Tacheometric Surveying

Generally, horizontal distances are measured by direct methods, i.e. laying of chains or tapes on ground.

These methods are not always convenient if the ground is undulating, rough, difficult and inaccessible. **Under these circumstances, indirect methods are used to obtain distances. One such method is "Tacheometry".**

It is best adapted in obstacles such as steep and broken ground, deep ravines, stretches of water or swamp and so on, which make chaining difficult or impossible

Using tacheometric methods, elevations can also be determined.

What is Tacheometry?

Tacheometry is a method of surveying in which ***horizontal and vertical distances of points are obtained by optical means as opposed to the ordinary slower processes like measurement by chan, tape etc.***

The method is **very rapid and convenient.**

The measurement taken are of **lower degree of accuracy.**

Advantages or Uses of Tacheometry

- Tacheometry is **used for preparation of topographic map** where both horizontal and vertical distances are required to be measured
- This method is **used in difficult terrain** where direct methods of measurements are inconvenient
- It is **convenient for reconnaissance surveys** of routes, for hydrographic surveying and for filling in details in a traverse.
- There is **considerable saving in time and money** with the use of tacheometric methods.
- It is **used for checking distances measured by chains, tapes, dumpy levels**.

Instrument Used

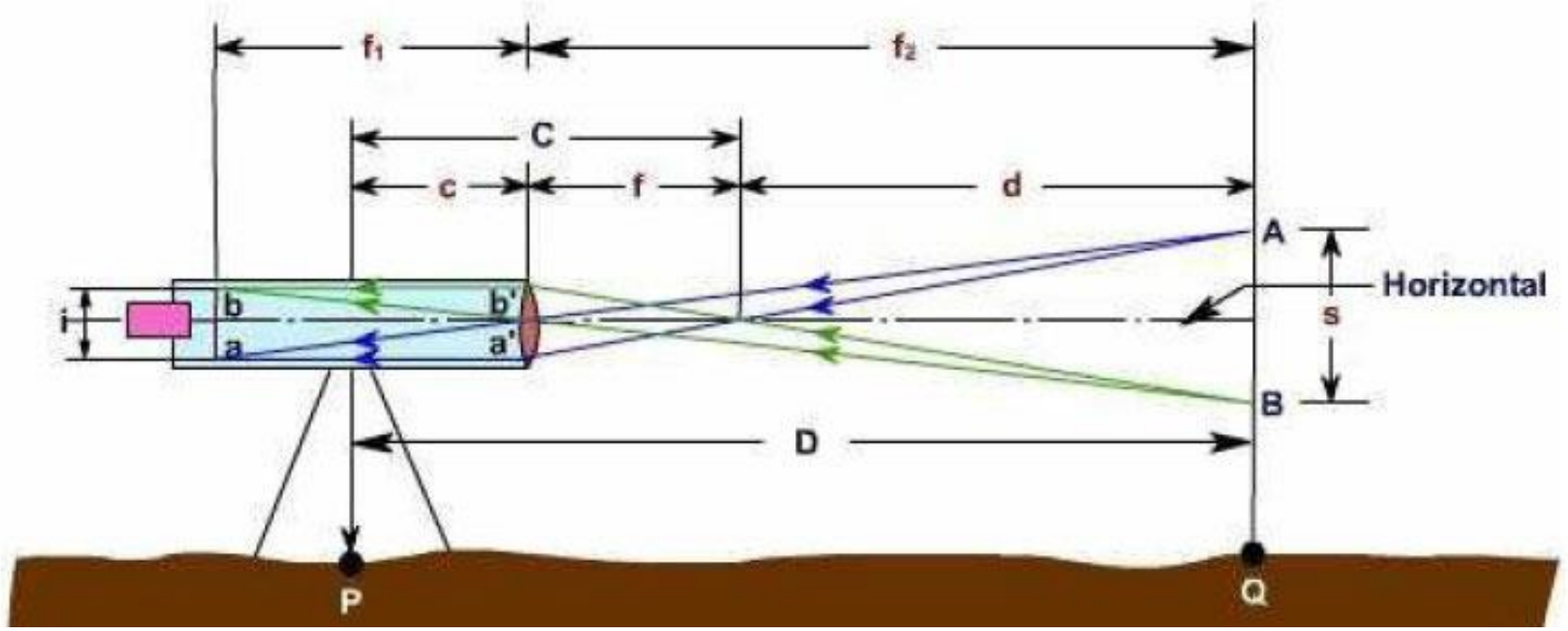
The instruments employed in tacheometry are:

- **Tacheometer / Engineer's transit/ Theodolite**
- **Leveling rod or stadia rod**
- **Subtense bar**

For **short sights** (100 m or less), an **ordinary levelling staff**.

For **long sights**, special staff called **stadia rod**.





