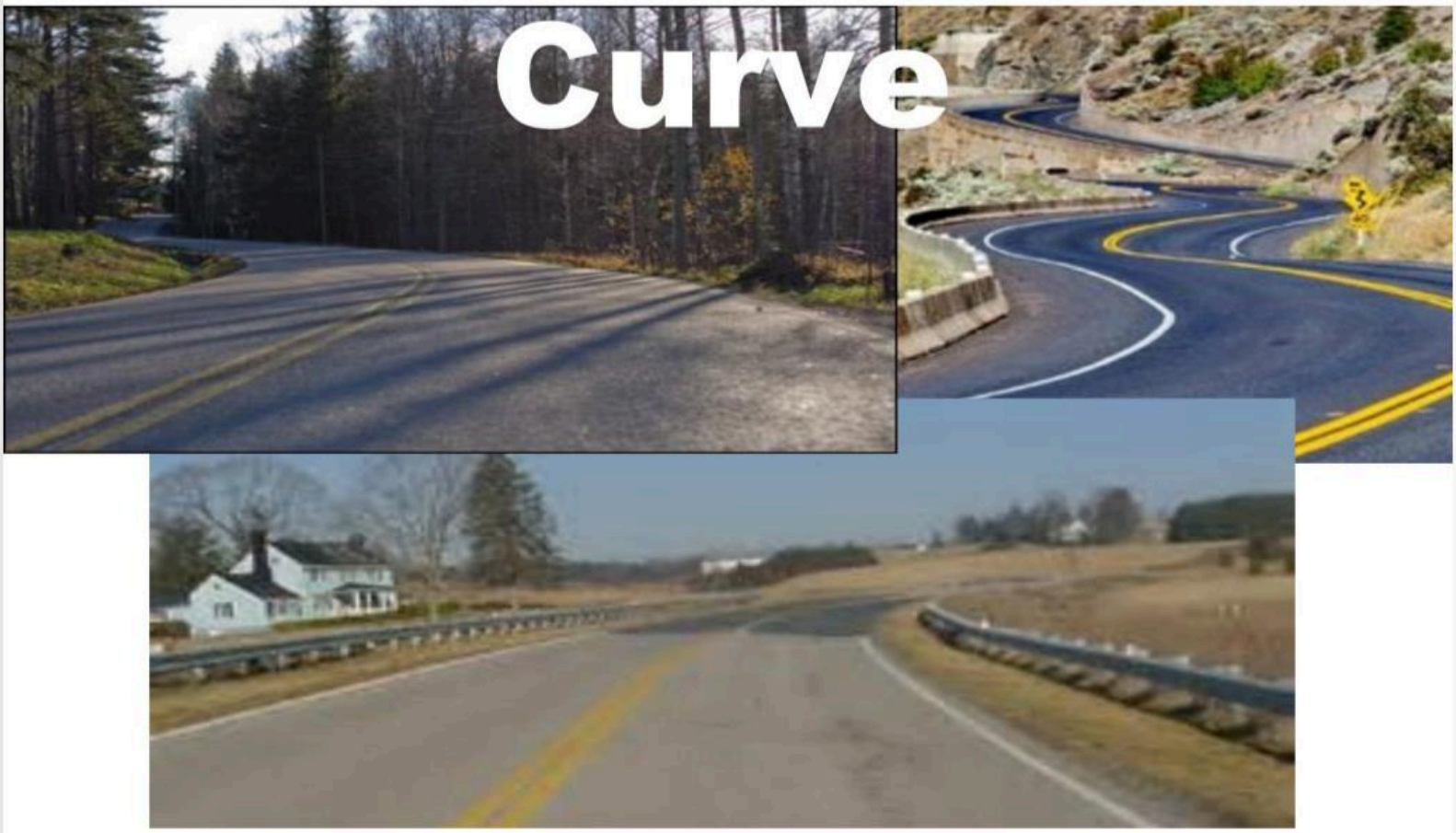


Curve

Curve are generally used on **highway and railways** where it is necessary **to change the direction** of motion.

A curve may be Circular, Parabolic or Spiral.

Curve may be vertical and horizontal

The most common type of horizontal curve used to connect intersecting tangent (or straight) sections of highways or railroads are **Circular curves**.

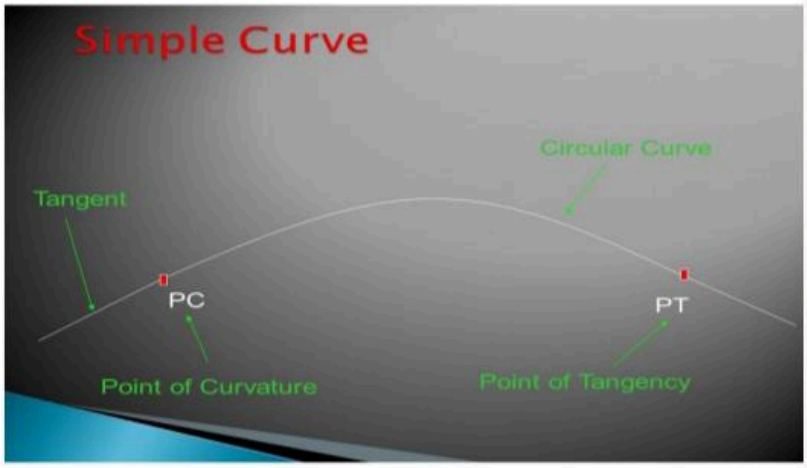
Lesson Outline

- ✔ Types of Curve
- ✔ Notation of Curve and Elements of Curve
- ✔ Designation of Curve-by D and R
- ✔ Setting out of Simple circular curve
- ✔ Transition Curve-function and Characteristics
- ✔ Length of transition curve
- ✔ Super elevation on highway
- ✔ Combined curve-Length and Setting out
- ✔ Problem -Circular and Combined curve setting and superelavation
- ✔ Various types of vertical Curve

Circular Curve

Circular curve are subdivided into three classes: Simple, Compound and Reverse

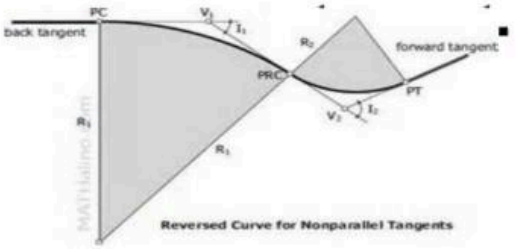
Simple Circular curve
Consists of a single arc of a circle. It is tangential to both the straight lines.

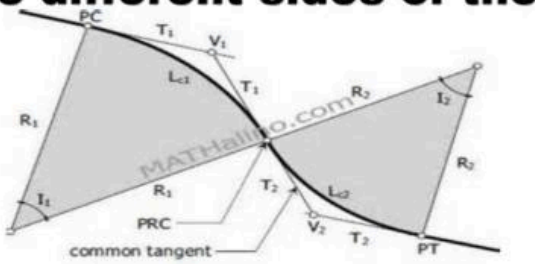


Circular Curve

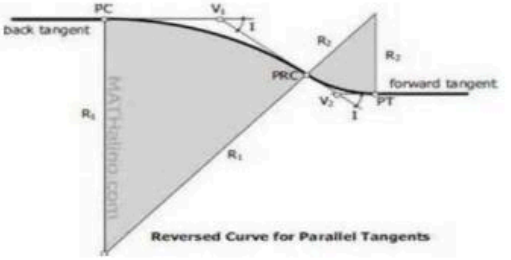
Reverse curve

Consists of two or more circular arcs of same or different radii, having their centres to the different sides of the





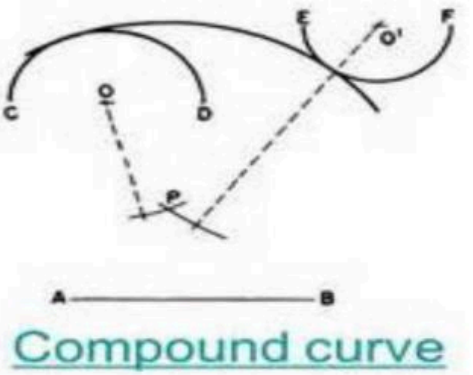
Reversed Curve



Circular Curve

Compound curve

Consists of two or more simple arcs that turn in the same direction and join at common tangent point.



Compound curve

Designation of Curve

Sharpness or flatness of the curve is designated either by its **radius of curve (R)** (Great Britain) or degree of curvature (D) (America, Canada, India etc).

Large radius curves are relatively flat, whereas small radius curves are relatively sharp. The larger the degree of curve, the "sharper" the curve and the shorter the radius

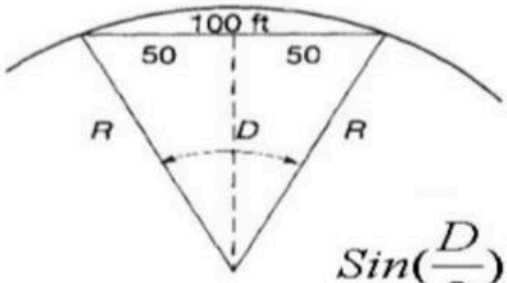
In most countries, two methods of defining **degree of curve** are in use:

- Chord Definition
- Arc Definition

Designation of Curve

Chord Definition

Degree of curve is the central angle subtended by a chord of 100 ft (30.48 m) in length.



$$Sin(\frac{D}{2}) = \frac{50\,ft}{R}$$

This definition is used primarily for civilian railroad construction and is used by the military for both roads and railroads.

If D is small, D/2 may be taken approx as D/2 radians.

