Calculation of Areas and Volume

Introduction

For Engg projects, earthwork involve cutting, removal and filling, therefore it is required to make a good estimate of volume of earth work.

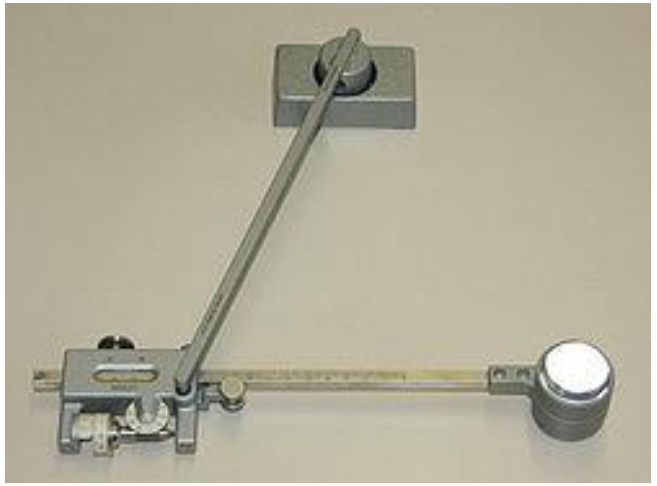
Volume computation also required for the determination of reservoir capacity

In this topic, we will be learning:

- Method of calculation of Area
- Method of calculation of volume
- Related problem: Area and Volume Calculation

Methods of Calculation of Areas

- When the area is bounded by straight lines
  a. By Trigonometry
  b. By Coordinate method
  c. By Latitude and Departure
- When the area is bounded by curved lines
- Computation of area from map
  a. By dividing the area into geometric figures
  b. By dividing into squares
  c. By dividing into trapezoids
  d. Area by plainimeter

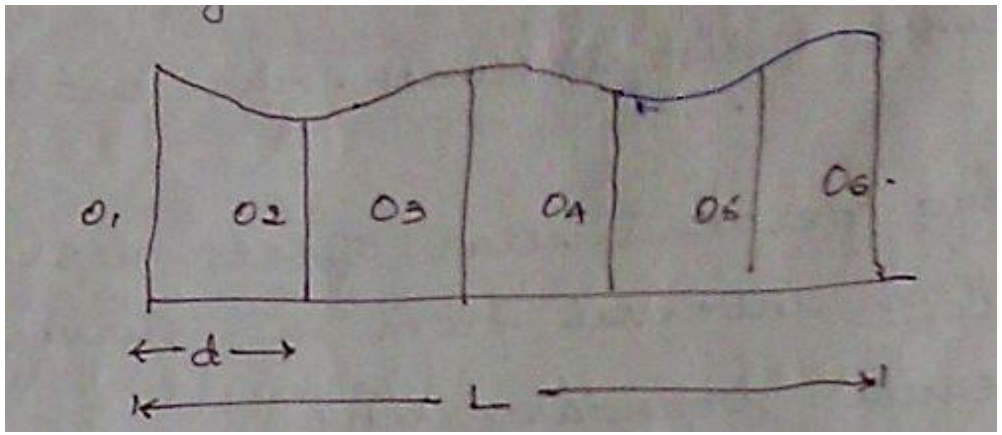


Methods of calculation of areas
When area is bounded by curved/irregular lines

a. **Average ordinate rule**
b. **Trapezoidal rule**
c. **Prismoidal or Simpson's rule**
d. **Mid-ordinate rule**
e. Offset at irregular interval
f. Graphical rule

Average Ordinate Rule

- Area is divided into number if division of equal length
- Average of Ordinates of all division are determined
- Multiply by the division length



Mathematically

Area = $[(O_1+ O_2+ O_3+ \ldots. + O_n)*L]/(n+1)$

where

$O_1, O_2, O_3, O_4 \ldots O_n$ are ordinate taken at each division.