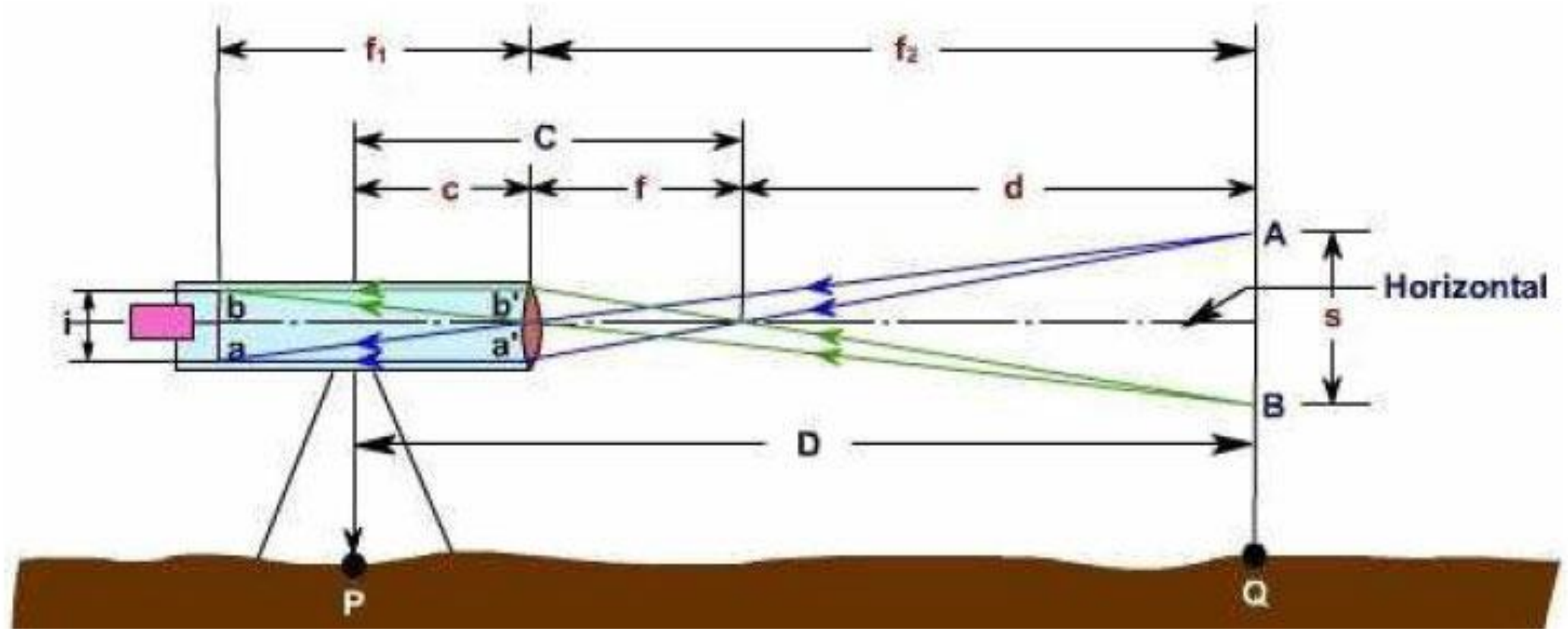
Tacheometer

The telescope used in stadia surveying are of three kinds:

(1) The simple external-focusing telescope

(2) the external-focusing anallactic telescope

(3) the internal-focusing telescope.

