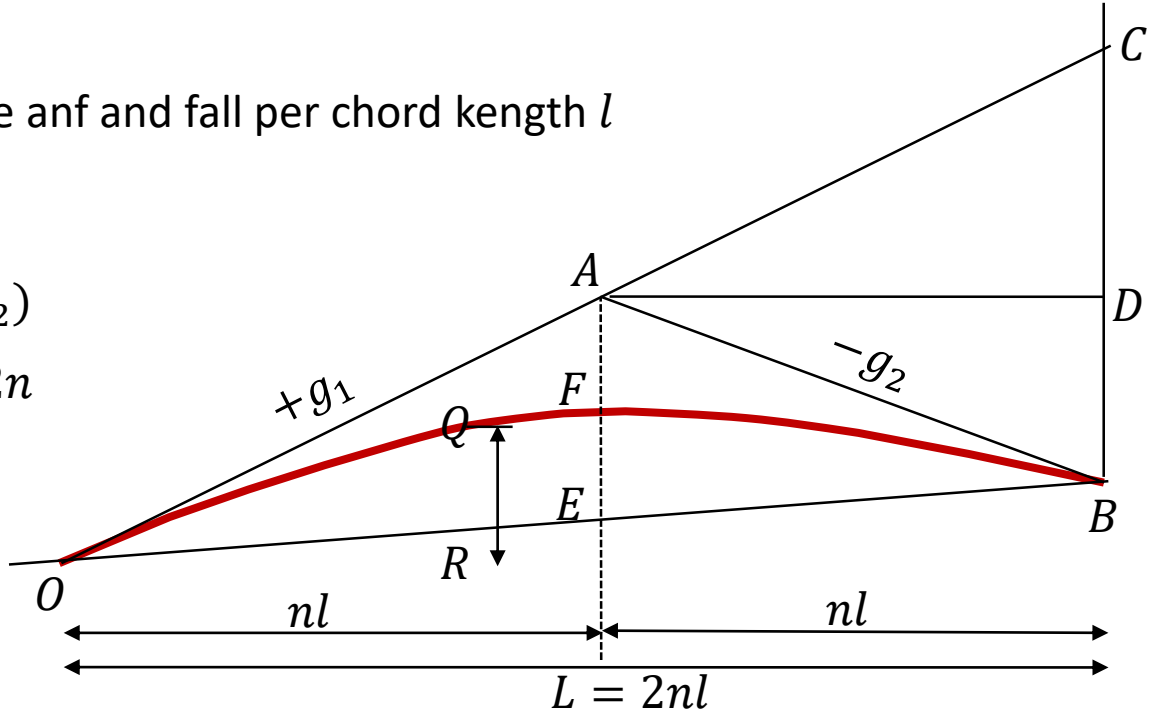
$$CD = +ne_1 \qquad BD = -ne_2$$

$$CB = ne_1 - ne_2 = n(e_1 - e_2)$$

$$But, CB = kN^2 \text{ where } N = 2n$$

$$or, n(e_1 - e_2) = k4n^2$$

$$or, k = \frac{e_1 - e_2}{4n}$$



Set out of Vertical Curve

Prob: A parabolic vertical curve is to be set out connecting two uniform grades of$+0.8\%\ and\ -0.9\%$. The chainage and RL of P.I are 1664 m and 238.755 m respectively. The rate of change of grade is 0.05%per chain of 20 m. Calculate the RL of various station pegs.

The length of the vertical curve $(L) = \frac{g_1 - g_2}{r} = \frac{+0.8-(-0.9)}{0.05}$

$= 34\ chains\ having\ 17\ chain\ on\ both\ sides\ of\ apex.$

The ch. of first tangent point $= 1664\ - 17 \times 20 = 1324\ m$

The ch. of second tangent point $= 1664 + 17 \times 20 = 2004\ m$

$RLof\ P.I\ = 238.755\ m$

∴ RL of beginning of curve $(O) = 238.755\ - (17 \times 20)e_1$

$= 238.755\ - (17 \times 20) \times \frac{0.8}{100} = 236.035\ m$

Set out of Vertical Curve

and RL of the end of curve $(B) = 238.755 - (17 \times 20)e_2 = 238.755 - (17 \times 20) \times \frac{0.9}{100} = 235.695\ m$

RL of the mid point $(E) = \frac{RL\ of\ O + RL\ of\ B}{2} = \frac{236.035 + 235.695}{2} = 235.865\ m$

Since F is midway between A and E, RL of vertex $(F) = \frac{RL\ of\ A + RL\ of\ E}{2} = \frac{238.755 + 235.865}{2} = 237.310\ m$

Difference in elevation of A and $F = 238.755 - 237.310 = 1.445\ m$

Check: $AF = kN^2$ where $N = 2n$

$here, k = \frac{e_1 - e_2}{4n}$

$AF = kN^2 = \frac{1}{200}.17^2 = 1.445\ m$

$= \frac{0.8 - (-0.9)}{4 \times 17 \times 20}.\frac{1}{100} = \frac{1}{200}$

Set out of Vertical Curve

Station peg will be placed every after one chain, so Ch of first station peg will be $= 1324 + 20 = 1344\ m$

$$h_1 = kN_1{}^2 = \frac{1}{200}.1 = 0.005\ \text{m}$$

Tangent elevation of first station point RL of $\text{O} + e_1 = 236.035 + 1 \times 20 \times \frac{0.8}{100} = 236.195\ m$

RL of first station on curve $= tangent\ elevation - tangent\ correction$

$$= 236.195 - 0.005 = 236.490\ m$$

Set out of Vertical Curve

Station	Chainage	Tangent Elev.	Tangent Correction	Curve Elev.	Remarks
1	1324	236.035	0.000	236.035-0.000	Begining
2	1324+20	236.195	0.005	236.190	
3	1364	236.355	0.020	236.335	
4	1384		0.045		
5	1404		0.080		

31	1944				
32	1964				
33	1984				
34	2004				End point

Set out of Vertical Curve