L = length of baseline, n = number of equal division

Example: The following perpendicular offsets were taken at 10m interval from a survey line to an irregular boundary line ; 9, 12, 17, 15, 19, 21, 24, 22, 18

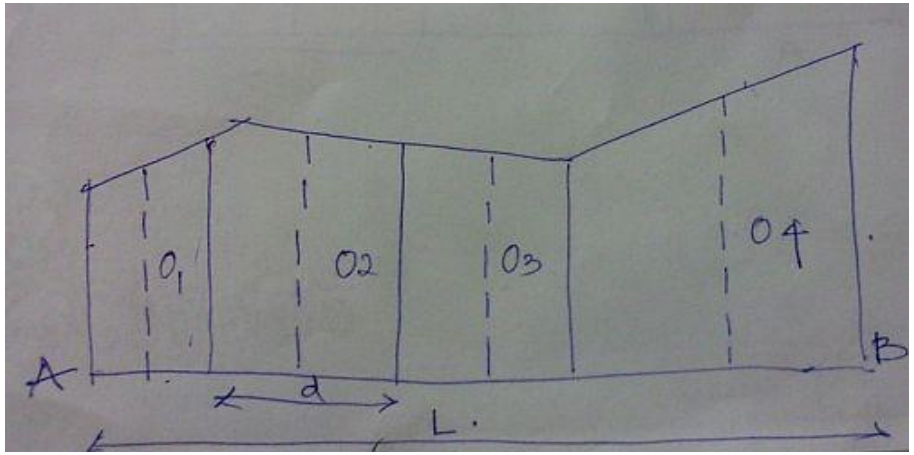
Calculate area enclosed between the survey line and irregular boundary line.

Area = [(O1+ O2+ O3+ …. + O9)*L]/(n+1)

= [(9+12+17+15+19+21+24+22+18)*8*10]/(8+1) = 139538 sqm

Midpoint-ordinate rule

- Ordinates are taken at midpoints of each division
- Average of the ordinates are determined
- Multiply by the length of the division



In this, base line AB is divided into equal parts and the ordinates are measured in the midpoints of each division.

Area = $[(O_1 + O_2 + O_3 + \ldots.. + O_n)]*L/n$, where, L = length of baseline, n = no of equal divisions

Example: The following perpendicular offsets were taken at 10m interval from a survey line to an irregular boundary line. The ordinates are measured at midpoint of the division are 10, 13, 17, 16, 19, 21, 20 and 18m. Calculate the are enclosed by the midpoint ordinate rule.

Given: Ordinates O1 = 10, O2 = 13, O3 = 17, O4 = 16, O5 = 19, O6 = 21, O7 = 20, O8 = 18
Common distance, d =10m, Number of equal parts of the baseline, n = 8
Length of baseline, L = n *d = 8*10 = 80m
Area = [(10+13+17+16+19+21+20+18)*80]/8 = 1340sqm

Area Calculation - Simpson's Rule

Steps:

- Add first and last ordinates
- Add twice the sum of remaining odd ordinates
- Add four times the sum of even ordinates
- Multiply to this total sum by 1/3 of the common dist. between the ordinates

Mathematically,

Area A = $d/3[(O_1 + O_n) + 2(O_3 + O_5 + \ldots\ldots) + 4(O_2 + O_4 + O_6 + \ldots..)]$

Where, $O_1, O_2, O_3, \ldots O_n$ are the ordinates

d = common distance

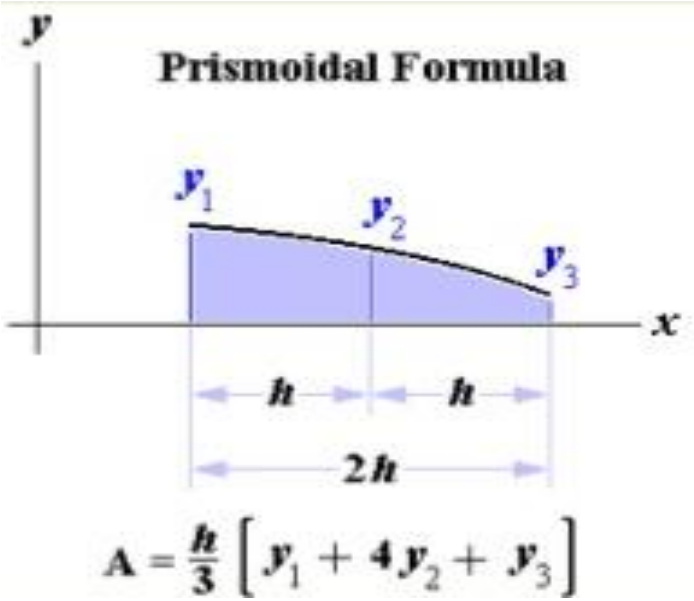
n = number of divisions

Area Calculation - Simpson's Rule

This rule is applicable only if ordinates are odd, i.e. even number of divisions.