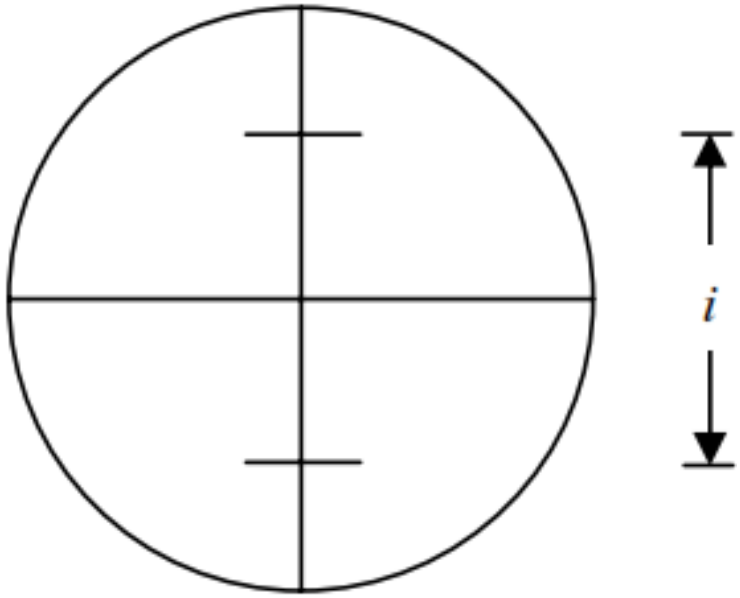


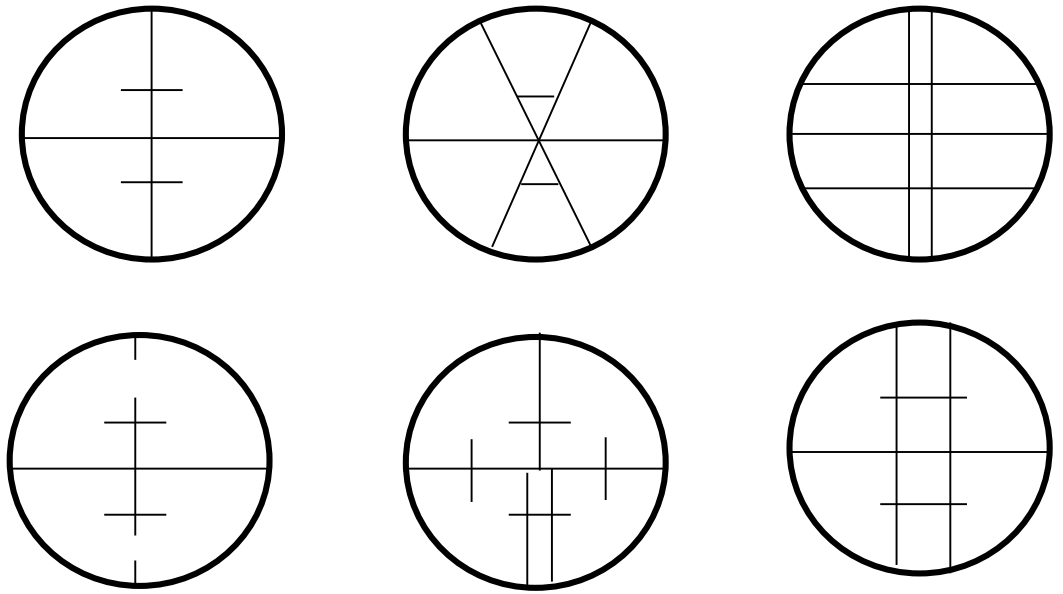
Instrument - Tacheometer

A tacheometer is similar to **a transit theodolite, fitted with two stadia wires in addition to the central cross-hair**.

The **upper and lower stadia hairs are equidistant from the central horizontal hair**.

Stadia hairs are sometimes called **stadia lines**.





Different forms of stadia diaphragm commonly used

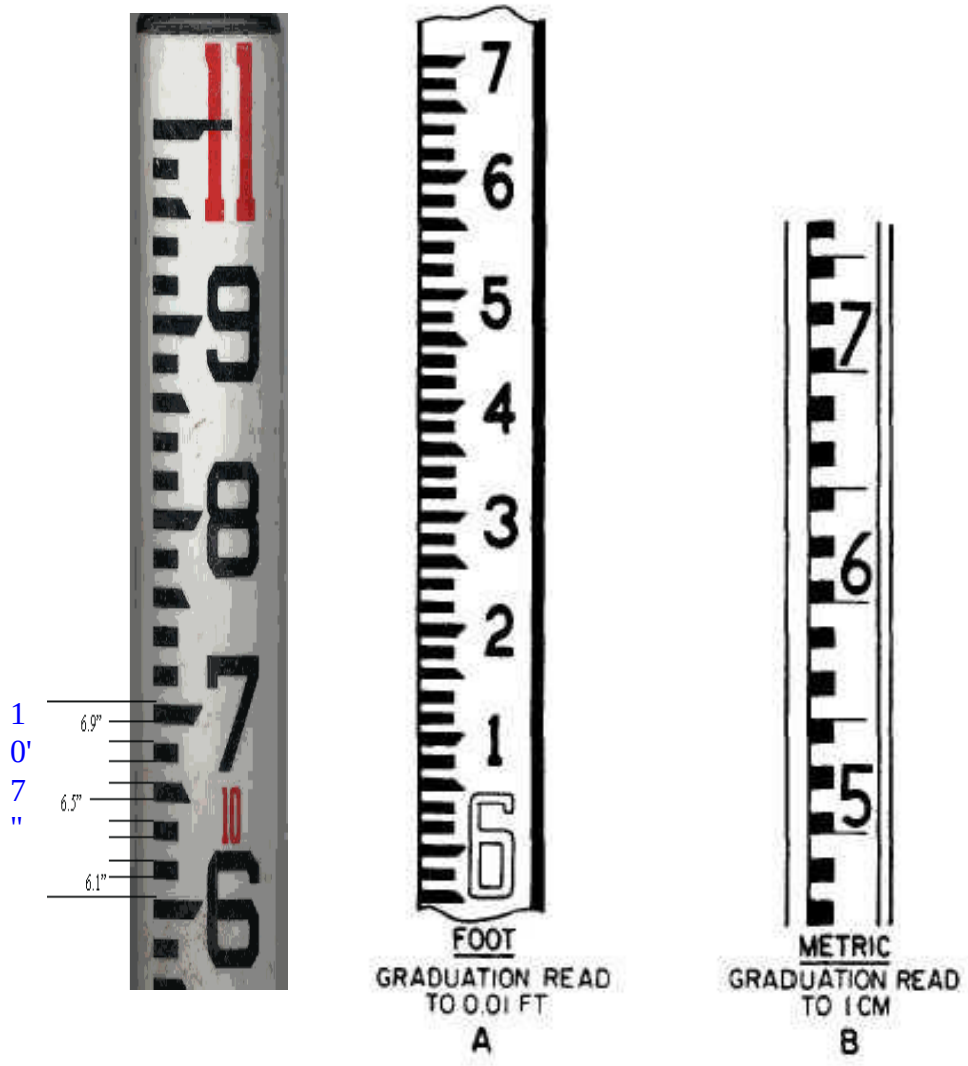
Instrument Used - Stadia Rods

It is an special staff somewhat similar to levelling staff.

The **graduations are in bold** type. A stadia rod **graduated in 5 mm** (i.e. 0.005 m) **for smaller distances and in 1 cm (i.e. 0.01 m) for longer distances.**

It is **3 m to 5 m long**. It is **hinged** to fold up.

Fitted with a small circular spirit level on its backside to keep the staff/rod vertical.