So, R = 50/(D/2) (π/180)

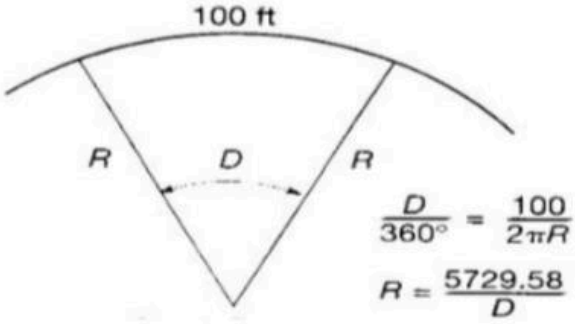
= 5729.578/D

= 5730/D where D is in degree.

Designation of Curve

Arc Definition

Degree of curve is the central angle subtended by an arc of 100 ft (30.48 m) in length.



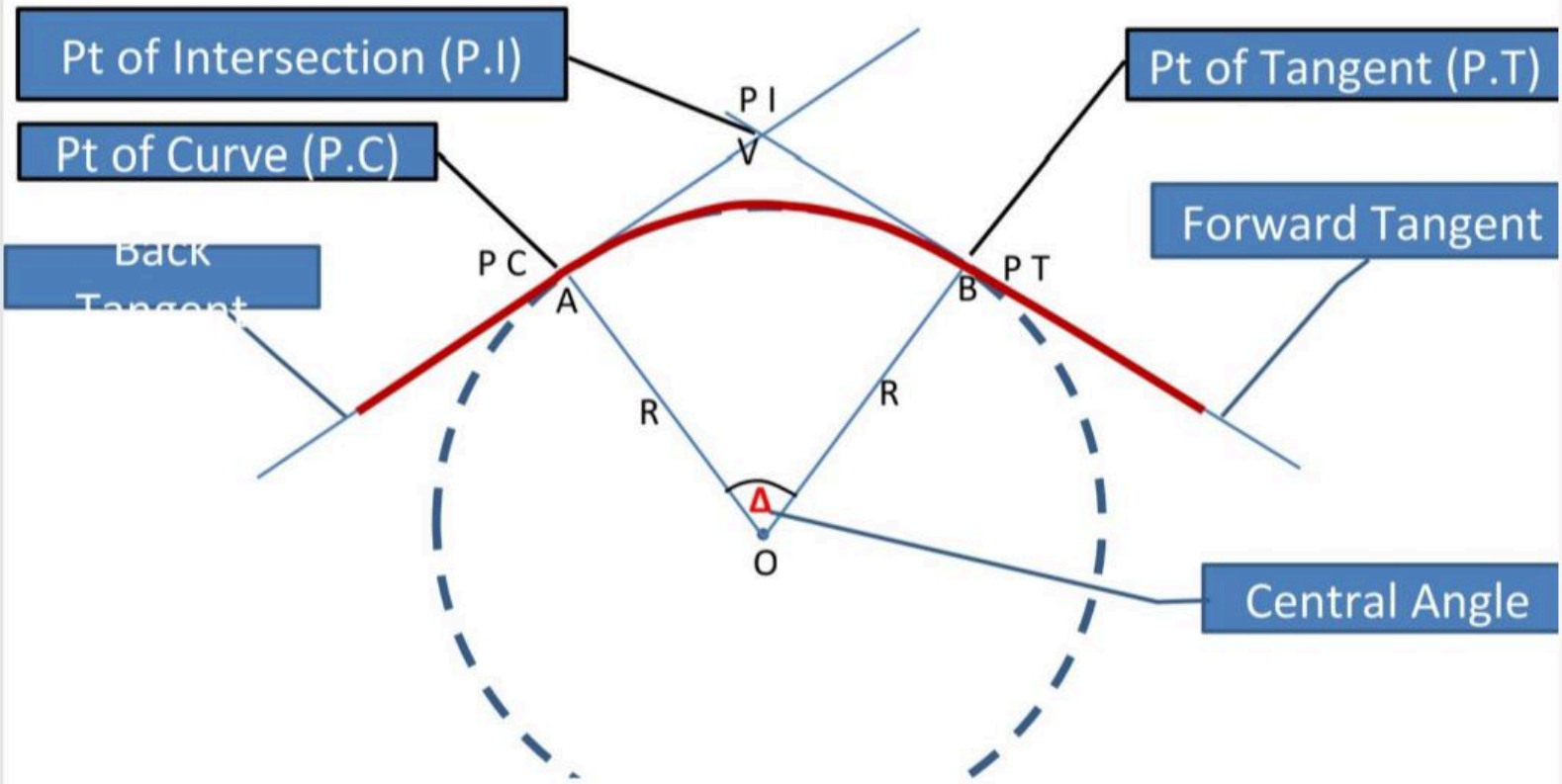
This definition is used primarily for highways and streets.

With approximation
R = 5730 ft / D.
R is in ft, D is in degree.

Metric Degree of Curve

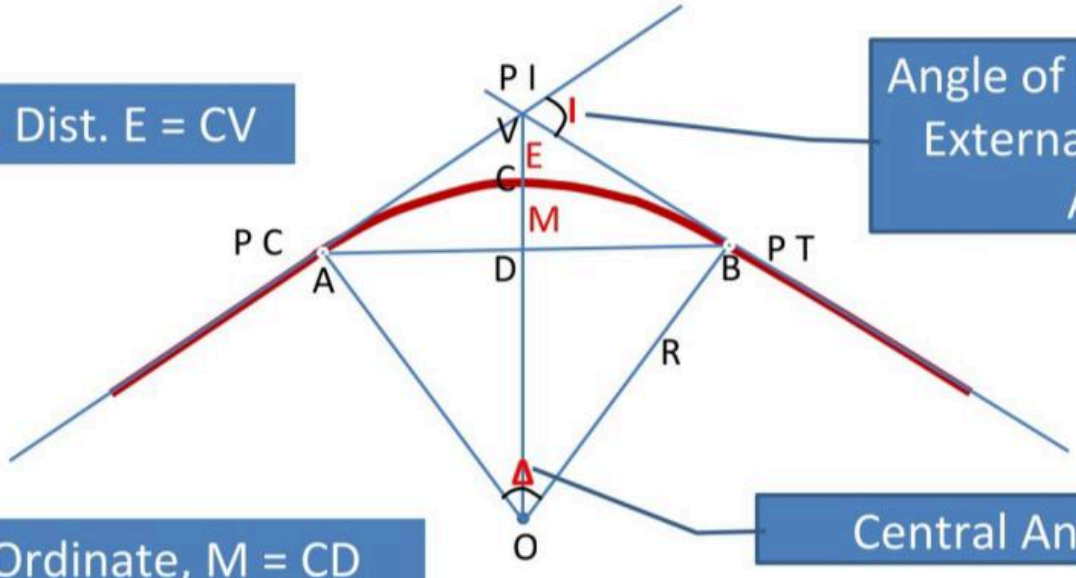
Angle subtended at the centre by an arc of 20 m. R= 1416/D meters
Angle subtended at the centre by an arc of 10 m. R=573/D meters