If the number of ordinates are even, the area of last division maybe calculated separated and added to the result obtained by applying Simpson's rule to two remaining ordinates.

Even if first or last ordinate happens to be zero, they are not to be omitted from Simpson's rule.



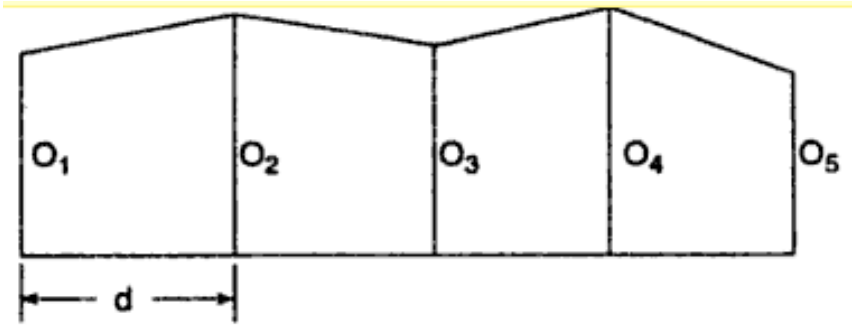
Example: The following offsets are taken from a chain line to an irregular boundary towards right side of the chain line.

Chainage	0	25	50	75	100	125	150
Offset in m	3.6	5.0	6.5	5.5	7.3	6.0	4.0

Area A = d/3[(O_1+ O_7)+ 2 (O_3+O_5) + 4 (O_2+O_4)]

Area = 25/3[(3.6+4)+2(6.5+7.3)+4(5+5.5+6)] = 843.33 sqm

Trapezoidal rule



$O_1, O_2, \ldots, O_n$ = ordinates at equal intervals

d = common distance

$$\text{1st area} = \frac{O_1 + O_2}{2} \times d$$

$$\text{2nd area} = \frac{O_2 + O_3}{2} \times d$$

$$\text{3rd area} = \frac{O_3 + O_4}{2} \times d$$

$$\text{Last area} = \frac{O_{n-1} + O_n}{2} \times d$$

$$\text{Total area} = \frac{d}{2}\{O_1 + 2O_1 + 2O_2 + \ldots + 2O_{n-1} + O_n\} = \frac{\text{common distance}}{2}\{(\text{1st ordinate} + \text{last ordinate} + 2\ (\text{sum of other ordinate})\}$$

Example: The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m. 0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m. Compute the area between the chain line, the irregular boundary line and the end offsets by: (a) Trapezoidal Rule and (b) Simpson's Rule

(a) Trapezoidal Rule $\text{Area} = \frac{10}{2}\{0+0+2(2.50+3.50+5.00+4.60+3.20)\} = 5 * 37.60 = \mathbf{188\ m^2}$

(b) Simpson's Rule $\text{Area} = \frac{10}{3}\{0+0+4(2.50+5.00+3.20)+2(3.50+4.60)\} = \frac{10}{3} * 59.00 = \mathbf{196.66\ m^2}$

Example: The following offsets were taken from a survey line to a curved boundary line. Find the area between the survey line, the curved boundary and the first and last offsets by: (a) Trapezoidal Rule and (b) Simpson's Rule

Distance (m)	0	5	10	15	20	30	40	60	80
Offset (m)	2.50	3.80	4.60	5.20	6.10	4.70	5.80	3.90	2.20

(a) Trapezoidal Rule

$\Delta_1 = \frac{5}{2}\{2.50+6.10+2(3.80+4.60+5.20)\} = 89.50\ \text{m}^2$

$\Delta_2 = \frac{10}{2}\{6.10+5.80+2(4.70)\} = 106.50\ \text{m}^2$

$\Delta_3 = \frac{20}{2}\{5.80+2.20+2(3.90)\} = 158.00\ \text{m}^2$

Total Area = 89.50 + 106.50 + 158.00 = **354.00 m²**

(b) Simpson's Rule

$\Delta_1 = \frac{5}{3}\{2.50+6.10+4(3.80+5.20)+2(4.60)\} = 89.66\ \text{m}^2$

$\Delta_2 = \frac{10}{3}\{6.10+5.80+4.70\} = 102.33\ \text{m}^2$

$\Delta_3 = \frac{20}{3}\{5.80+2.20+4(3.90)\} = 157.33\ \text{m}^2$

Total area = 89.66 + 102.33 + 157.33 = **349.32 m²**

Calculation of Volume

There is 4 different Volume calculation methods that are in Practice for Earthwork Calculation.