Systems of Tacheometric Measurements

There are basically three systems of tacheometric measurements such as:

1. **The stadia System**

   (a) Fixed Hair method, or Stadia method

   (b) Movable hair method, or Subtense method

2. **The tangential system**

3. **Measurements by means of special instruments (Subtense bar system).**

Stadia Systems – Fixed Hair method

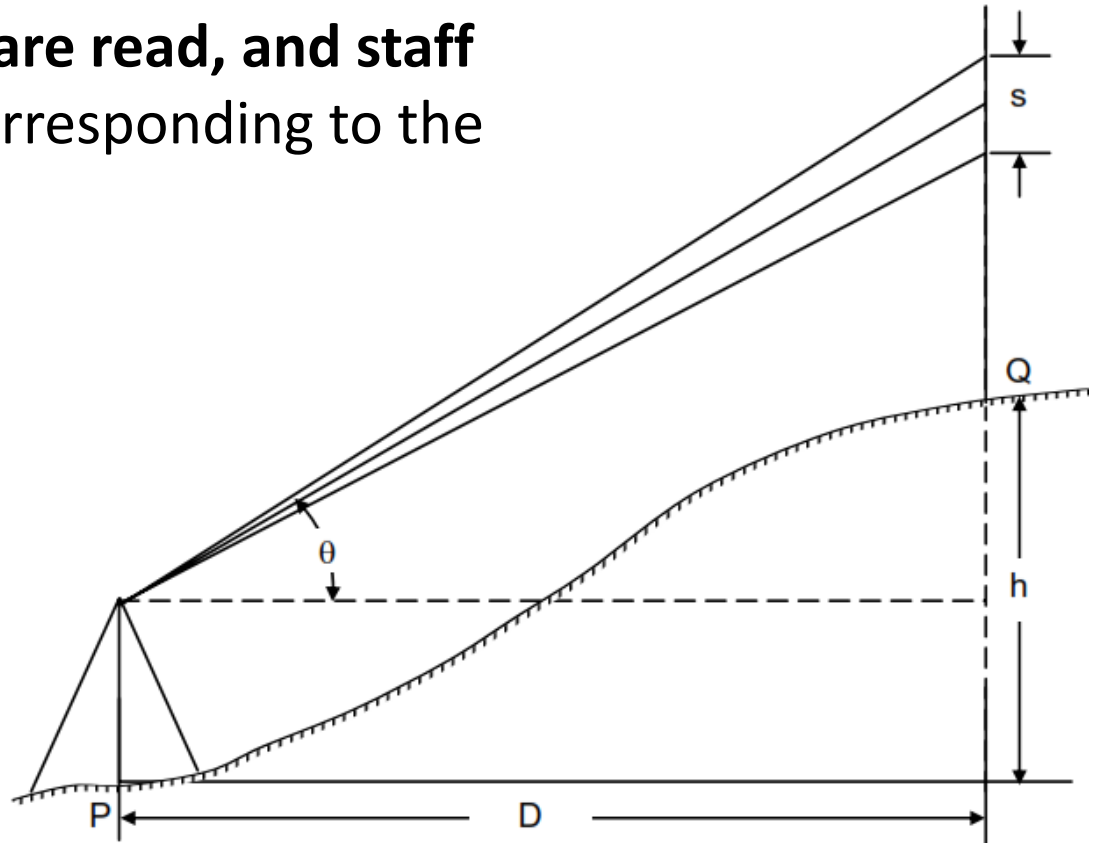
- In this method, **stadia hairs are kept at fixed interval** and the **staff interval or intercept** (corresponding to the stadia hairs) **on the leveling staff varies**.
- Staff intercept depends upon the distance between the instrument station and the staff.
- The fixed hair method is the one which is **commonly used** and, unless otherwise mentioned, **stadia method means fixed hair method.**

Stadia System - Fixed Hair Method

➢All the three hairs including **central cross hair are read, and staff intercept 's' is determined. Vertical angle 'θ',** corresponding to the central hair is also measured.

➢These measurements enable **determination of horizontal distance between P and Q and their difference in elevation.**

➢**When the staff intercept is more than the length of the staff, only half intercept is read.**

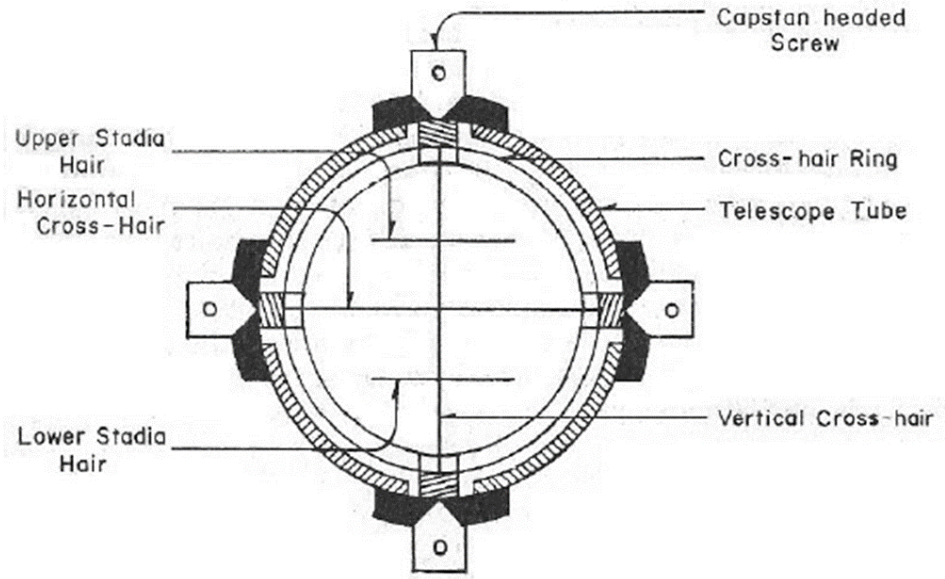


Stadia Systems – Movable Hair method

In this method, the **staff interval is kept constant** and the **stadia hairs are adjusted** to bisect the upper target at the upper hair and the lower target at the lower hair.

Instruments **need to have the provision for measurement of the variable interval** between the stadia hairs.