ELEMENTS of SIMPLE CIRCULAR CURVE



ELEMENTS of SIMPLE CIRCULAR CURVE

External Dist. E = CV



Angle of Intersection / External Deflection Angle

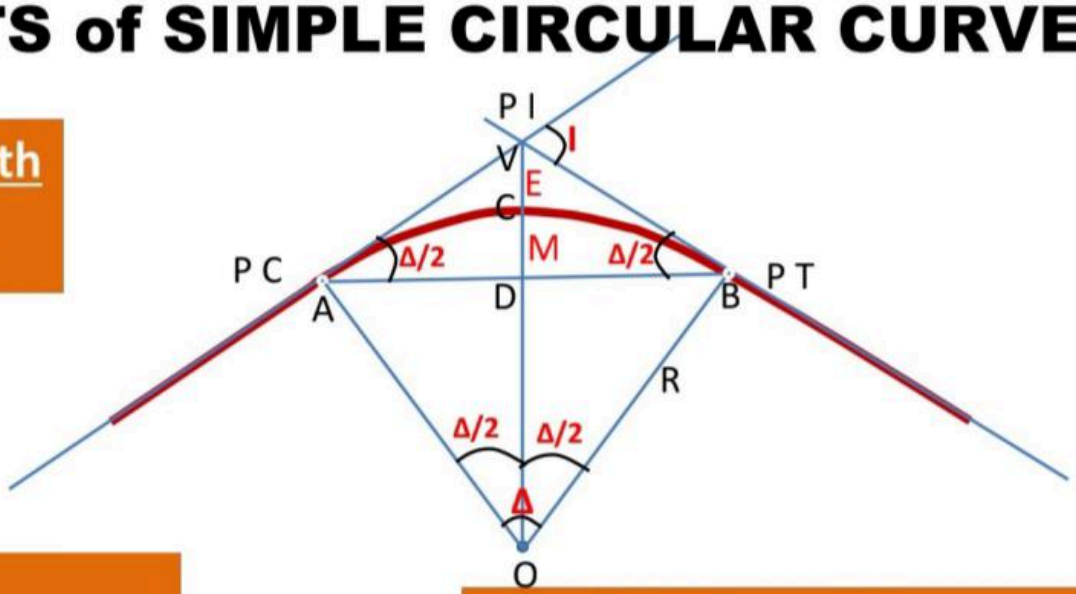
Middle Ordinate, M = CD

Central Angle

ELEMENTS of SIMPLE CIRCULAR CURVE

Tangent Dist /Length

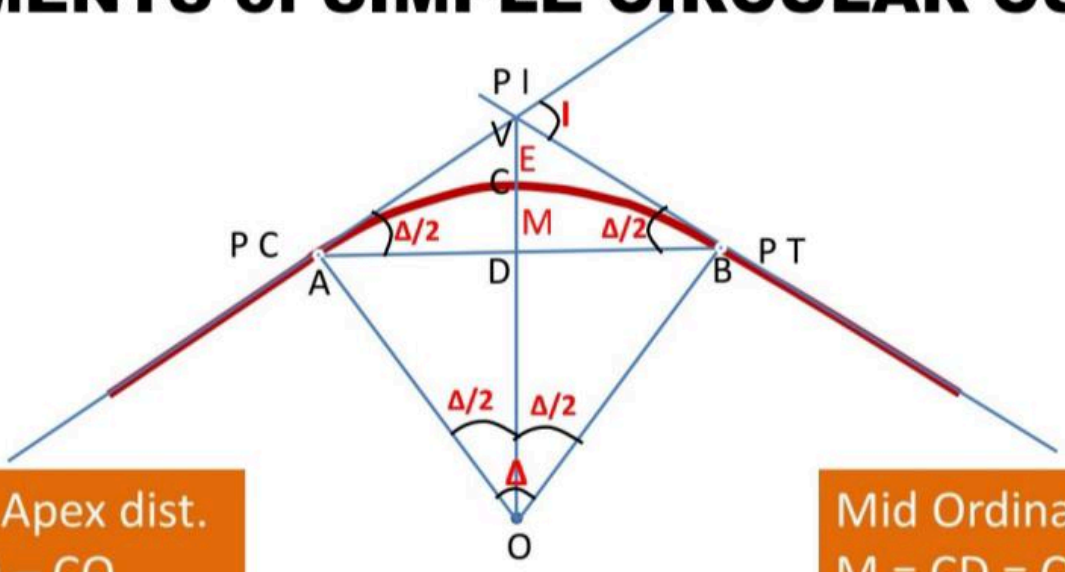
T = AV = $R.\tan(\frac{\Delta}{2})$



Long Chord (LC):
Dist. along the line joining the PC and the PT, here
AB = $2.R.\sin(\frac{\Delta}{2})$

Length of Curve (L):
Dist. from PC to PT measured along the curve, here ACB = $R\Delta = \pi R\Delta/180$

ELEMENTS of SIMPLE CIRCULAR CURVE



External or Apex dist.

$E = CV = VO - CO$

$= R.\sec(\frac{\Delta}{2}) - R$

$= R\{\sec(\frac{\Delta}{2}) - 1\}$

Mid Ordinate

$M = CD = CO - DO$

$= R - R.\cos(\frac{\Delta}{2})$

$= R\{1 - \cos(\frac{\Delta}{2})\}$

ELEMENTS of SIMPLE CIRCULAR CURVE

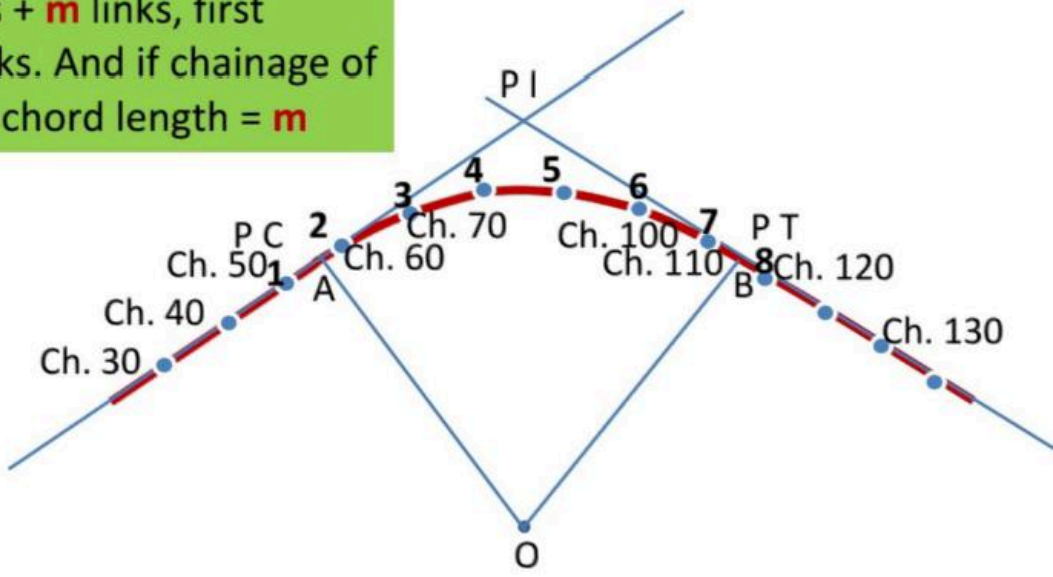
Peg interval – dist between subsequent pegs

Full chord or normal chord -100 ft in English unit, 20m in metric unit

Full station- station having chainage in the multiple of chain length. Usually Tangent points will not be full stations.

Sub chord-first chord joining the P.C to the first peg and the last chord joining the last peg to the P.T. All intermediate chord will be Normal Chord.

If chainage of P.C is **n** chains + **m** links, first chord length = **(100 – m)** links. And if chainage of P.T is **n** chains + **m** links last chord length = **m** links.



Length of the chord should be so selected that there is no appreciable difference between the length of the chord and arc. If l is 1/10 of R there will be error of 8 mm in 20m, May be taken as 1/20 of R, so that the error is only 2 mm in 20 m.

ELEMENTS of SIMPLE CIRCULAR CURVE

Back Tangent/ First Tangent:

Tangent previous to the curve. Straight line that connects the PC to the PI

Forward Tangent / Second Tangent:

The tangent following the curve. Straight line that connects the PI to the PT

Point of Intersection (PI):

at which the two tangents (back and forward tangents) to the curve intersect

ELEMENTS of SIMPLE CIRCULAR CURVE

Point of Curvature (PC):

The beginning point of the curve.
The back tangent is tangent to the curve at this point.

Point of Tangency (PT):

the end point of the curve.
The forward tangent is tangent to the curve at this point.

Tangent Distance (T):

the distance along the tangents from the PI to the PC or PT.
These distances are equal on a simple curve.

ELEMENTS of SIMPLE CIRCULAR CURVE

External Distance (E):

the distance from the PI to the middle point of the curve.
The external distance bisects the interior angle at the PI.

Middle Ordinate (M):

the distance from the midpoint of the curve to the midpoint of the long chord. The extension of the middle ordinate bisects the central angle.

ELEMENTS of SIMPLE CIRCULAR CURVE

The Central Angle (Δ):

the angle formed by two radii drawn from the center of the circle (O) to the PC or PT. The central angle is equal in value to the intersecting angle (Δ = I).

Intersection Angle (I):

The angle between produced back tangent and forward tangent at PI. Also called external deflection angle between two tangents.

Deflection Angle (Δ):

The angle at PC between the back tangent and the chord from that point to PC.

Setting out Simple Curves

Before setting out a curve it is essential to locate

- **Tangents**
- **Point of intersection**
- **Point of curve**
- **Point of tangency**

Setting out Simple Curves

Location of Tangents and Point of intersection

Obtain deflection angle (Δ) from P.I.

Location P.C and P.T

Determine tangent length from eqn T = R Tan (Δ/2)

Locate P.C and P.T. by measuring back dist T from P.I.

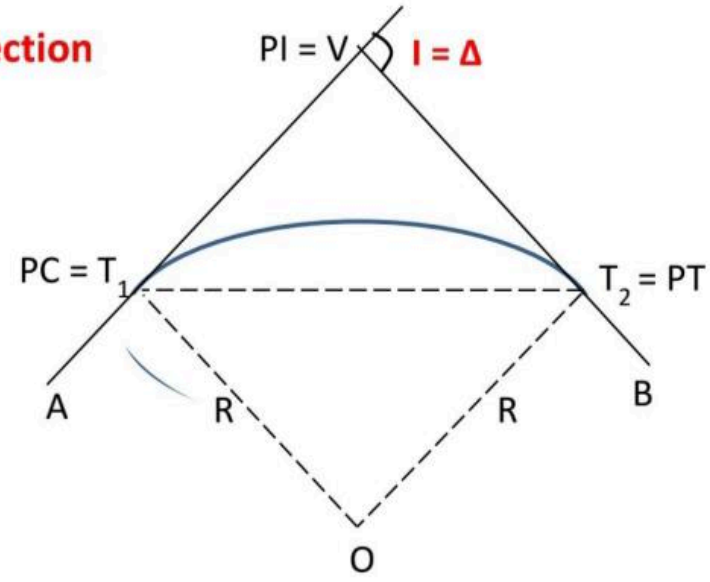
Determine the chainage of P.C. and P.T.

Chainage of P.C. = Chainage of P.I. – T (length of Tangent)

Chainage of P.T = Chainage of P.C. + length of the curve.

Set the pegs on the curves.

Pegs should be on regular interval.



Setting out Simple Curves

Determine the chainage of P.C. and P.T.

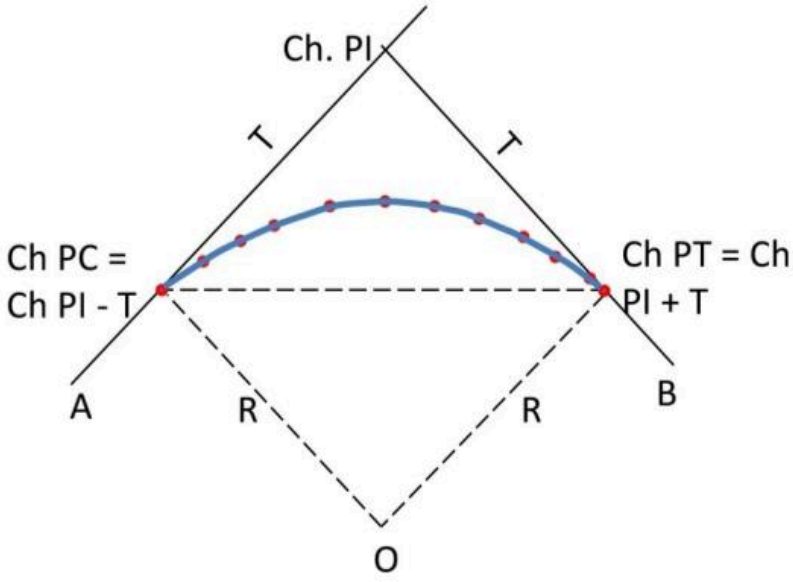
Chainage of P.C. = Chainage of P.I. – T

Ch. of P.T = Ch. of P.C. + length of curve.