- **From Cross section**
  1. **Prismoidal rule or Simpsons 1/3rd Rule**
  2. **Trapezoidal rule or Average end area Method**
- Average method
- From Spot level
- **From Contour (usually for the calculaton of reservoir capacity)**

Vol Calculation from Cross section

- Divide the total length into no. of sec
- Cross sec are taken at right angles to a fixed line, which runs continuously through the earth work
- Spacing of sec depends on the nature of ground and the accuracy required
- calculate the cross sec area of each sec
- Then determine the volume by using Prismoidal or Trapezoidal rule

Prismoidal or Simpson 1/3rd Rule

Prismoidal formula is accurate in finding the volume of prisms, pyramids, wedges, and prismoids having irregular end sections. The estimation of earthwork gives nearly an accurate volume.

The formula given is **V=d/3[Sum of areas of end two sections+ 4(sum of the even sections) +2(sum of the remaining sections)]**

$$V = \frac{d}{3}\{(A_0 + A_n) + 4\,(A_2 + A_4 + \cdots) + 2(A_1 + A_3 + \cdots)\}$$

Where d is the common dist between sections and A are the cross sectional areas.

Notes:
No of Sec to be oddfor Simpson rule.
For even no of sec, volume of last sec is calculated by Trapezoidal rule.

Average End area Method or Trapezoidal Rule

The average end area calculation is used to calculate volume between two cross sections, where, Volume = d x 1/2 (A1 + A2)

d is the distance and A1 and A2 are areas.

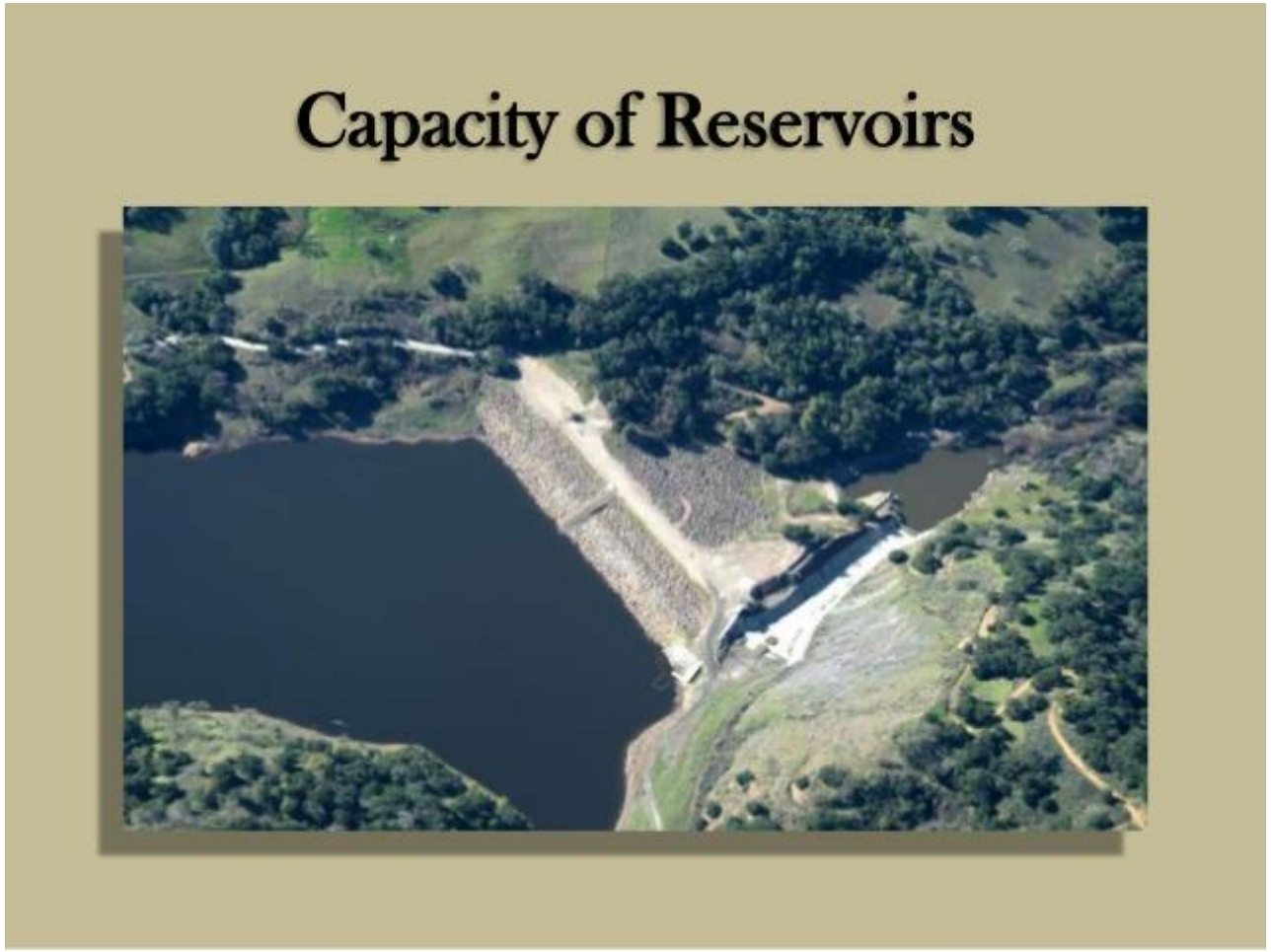
If there are a series of areas A1, A2, A3,….An at regular interval L, then the formulae become as V=d/2[(A1+An)+2(A2+A3+…..+An-1)]
This is known as Trapezoidal Formula