Movable hair method is **not in common use due to difficulties in determining the value of i (**stadia interval) **accurately.**



Basic Principle of Stadia Method

Horizontal Sights

***O* is the optical centre** of the objective glass of the telescope

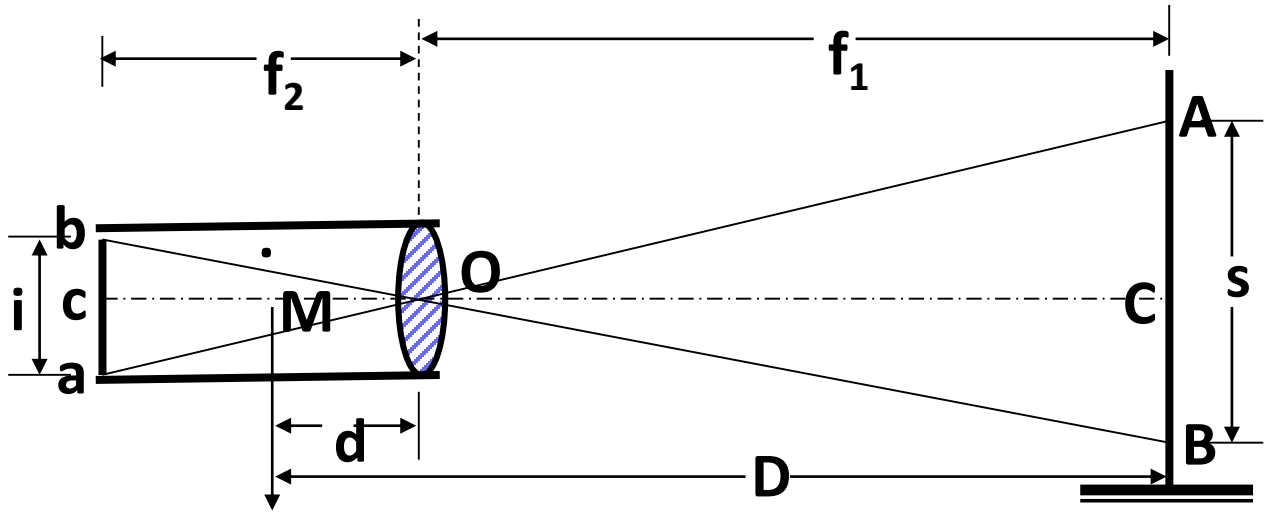
f_1 = horizontal dist. of the staff from O

f_2 = hor. dist. of the cross-wires from O

d = dist. of the vert. axis of the instr. from O.

M = Centre of the instr.

D = hor. dist. of the staff from the vert. axis of the instr.



b, c, and a = top, axial and bottom hairs of the diaphragm.

ab = i = interval betweenthe stadia hairs (stadia interval)

AB = s = staff intercept;

f = focal length of the objective

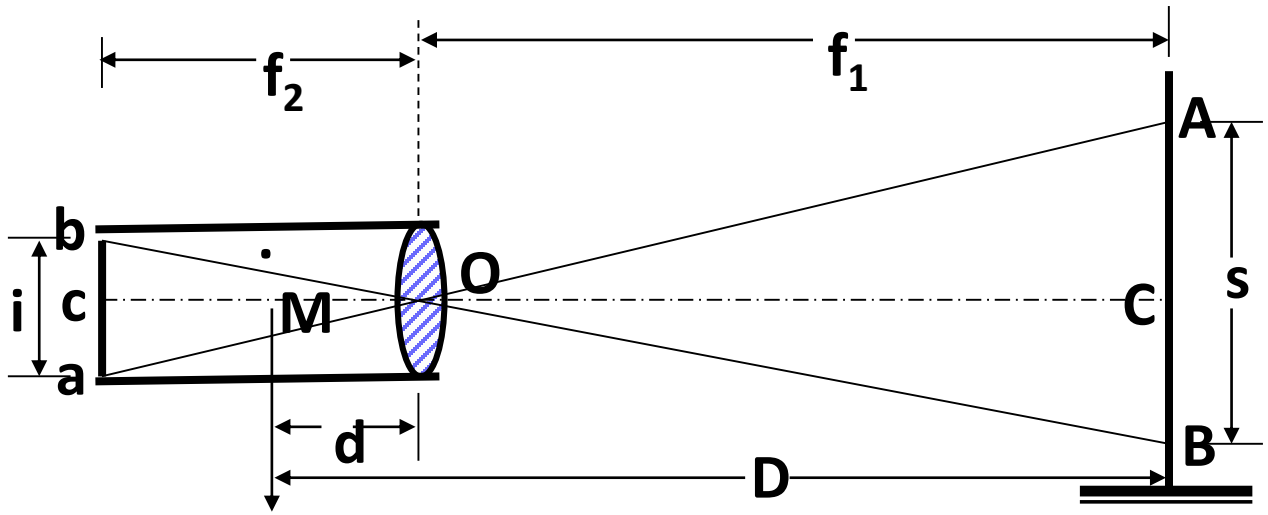
***Let A, C, and B* = the points cut by the three L of S corresponding to three wires.**

Basic Principle of Stadia Method

Horizontal Sights

From figure, ***aOb*** **and** ***AOB*** **are similar.**

Hence, $\frac{f_1}{f_2} = \frac{s}{i}$



Again, since f_1 **and** f_2 **are conjugate focal dist., from lens formula,** $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Multiplying throughout by ff_1**, we get,** $f_1 = \frac{f_1}{f_2} f + f$