Set the pegs on the curves.

Pegs should be on regular interval.

Connect the pegs and develop the curve



Setting out Simple Curves

Depending upon instrument used, two types:

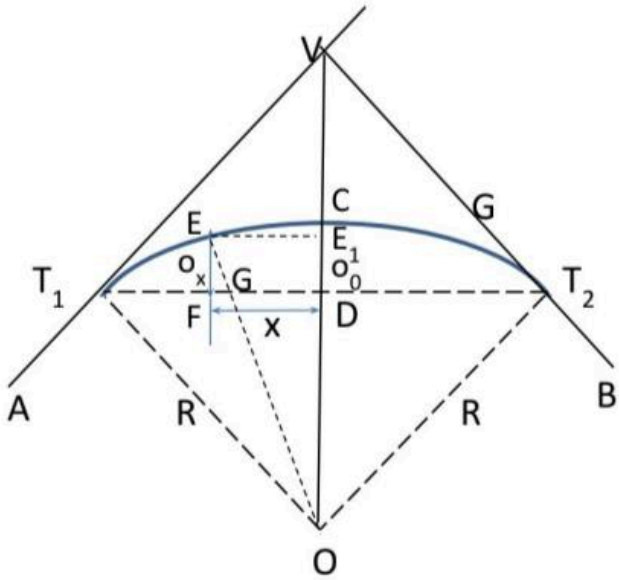
1. **Linear method:** Only chain or tape used. Used when high degree of accuracy is not required and when the curve is short.
   a. By ordinate or offset from long chord
   b. By successive bisection of arcs
   c. By offsets from tangents
   d. By offsets from chord produced or by deflection distances
2. **Angular Method:** Theodolite is used with or w/o chain or tape
   a. By Rankine's method of tangential / deflection angles
   b. By Two theodolite method
   c. By Tachemetric method

Setting out by Ordinate from Long Chord

Locate mid ordinate O_0

$$O_0 = CD = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

In order to calculate ordinate O_x to any point E (at a dist. x from the mid point of the chord), draw the line EE_1, parallel to the long chord T_1T_2. Join EO to cut the long chord at G



Then, $O_x = EF = E_1D = E_1O - DO$

$$= \sqrt{EO^2 - EE_1^2} - (CO\text{-}CD)$$

$$= \sqrt{R^2 - X^2} - (R - O_0)$$

Setting out by Offsets from the Tangents

Radial Offset

From triangle T_1DO

$DO^2 = T_1D^2 + T_1O^2$

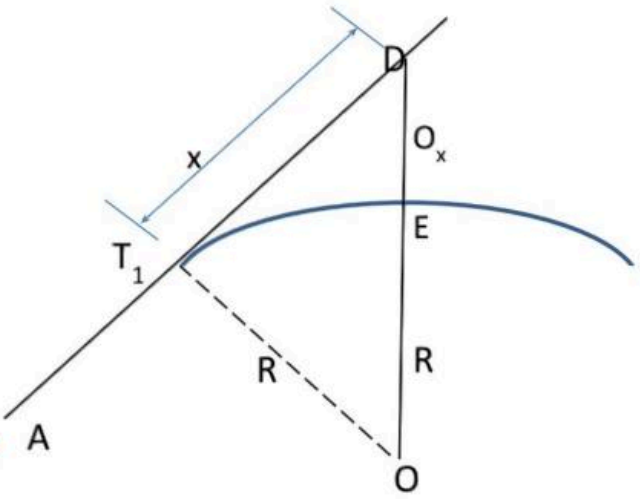
or, $(DE + EO)^2 = T_1D^2 + T_1O^2$

or, $(O_x + R)^2 = R^2 + x^2$

$O_x = \sqrt{(R^2 + x^2)} - R$ (exact)

$O_x = x^2/2R$ (approximate)

When R is large as compared to L, approximate formulae may be used.



Setting out by Offsets from the Tangents

Perpendicular Offset

From triangle EE_1O

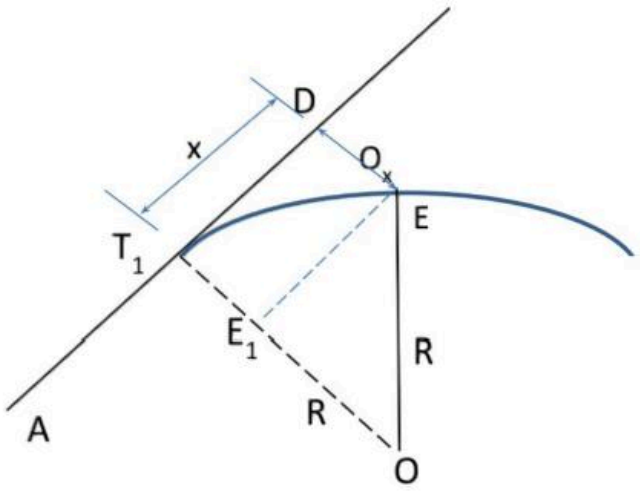
$E_1O^2 = EO^2 - EE_1^2$

or, $(T_1O - T_1E_1)^2 = EO^2 - EE_1^2$

or, $(R - O_x)^2 = R^2 - x^2$

$O_x = R - \sqrt{(R^2 - x^2)}$ (exact)

$O_x = x^2/2R$ (approximate)



Setting out by Offsets from Chords

Perpendicular Offset

From triangle EE_1O

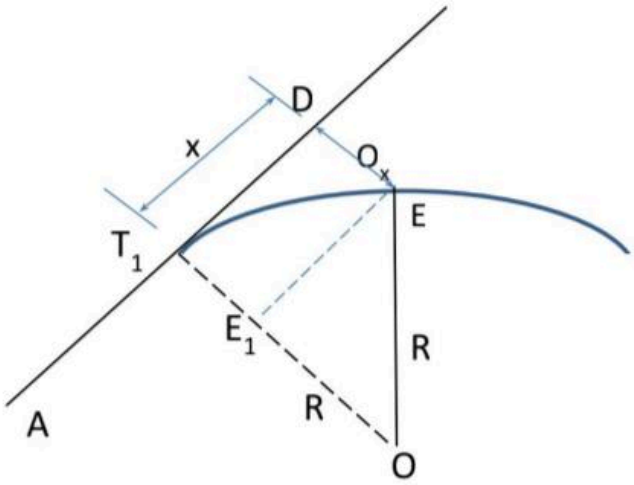
$E_1O^2 = EO^2 - EE_1^2$

or, $(T_1O - T_1E_1)^2 = EO^2 - EE_1^2$

or, $(R - O_x)^2 = R^2 - x^2$

$O_x = R - \sqrt{(R^2 - x^2)}$ (exact)

$O_x = x^2/2R$ (approximate)



Setting out by Offsets from the Chord Produced

The offset for the first chord (usually a sub chord), $O_1 = C_1^2/2R$

The offset for the Second chord (first intermediate chord)

$O_2 = C_2 (C_1+C_2)/2R$

Offsets for others,

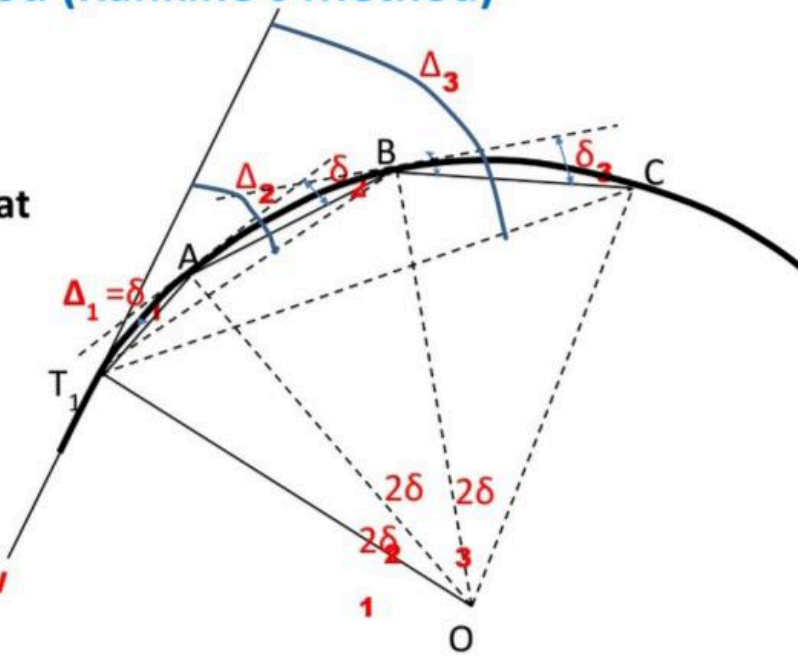
$O_3,....O_{n-1} = C^2/R$ where $C_2=C_3=C_4= C_{n-1}$.

The offset for the last chord (usually a sub chord), $O_n = C_n(C_{n-1}+C_n)/2R$

Setting out by Instrumental Method (Rankine's Method)

$\Delta_1, \Delta_2, \Delta_3, \Delta_4, \ldots\ldots\ldots \Delta_n$ are the total tangential angles or deflection angles of the chords with respect to back tangent at P.C.