The formula given is **V=d/2[Sum of end two sections+ 2(sum of the remaining sections)]**

$$V = \frac{d}{2}\{(A_0 + A_n) + 2(A_1 + A_2 + A_3 + A_4 + \ldots\ldots\ldots + A_{n-1})\}$$

where d is the common distance between sections and A are the cross sectional areas.

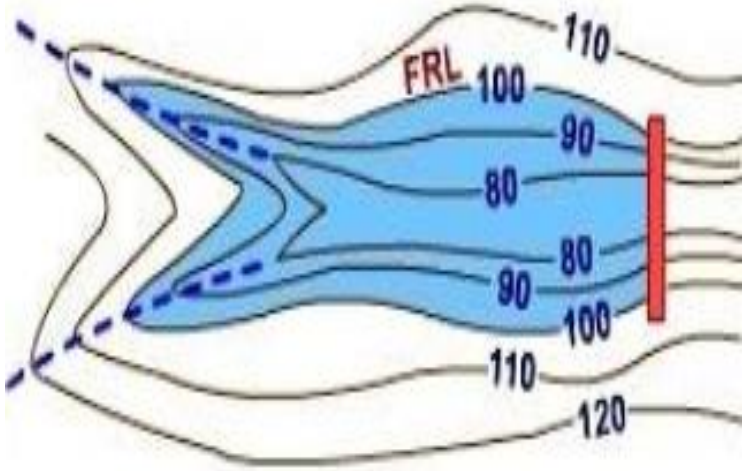
Problem: Volume Calculation



Capacity of Reservoir

The plane containing any contour represents a hor plane and the area bounded by a contour is treated as the area of the cross sec. The contour interval is the vertical dist between any two adjacent cross sec bounded by the contours. Area is measured by a planimeter.

Reservoir are made for water supply, irrigation and hydropower etc. A contour map is veru useful to study the possible loc of reservoir and the vol of water to be confined. All the contours ae closed lines within the reservoir area.



Problem: Reservoir Volume

- From a contour plan of a **proposed reservoir area**, the following data were found

Contour (m)	100	105	110	115	120	125
Area of contour (ha)	3	8	13	17	23	29

Determine

- The capacity of the reservoir if the FRL is 125 m. The volume **below the contour of 100** may be ignored.

Volume by the trapezoidal formula

- Volume by the trapezoidal formula
- $V = \frac{h}{2}$ [First Area + Last Area + 2 (Sum of area of other section)]

$= \frac{5}{2} \times [3 + 29 + 2 (8 + 13 + 17 + 23)]$

$= \frac{5}{2} \times (32 + 122)$

$=$ **385 ha-m**

Prismoidal formula can be used **when odd number of section are there** So, the volume up to the contour 120 m is

$V_{120} = \frac{d}{3} [(A_1 + A_5 + 4 (A_2 + A_4) + 2 (A_3)]$

$= \frac{5}{3} [(3 + 23) + 4 (8 + 17) + 2 \times 13)]$

$= \frac{5}{3} [26 + 100 + 26]$

$=$ **255.33 ha.m**

And by **trapezoidal formula**, the volume, the formula, the **volume between 120 and 125 m** is