Substituting the values of $\frac{f_1}{f_2} = \frac{s}{i}$ **in the above, we get ,** $f_1 = \frac{s}{i} f + f$

Horizontal distance between the axis and the staff is D = $f_1 + d$

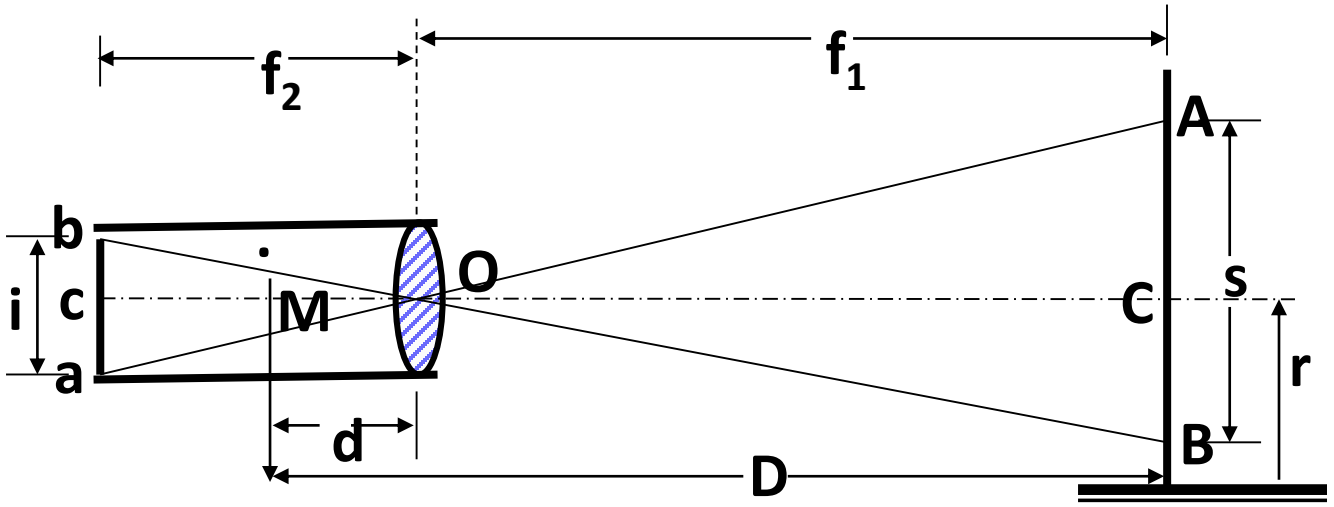
$$D = \frac{s}{i} f + f + d = ks + C$$

Where k = $\frac{f}{i}$**, known as Multiplying constant and**

C = (f+d), which is known as Additive constant

Basic Principle of Stadia Method

Horizontal Sights



$$\textbf{Formulae for Dist, } D = \text{ks} + \text{C}$$

$$\textbf{Formulae for Elev.} = \text{Elev of Sta} + \text{h (HI)} - \text{r (central reading)}$$

Determination of Tacheometric Constants

Step 1 : Set up the tacheometer at any station say P on a flat ground.

Step 2 : Select another point say Q about 200 m away. Measure the distance between P and Q accurately with a precise tape. Then, drive pegs at a uniform interval, say 50 m, along PQ. Mark the peg points as 1, 2, 3 and last peg-4 at station Q.

Step 3 : Keep the staff on the peg-1, and read staff intercept say s_1

Step 4 : Likewise, obtain staff intercepts say s_2, at peg-2,

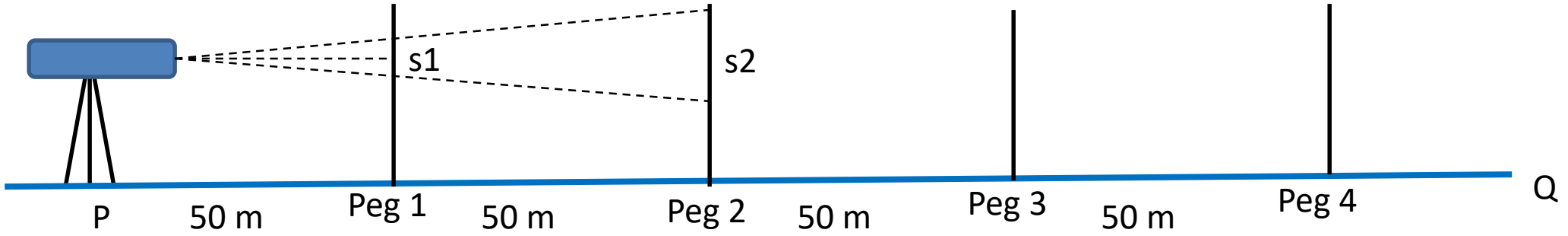
Step 5 : Solving Equations (i) and (ii), determine the values of K and C say K_1 and C_1

$$\mathbf{D_1 = K.\ s_1 + C \text{ ----------(i)}}$$

$$\mathbf{and\ D_2 = K.\ s_2 + C \text{ ----------(ii)}}$$

Step 6 : Form another set of observations to the pegs 3 & 4, for staff intercepts s_3 and s_4, solving equations, determine the values of K and C again say K_2 and C_2.

Step 7 : The average of the values obtained in steps (5) and (6), provide the tacheometric constants K and C of the instrument.



Anallactic Lens

It is a **special convex lens**, fitted in **between the object glass and eyepiece**, at a fixed dist from the object glass, **inside the telescope of a tacheometer**.