$\delta_1, \delta_2, \delta_3, \delta_4, \ldots\ldots\ldots\delta_n$ are the tangential angles or the angles which each of the successive chords makes with the respective tangents. That is the angle b/w tangent drawn at the beginning of each chord and chord itself



Deflection angle or Tangential angle is half the angle subtended at the center by the chord

Setting out by Rankine's Method of Tangential or Deflection Angle)

For first chord $\Delta_1 = \delta_1$,

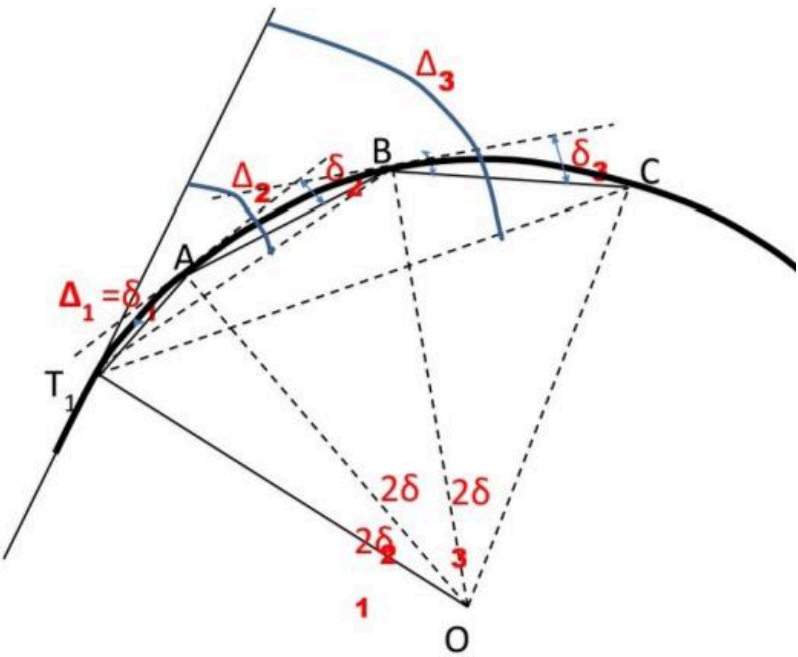
$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$

$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$

For last chord, $\Delta_n = \delta_1 + \delta_2 + \ldots\ldots$

$\delta_n = \Delta_{n-1} + \delta_n$

Check: Deflection angle of long chord = ½(angle of intersection)



Setting out by Rankine's Method of Tangential or Deflection Angle)

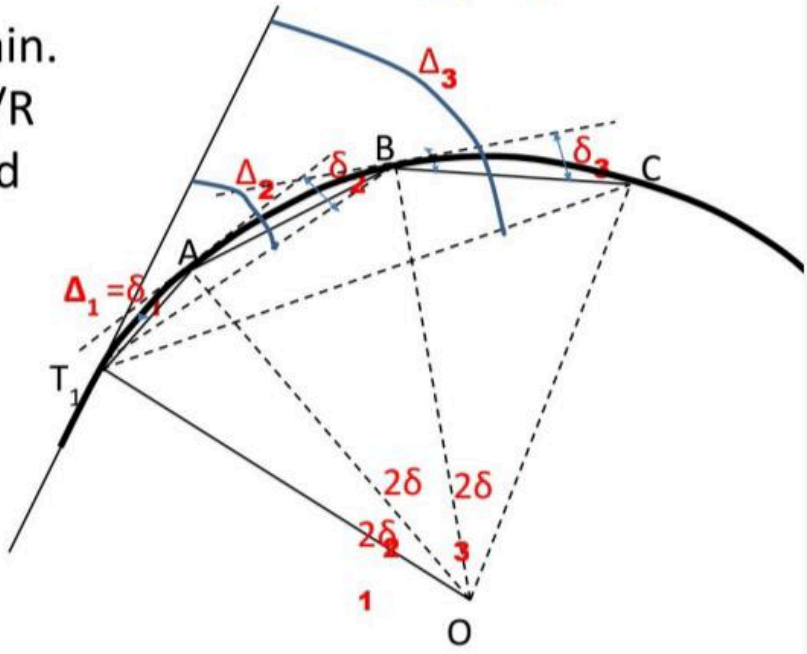
Need Theodolite and Tape or chain.

$\delta_1=\delta_2=\delta_3=\delta_4.....=\delta_n$ = 1718.9 C/R where C is the length of the chord and R is the radius of curvature.

$$\frac{T_1OA}{360}=\frac{T_1A}{2\pi R}$$

$$2\delta_1=\frac{180C_1}{\pi R}\text{ degree}$$

$$\delta_1=\frac{90C_1}{\pi R}\text{ degree}=1718.9\frac{C_1}{R}\text{ minutes}$$



Example

Prob: For a circular curve having radius R 250m,500m,and 1200m, what is their degree of curve by arc definition and by chord definition?

Sol:

Arc definition, $20/2\pi R = D^0/360^0$

Chord Definition, $\text{Sin}(D/2) = 10/R$

Example

Prob: Calculate the ordinates from chord at 25 ft intervals for a circular curve having a long chord of 200 ft and a versed sine (Mid ordinate) of 10 ft.

Sol:

$$o_x = \sqrt{(R^2 - x^2)} - (R - O_0)$$

Note: x is measured from the mid point of the chord

Example

Prob: determine the offset to be set out at half chain interval along the tangent to locate a 16 –chain curve, the length of each chain being 20m.

Sol:

Radial Offset: $o_x = \sqrt{(R^2 + x^2)} - R$

Or, Perpendicular Offset: $o_x = R - \sqrt{(R^2 - x^2)}$

Or, Approximate Offset: $o_x = \frac{x^2}{2R}$

Note: x is measured from the P.C

Example

Prob: Two tangents intersect at chainage 59+60, the deflection angle being 50°30'. Calculate the necessary data for setting out a curve of 15 chains radius to connect the two tangents if it is intended to set out the curve by offset from chords. Take peg interval equal to 100 links, length of the chain 20m (100 links)

Determine Tangent Length (T)
Determine Length of curve (l)
Obtain P.C (Chainage of P.I – Tangent length T)
Obtain P.T (Chainage of P.C + length of Curve l)
Determine Chainage of each peg: Offset from Chord produced:

$$o_n = \frac{C_n\,(C_{n-1} + C_n)}{2R}$$

This can also be determined from long chord by knowing the long chord

Example

Prob: Solve the previous problem by Rankine's method of Tangential angle. The theodolite has a least count of 20". Tabulate the actual theodolite readings of deflection angles to be set out.

Determine all data as previous.
Calculate deflection angles

$\delta = 1718.9\frac{c}{R}$min.

$\Delta_1 = \delta_1, \Delta_2 = \Delta_1 + \delta_2, \Delta_3 = \Delta_2 + \delta_3$, and so on.

Check: $\Delta_n = \frac{\Delta}{2}$

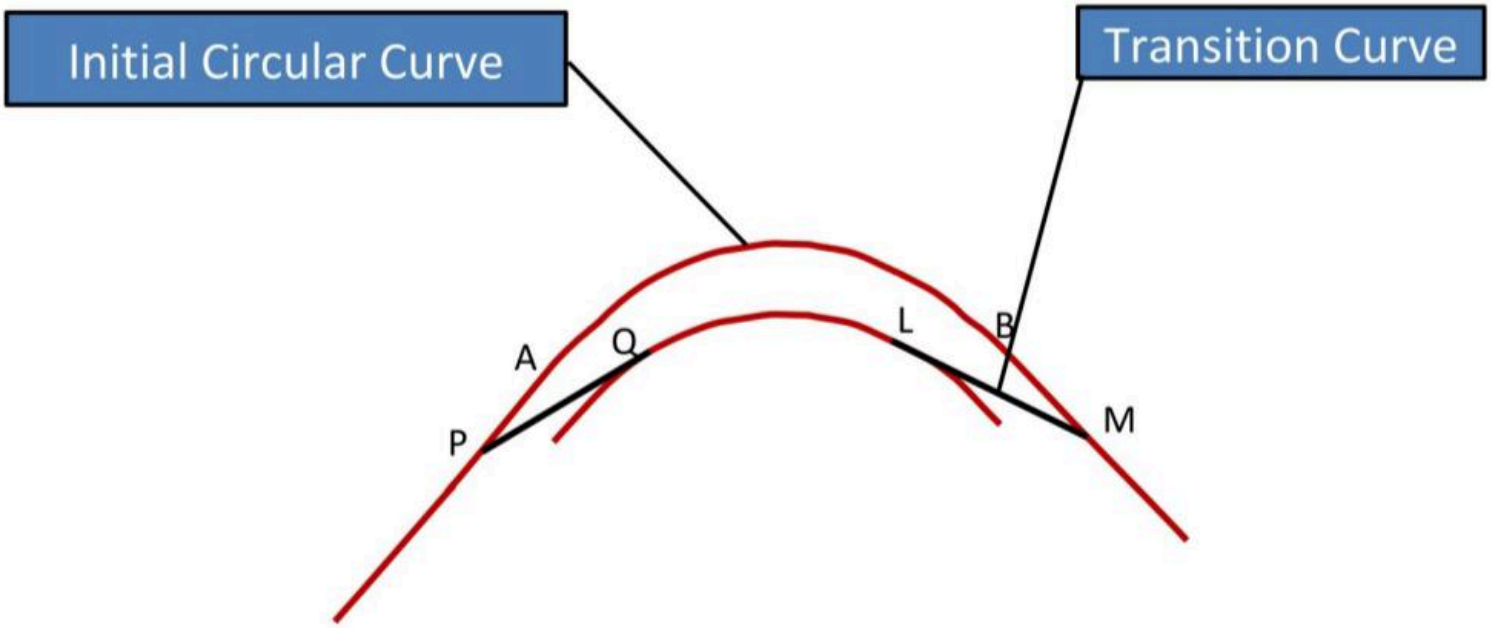
Set pegs at each deflection angles fixed by Theodolite

Transition Curve

Transition Curve

- Function
- Condition to be fulfilled by a transition curve
- Super elevation or cant
- Super elevation and side friction on highway
- Length of transition curve
- Problems