$V_{120\text{-}125} = h \times \frac{A_5 + A_6}{2}$

$= 5 \times \frac{23 + 29}{2}$

$= 130$ ha.m

Total Volume, $V = V_{120} + V_{120\text{-}125}$

$= 253.33 + 130$

$=$ **383.33 ha.m**

Problem: Reservoir Volume

- The **area enclosed by the contour in a reservoir** are as follows

Contour (m)	175	180	185	190	195
Area (m^2)	460	750	2500	3500	3950

- The **top water level is 195 m** and the **lowest point in the reservoir is 175 m**. Find the volume of water (reservoir capacity) between 175 m and 195 m

By

- **Trapezoidal formula**
- **Prismoidal formula**

Ans: V_T=44775 m^3
V_P= 44016.66 m^3

- **Contour Interval= 5 m**
- (i) Volume according to **trapezoidal formula**
- $V = \frac{h}{2}[A_1 + A_5 + 2(A_2 + A_3 + A_4)]$

$= \frac{5}{2}[460 + 3950 + 2(750 + 2500 + 3500)]$

$= 44775\ m^3$

Volume according to **Prismoidal formula**

- $V = \frac{h}{3}[A_1 + A_5 + 4(A_2 + A_4) + 2(A_3)]$

$= \frac{5}{3}[(460 + 3950) + 4(750 + 3500) + 2(2500)]$

$= \frac{5}{3}[4410 + 1700 + 5000]$

$= 44016.66\ m^3$

Example

An embankment of **width 10 m** and **side slope 1 ½ : 1** is required to be made on a ground which is level in a direction traverse to centre line. The central height at **20 m intervals** are as follows:

0.8, 1.2, 2.25, 2.6, 1.9,1.4 and 0.9

Calculate the volume of earth work according to

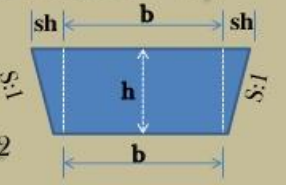
(1) The trapezoidal formula

(2) The prismoidal formula

Level Section : Ground is level along the traverse direction

Here, **b= 10 m, s= 1.5, interval = 20 m**

The cross-Sectional Area are calculated by equation



Area= (b + sh) h

$\Delta_1 = (10 + 1.5 \times 0.8) \times 0.8 = 8.96\ m^2$

$\Delta_2 = (10 + 1.5 \times 1.2) \times 1.2 = 14.16\ m^2$

- $\Delta_1 = (10 + 1.5 \times 0.8) \times 0.8 =$ **8.96 m²**
- $\Delta_2 = (10 + 1.5 \times 1.2) \times 1.2 =$ **14.16 m²**
- $\Delta_3 = (10 + 1.5 \times 2.25) \times 2.25 =$ **30.09 m²**
- $\Delta_4 = (10 + 1.5 \times 2.6) \times 2.6 =$ **36.14 m²**
- $\Delta_5 = (10 + 1.5 \times 1.9) \times 1.9 =$ **24 .42 m²**
- $\Delta_6 = (10 + 1.5 \times 1.4) \times 1.4 =$ **16.94 m²**
- $\Delta_7 = (10 + 1.5 \times 0.9) 0.9 =$ **10.22 m²**

Volume according to Prismoidal Formula

$$V = \frac{20}{3}\,[8.96 + 10.22 + 4\,(14.16 + 36.14 + 16.94) + 2\,(30.09 + 24.42)]$$

$$= \frac{20}{3} \times (19.18 + 268.96 + 109.02)$$

$$= 2647.73\ m^3$$

Volume according to Trapezoidal Rule

- $V = \frac{20}{2}\,[8.96 + 10.22 + 2\,(14.16 + 30.09 + 36.14 + 24.42 + 16.94)]$

$$= 10\,[19.18 + 242.10]$$

$$= 2612.80\ m^3$$

Example: Calculate the volume of earthwork of a railway embankment of 400 m long and 12 m wide at the formation level with side slope 2 to 1. the ground level at every 100 m along the center line as follows. The formation level at zero chainage is 207.00 m and the embankment has rising gradient of 1 in every 100. The ground is level across the center line.