The f**unction** of the anallactic lens is **to reduce the stadia constant to zero**.

Thus, when tacheometer is fitted with anallactic lens, the distance measured between instrument station and staff position (for line of sight perpendicular to the staff intercept) becomes directly proportional to the staff intercept. $\boldsymbol{D = \mathrm{ks} + 0}$

Anallactic lens is **provided in an external focusing type telescopes only**.

Disadvantage - Brightness of the image is much reduced due to absorption of light.

Theory of Tacheometry (Stadia Method)

Observations may be made with either **horizontal L of S or with inclined L of S**

For **Staff Normal to Inclined L of S**

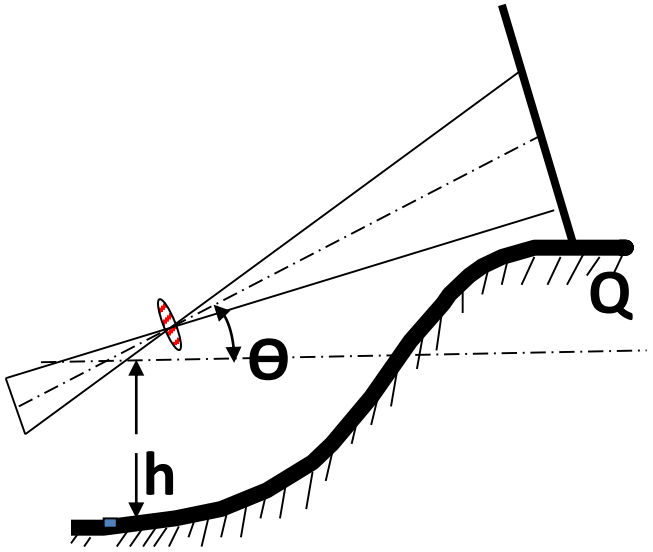
Line of Sight at an angle of elevation

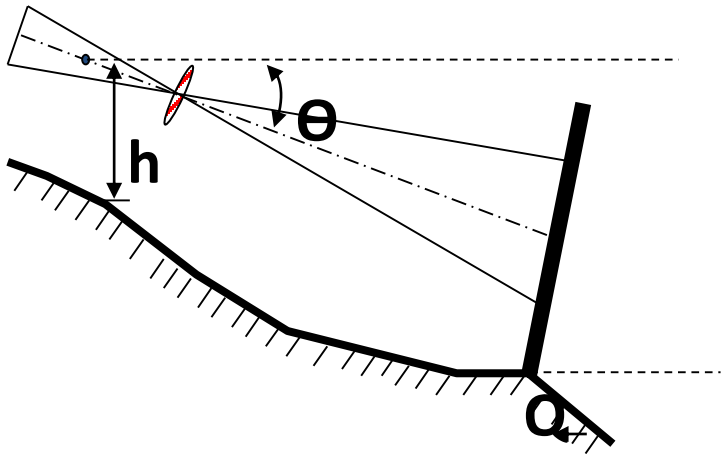
Line of Sight at an angle of depression

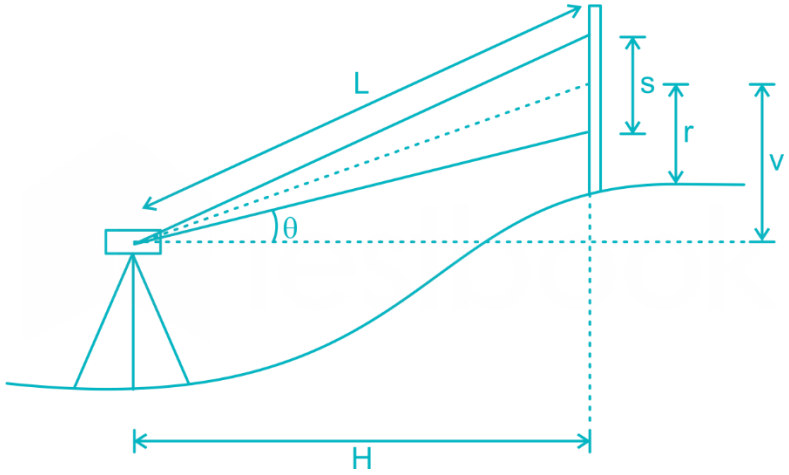
For **Staff Vertical to Inclined L of S**

Line of Sight at an angle of elevation

Line of Sight at an angle of depression







Distance and Elevation formulae for Staff Normal to Inclined L of S

Case (a): Line of Sight at an angle of elevation θ

Let *AB* = s = staff intercept;