Transition Curve



Transition Curve

A transition or easement curve is a curve of varying radius introduced between a straight or a circular curve or between two branches of compound curve or reverse curve

Functions:

- **To accomplish gradually the transition from the tangent to circular curve (to increase or decrease curvature gradually)**
- **To provide required super-elevation**

Transition Curve

Conditions to be fulfilled by a transition curve:

1. **Should be tangential to the straight**
2. **Should meet the circular curve tangentially**
3. **Curvature should be zero at origin on straight**
4. **Curvature at junction with circular curve should be the same that of the circular curve**
5. **The rate of increase of curvature along the transition should be the same as that of the increase of cant or super elevation**
6. **Its length should be such that full cant or super elevation is attained at the junction with the circular curve**

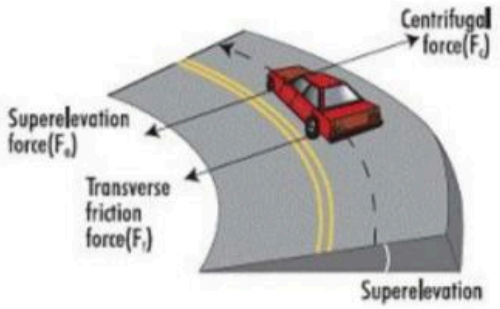
Transition Curve

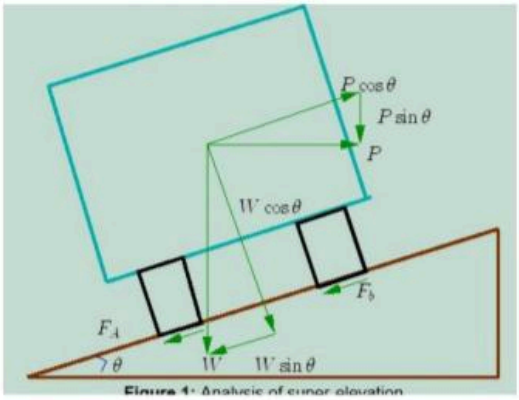
Super-elevation:

Rising of outer edge of railway or road surface above the inner edge in order to counteract or balance centrifugal force which is experienced by the train or veh when entered into a curve or turn in order to prevent derailment, overturning or side slipping of vehicles.









N = sum of the forces normal to the pavement = $P\sin\theta + W\cos\theta$

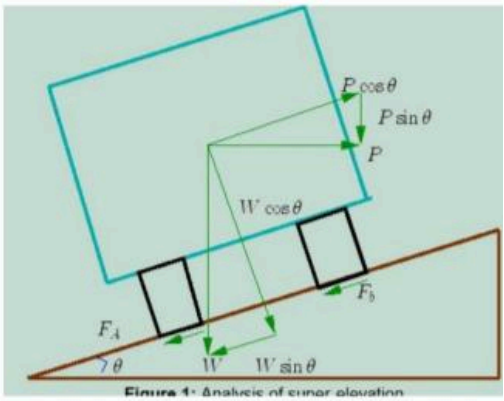
T = sum of the forces tangential to the pavement = $P\cos\theta - W\sin\theta$

If f be the friction of the pavement, then

$T = fN$

$P\cos\theta - W\sin\theta = f(P\sin\theta + W\cos\theta)$

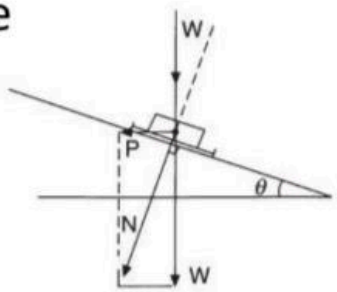
$$\frac{P}{W} = \frac{\sin\theta + f\cos\theta}{\cos\theta - f\sin\theta} = \frac{\tan\theta + f}{1 - f\tan\theta}$$



A vehicle of mass m, travelling at a constant speed v, along a curve of radius R, is subjected to a radial force

$$P = m\frac{v^2}{R} = \frac{W}{g}\frac{v^2}{R}, \text{ or } \frac{P}{W} = \frac{v^2}{gR}$$

$$\frac{P}{W} = \frac{\tan\theta + f}{1 - f\tan\theta} = \frac{v^2}{gR}$$



$$\frac{P}{W} = \frac{\tan\theta + f}{1 - f\tan\theta} = \frac{v^2}{gR}$$

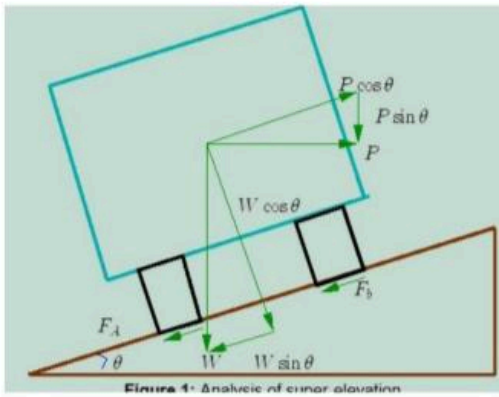


Figure 1: Analysis of super elevation

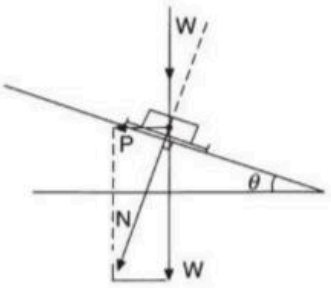
Thus the centrifugal force is balanced by the sum of the effect of the super elevation and the effect of friction.

For balancing fully by super elevation,

$$\tan\theta = \frac{v^2}{gR}$$

If to be balanced fully by friction, $f = \frac{v^2}{gR}$

Cant or Super elevation



A vehicle of mass m, travelling at a constant speed v, along a curve of radius R, is subjected to a radial force

$P = m\frac{v^2}{R} = \frac{W}{g}\frac{v^2}{R}$, or $\frac{P}{W} = \frac{v^2}{gR}$

Again e $= \tan\theta = \frac{P}{W} = \frac{v^2}{gR}$

If B width of the road then, $\tan\theta = \frac{e}{B} = \frac{v^2}{gR} = \frac{1}{4}$

If G gauge of the railway, $tan\,\theta = \frac{e}{G} = \frac{v^2}{gR} = \frac{1}{8}$

For road, v= √(gR/4)
For railways, v= √(gR/8)

Length of Transition Curve

Length should be such that the required s.e or cant is provided at a suitable rate. Three methods of determining length.

1. **By an arbitrary gradient**
2. **By the time rate**
3. **By the rate of change of radial acceleration**

Length of Transition Curve

Method 1: By an arbitrary gradient

If gradient is 1 in n, then L=ne
Value of n varies from 300 to 1200

Now put value of e and get the L

$$L = ne = nB.\tan\theta = B.v^2/gR \text{ on road}$$
$$L = ne = nG.\tan\theta = G.v^2/gR \text{ on railways}$$

Length of Transition Curve

Method 2: By the time rate

Let e is applied at an arbitrary time rate of r units per sec.

L=length of transition curve in m

v= speed of veh in m/sec

e amount of s.e.

Time taken by a veh to pass over a transition curve = t

=L/v sec.

Super elevation attained in this time = t x r cm = r.L/v

e= r.L/v

L = e.v/r meters

Length of Transition Curve

Method 3: By the rate of change of radial acceleration

In this method length of transition curve is decided on the basis of comfort of passengers. It is experienced that a rate of change of radial acceleration of 1 ft/sec (0.3 m/sec) will pass unnoticed.

If α is the rate of change of radial acceleration in m/sec and t is the time taken to travel over the transition curve having length L, Then acceleration attained in that time is = αt =αL/v sec.

But radial acceleration of a circular curve is v^2/R, so $v^2/R = \alpha L/v$

So $L = v^3/\alpha R$ where v is in m/sec

When α = 0.3 m/sec then $L = V^3/14R$ where V is in kmph

Length of Transition Curve

1. By an arbitrary gradient

   $L = ne = B.v^2/gR$

1. By the time rate

   $L = e.v/r$

2. By the rate of change of radial acceleration

   $L = v^3/\alpha R$ or $L = V^3/14R$

Prob 3.1: A transition curve is required for a circular curve of 200m radius, the gauge being 1.5 m and max super elevation restricted to 15 cm. The transition is to be designed for a velocity such that no lateral pressure is imposed on the rails and the rate of gain of radial acceleration is 30 cm/sec^3. calculate the required length of the transition curve and the design speed.

$$\text{Again } e = \tan\theta = \frac{e}{G} = \frac{v^2}{gR}$$

$$\frac{15}{150} = \frac{v^2}{9.81 \times 200}$$

Thus v = 14 m/sec

$$L = \frac{v^3}{\alpha R} = \frac{14^3}{0.3 \times 200} = 46 \text{ m}$$

Prob 3.2: A Road bend which deflects 80° is to be designed for a max speed of 100 km per hr, a max centrifugal ratio of ¼ and a max rate of change of acceleration of 30 cm/sec^3. the curve consisting of a circular arc combined with two cubic spirals. Calculate (a) radius of circular arc, (b) te requisite length of transition, © the total length of composite curve (d) The cahaonage of the beginig and the end of transition curve and of the junctions of the transition curves with the circular arc if the chainage of the P.I is 42862m.