Dist (m)	0	100	200	300	400
R.L (m)	204.8	206.2	207.5	207.2	208.3

Distance	Ground	Formation Level	Depth of Filling
0	204.8	207.0	2.2
100	206.2	208.0	1.8
200	207.5	209.0	1.5
300	207.2	210.0	2.8
400	208.3	211.0	2.7

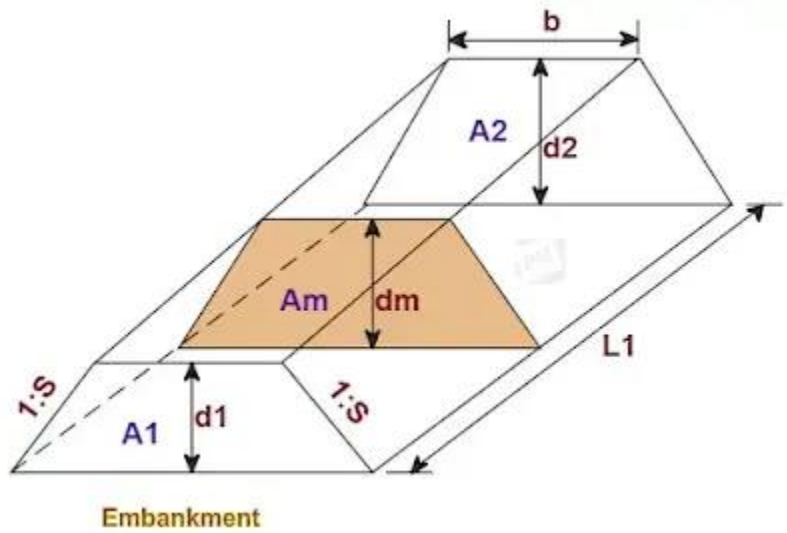
$A = (b + nh)h$
$(12 + 2 \times 2.2)2.2 = 36.08\ m^2$
$(12 + 2 \times 1.8)1.8 = 28.06\ m^2$
$(12 + 2 \times 1.5)1.5 = 22.50\ m^2$
$(12 + 2 \times 2.8)2.8 = 49.28\ m^2$
$(12 + 2 \times 2.7)2.7 === 46.98\ m^2$

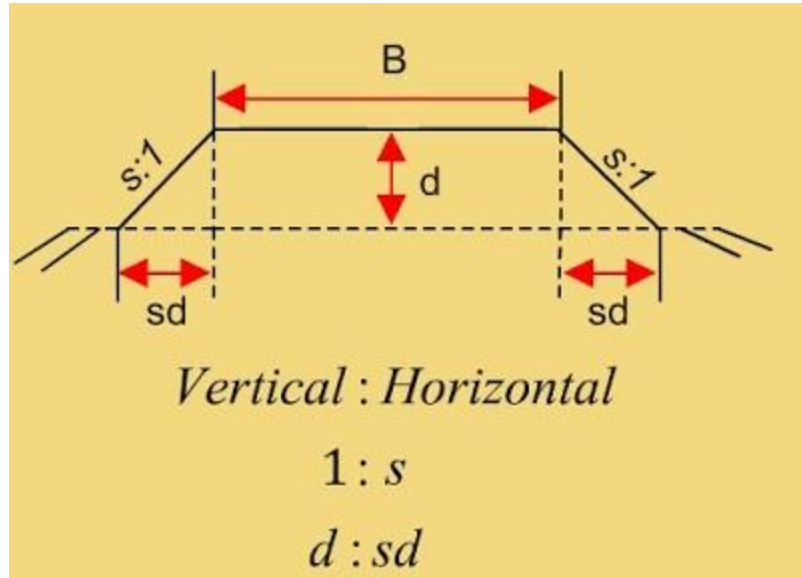
Volume by Trapezoidal Rule = $\frac{d}{2}\{(A_0 + A_n) + 2(A_1 + A_2 + A_3 + A_4 + \dots\dots\dots + A_{n-1})\}$

$=14{,}137\ m^3$

Volume of prismoidal rule = $\frac{d}{3}\{(A_0 + A_n) + 2(A_2 + A_4 + \cdots) + 2(A_1 + A_3 + \cdots)\}$

$=14{,}581\ m^3$


b
A2
d2
Am
dm
L1
1:S
A1
d1
1:S
Embankment


B
s:1
d
s:1
sd
sd
Vertical : Horizontal
1 : s
d : sd