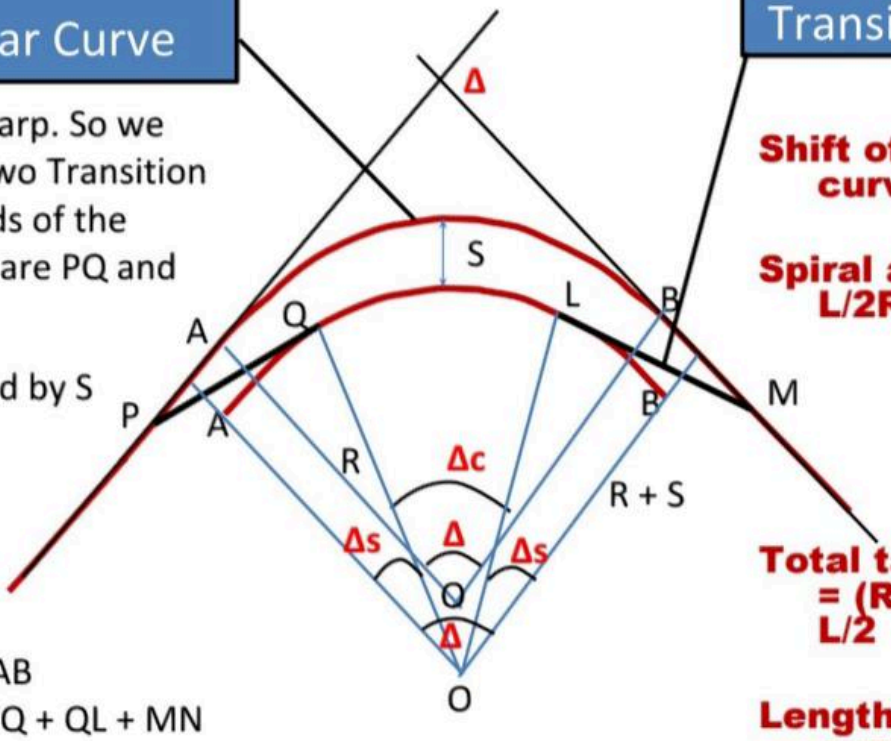
Combined Curve

Combined Curve: two transition curves on either sides of a circular curve.

Initial Circular Curve

This curve is very sharp. So we want to introduce two Transition curves on either ends of the circular curve. They are PQ and MN

Circular curve is shifted by S



Transition Curve

Shift of Circular curve, $s = L^2/24R$

Spiral angle, $\Delta_s = L/2R$

Total tangent length = $(R+s)\tan(\Delta/2) + L/2$

Length of combined curve = $\pi R (\Delta - 2\Delta_s)/180 + 2L$

Initial curve length = AB

New curve Length = PQ + QL + MN

$= L_C + L_T = + L_T$

$\Delta = \Delta c + 2\Delta s$

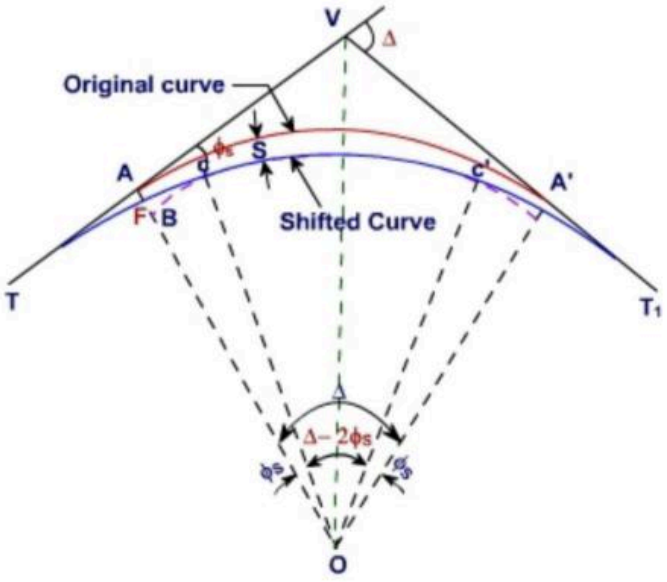
Setting out the Combined Curve

Shift of Circular curve, s = $L^2/24R$

Spiral angle, $\Delta_s = L/2R$

Total tangent length = $(R+s)\tan(\Delta/2) + L/2$

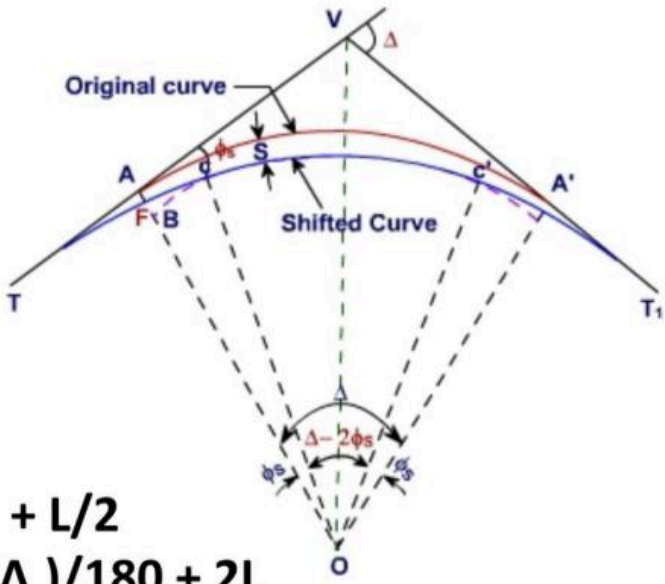
Length of combined curve = $\pi R\,(\Delta - 2\Delta_s)/180 + 2L$



Setting out the Combined Curve

Followings are required:

- **Deflection angle Δ**
- **Radius of circular curve R**
- **Length of transition curve**
- **Chainage of P.I**



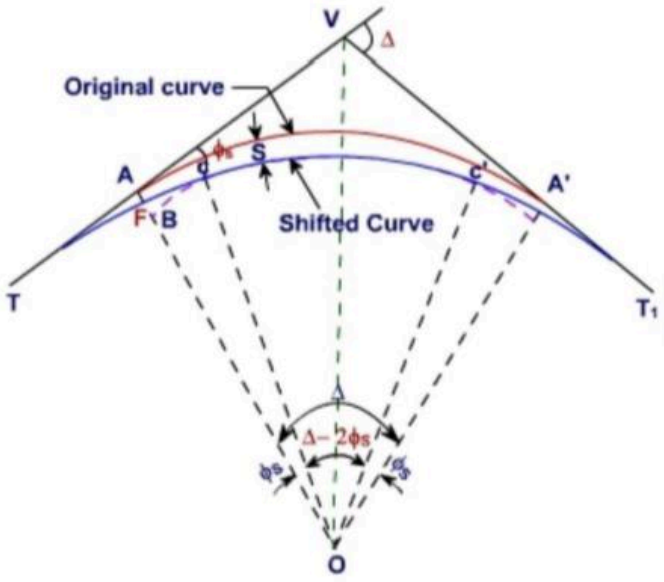
Following calculation to be made:

- ✔ Shift of Circular curve, $s = L^2/24R$
- ✔ Spiral angle, $\Delta_s = L/2R$
- ✔ Total tangent length = $(R+s)\tan(\Delta/2) + L/2$
- ✔ Length of combined curve = $\pi R\,(\Delta - 2\Delta_s)/180 + 2L$

Setting out the Combined Curve

Then follow the steps asbelow:

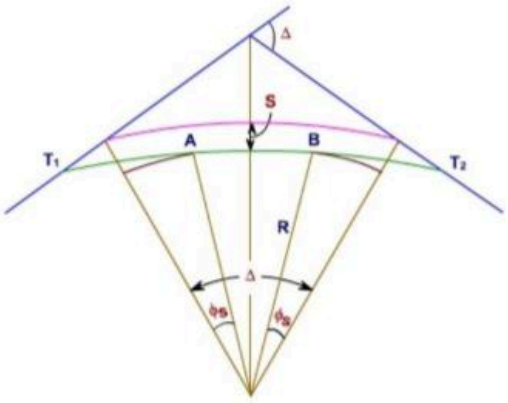
- ✔ Substract Length of tangent from chainage of P.I to get Chainage of T
- ✔ Add length of transition curve to to get chainage of junction
- ✔ Add length of circular curve to get chainage of other junction
- ✔ Add length of transition curve to get the chaiange of other Tangent point



Example: A road 10m wide is to deflect through an angle of 65° with the centre line radius 350 m, the chainage of the intersection point being 1006 m. A Transition curve is to be used at each end of the circular curve of such a length that the rate of gain of radial acceleration is 0.4 m^3/sec, when the speed is 60 kmph. Find out

(a) Length of the transition curve
(b) Chainage of all junction points
(c) offset at x = L/4, L/2, 3L/4 and L

Given, deflection angle D = 65°
Radius of the circular curve, R = 350 m
Road width, B = 10 m
Chainage of P.I = 1006 m
Speed of vehicle, V = 60 kmph = 16.67 m sec^{-1}
Radial acceleration, a = 0.4 m^3/sec



Length of the transition curve $L = \frac{V^3}{aR} = \frac{(16.67)^3}{(0.4 \times 350)}$

$$\text{Shift}, S = \frac{L^2}{24R} = \frac{(33.07)^2}{(24 \times 350)} = 0.13\text{m}$$

Spiral angle, Δ_s= L/2R
= 33.07/(2 X 350) = 0.047 rad x (180/3.14) deg = 2°42′

Length of the tangent = (R + S) tan (Δ/2) + L/2
= (350 + 0.13)tan(65/2)+33.07/2 = 239.59 m

Chainage of tangent T_1 = Chainage of P. I – Tangent length
= 1006.00 - 239.59 = 766.41 m

Length of circular curve = $\frac{\pi R\, \Delta c}{180} = \frac{\pi \times 350 \times (65° - 2 * 2°42')}{180}$ **= 375.5 m**

Length of combined curve = length of circular curve + length of transition curve = 397.06 + 33.07 = 430.13 m

Chainage of T_2 = 766.41+ 430.13 = 1196.54 m

Chainage of First Junc point = Chainage of tangent T_1 + length of transition curve = 766.41+ 33.07 = 799.48 m

Chainage of last Junc point = Chainage of First Junc point + length of circular curve = 799.48 + 375.5 = 1175 m

Example: A road bend which deflects 80^0 is to be designed for a max speed of 100 kmph, a max centrifugal ratio of ¼ and a max rate to the change of acceleration of 30 m/sec^3, the curve consisting of a circular arc combined with two cubic spirals. Calculate

a. The radius of circular arc
b. the requisite length of transition
c. the total length of composite curve
d. the ch. of the beginning and end of transition curve and the junctions of the transition curves with the circular arc if the ch. of the P.I is 42862 m.

Example: Two tangent AB and BC intersect at chainage 1000 m the deflection angle being 45°. It is proposed to insert a circular curve of 300m radius with a transition curve 90m long at each end. Make all calculation necessary for setting out the curve by the method of deflection angle using peg interval as 15m on the transition curve and 30m on the circular curve.

- ✔ Shift of Circular curve, $s = L^2/24R$
- ✔ Spiral angle, $\Delta_s = L/2R$ radians
- ✔ Total tangent length = $(R+s)\tan(\Delta/2) + L/2$
- ✔ Length of combined curve = $\pi R\,(\Delta - 2\Delta_s)/180 + 2L$

Radius of the circular curve, R = 300m

Length of the transition curve, L = 90m

(i) Shifts, $S = \frac{L^2}{24R} = \frac{90^2}{24 \times 300} = 1.125m$

(ii) Total tangent length of the combined curve

$$T_t = (R + S)\tan\left(\frac{\Delta}{2}\right) + \frac{L}{2}$$

$$= (300 + 1.125)\tan\frac{45°}{2} + \frac{90}{2}$$

$$= 169.73m$$

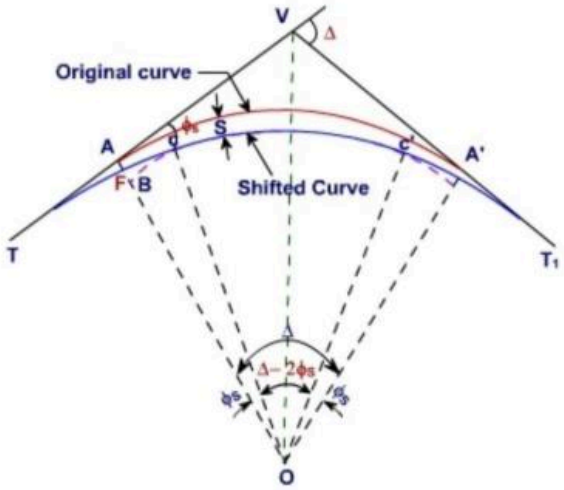


Figure 39.2 Simplified combined curve

(iii) Spiral angle,

$$\Delta_S = \frac{L}{2R} = \frac{180}{\pi} \times \frac{90}{2 \times 300} = 8°35'40"$$

(iv) Central angle for the circular curve = $\Delta - 2\Delta_S = 45° - 2 \times 8°35'40" = 27°48'40"$

(v) Length of the circular curve,

$$l = \frac{\pi R(\Delta - 2\Delta_S)}{180°}$$

$$= \frac{\pi \times 300 \times 27°48'40"}{180°} = 145.62m$$

(vi) Chainages at

T = chainage at V − T_t = 1000 − 169.73

= 830.27 m

C = chainage of T + L

= 830.27 + 90 = 920.27 m

C' = chainage of C + l

= 920.27 + 145.62 = 1065.89m

T' = chainage of C'+ L

= 1065.89 + 90 = 1155.89m

Vertical Curve

Provided where change of grade in the vertical plane to smoothen the change

A vertical curve should be designed so that

- **It gives smooth riding qualities**
- **Should have adequate sight distance before the veh reaches the summit**

Vertical Curve

Types

1. **Sag curve**
2. **Crest or summit curve**
3. **Rising curve**
4. **Falling Curve**

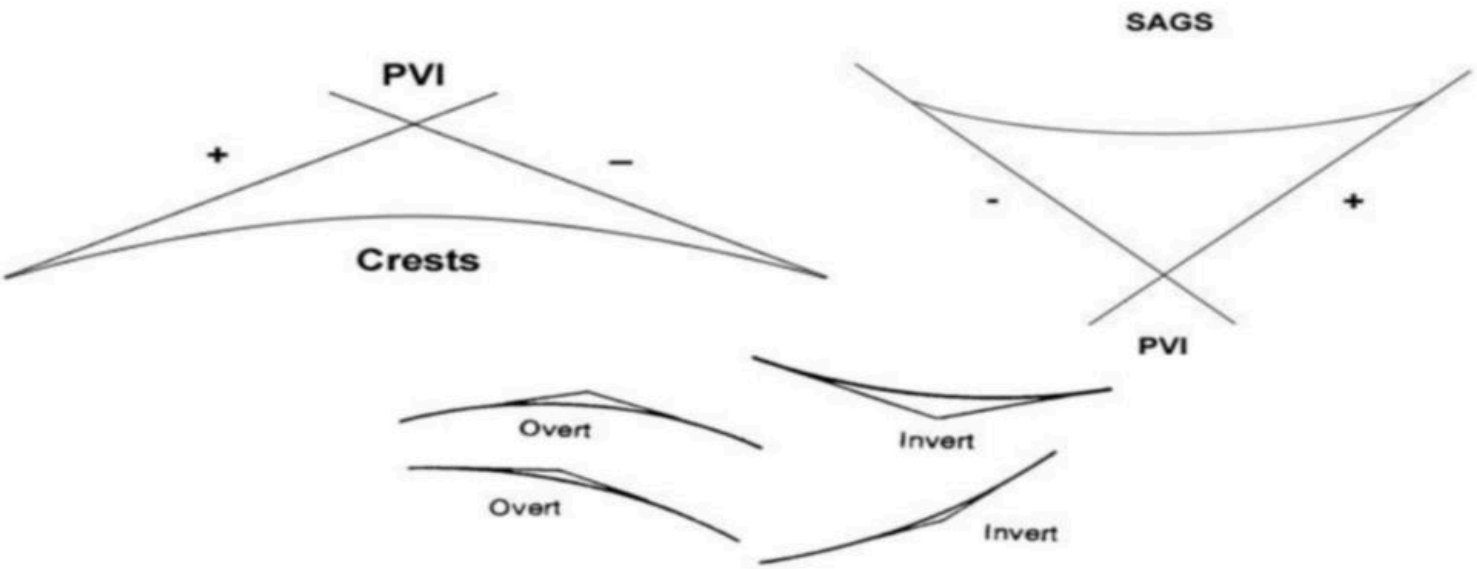
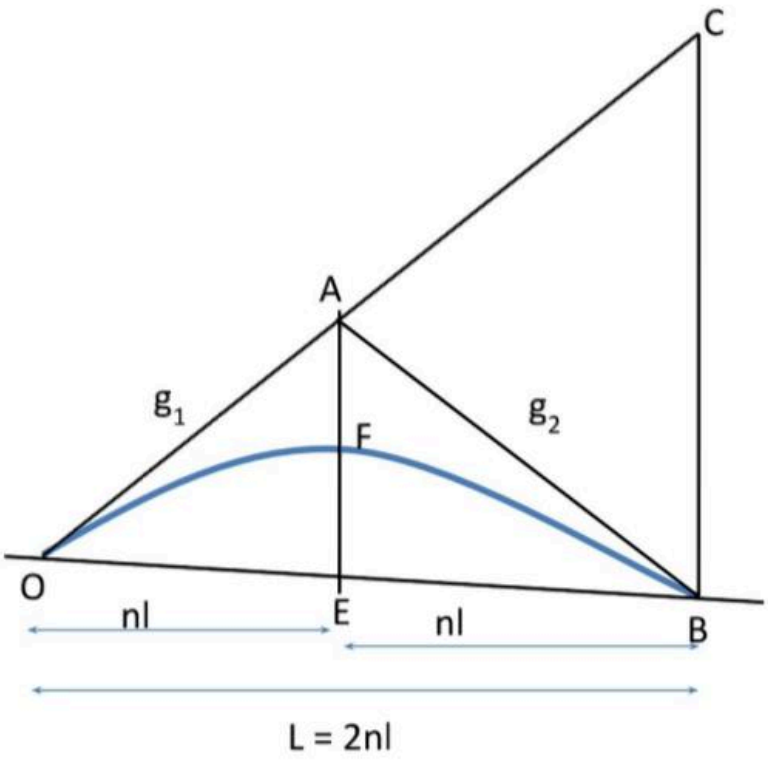


Figure 9-19. Types of vertical curves

- $Length\ of\ the\ Vertical\ Curve = \frac{total\ change\ of\ grade}{Rate\ of\ change\ of\ grade} = \frac{g_1 - g_2}{r}$
- But length of curve should be determined from the consideration of Sight distance.



$$Length\ of\ the\ Vertical\ Curve, L = \frac{total\ change\ of\ grade}{Rate\ of\ change\ of\ grade} = \frac{g_1 - g_2}{r}$$

Prob: A parabolic vertical is to be set out connecting two uniform grades of 0.8% and – 0.9%. The chainage and RL of the PI are 1664 m and 238.75 m respectively. The rate of change of grade is 0.05% per chain of 20 m. calculate the RL of various station pegs.

- $Length\ of\ the\ Vertical\ Curve = \frac{g_1 - g_2}{r} \frac{+0.8-(-0.9)}{0.05}$= 34 chains
- Length on either side of vertex = 34/2 = 17 chains
- Ch of 1st tangent point = 1664-17x20= 1324 m
- Ch of 2nd tangent point = 1664+ 17x20 = 2004 m
- RL of the beginning of curve = 238.755 – 17x(0.8x20/100) = 236.065 m
- RL of the end of curve = 238.755 – 17x(0.9x20/100) = 235.695 m
- RL of mid point between beginning and end = (236.065 + 235.695)/2 = 235.865 m.
- RL of vertex = (RL of PI + RL of mid point)/2 = (238.755 + 235.865)/2 = 237.31 m