CQ = r = axial hair reading

we have, $MC = D = K\,s + c$

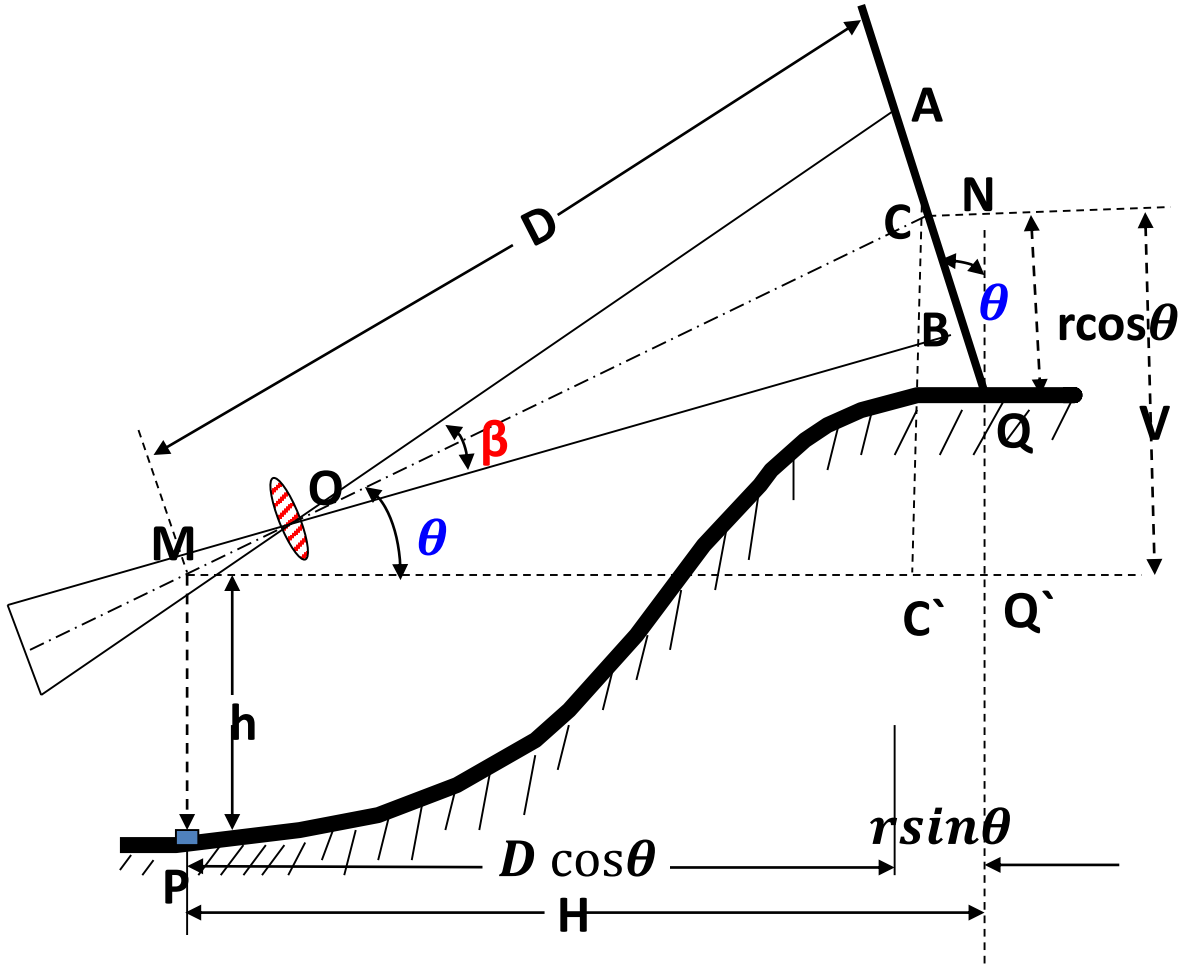
The horizontal dist, H = MC' + C'Q'

$= D\,cos\theta + r\,sin\theta$

$= (k\,s + C)\,cos\theta + r\,sin\theta$

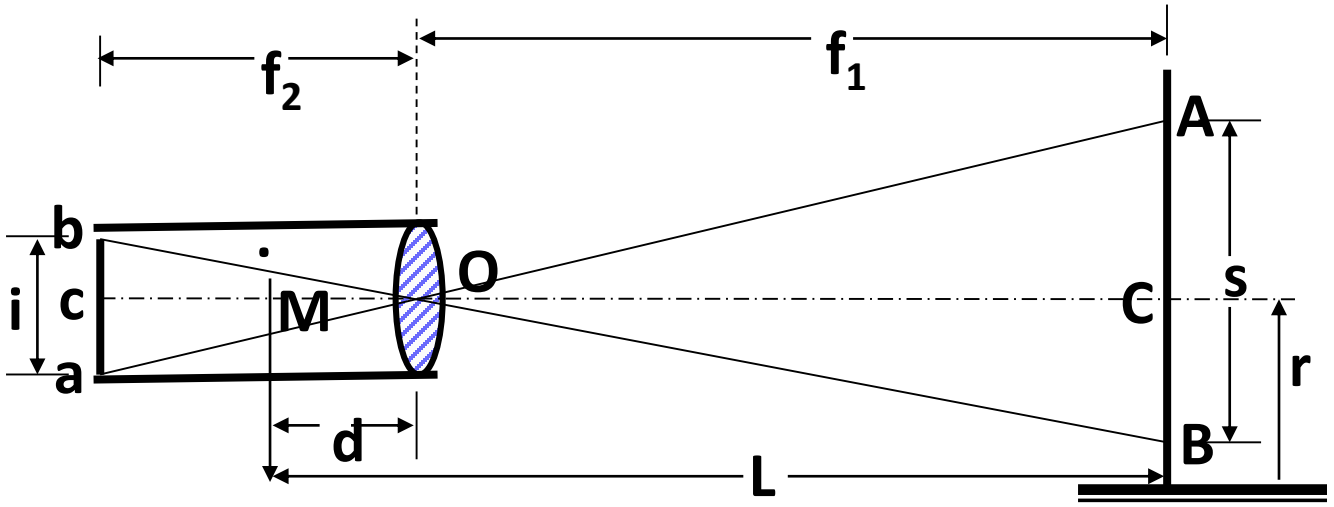
Similarly, V = D sin θ = (k s + C) sin θ

Elevation of Q = Elevation of P + h + V – r cosΘ



Basic Principle of Stadia Method

Horizontal Sights

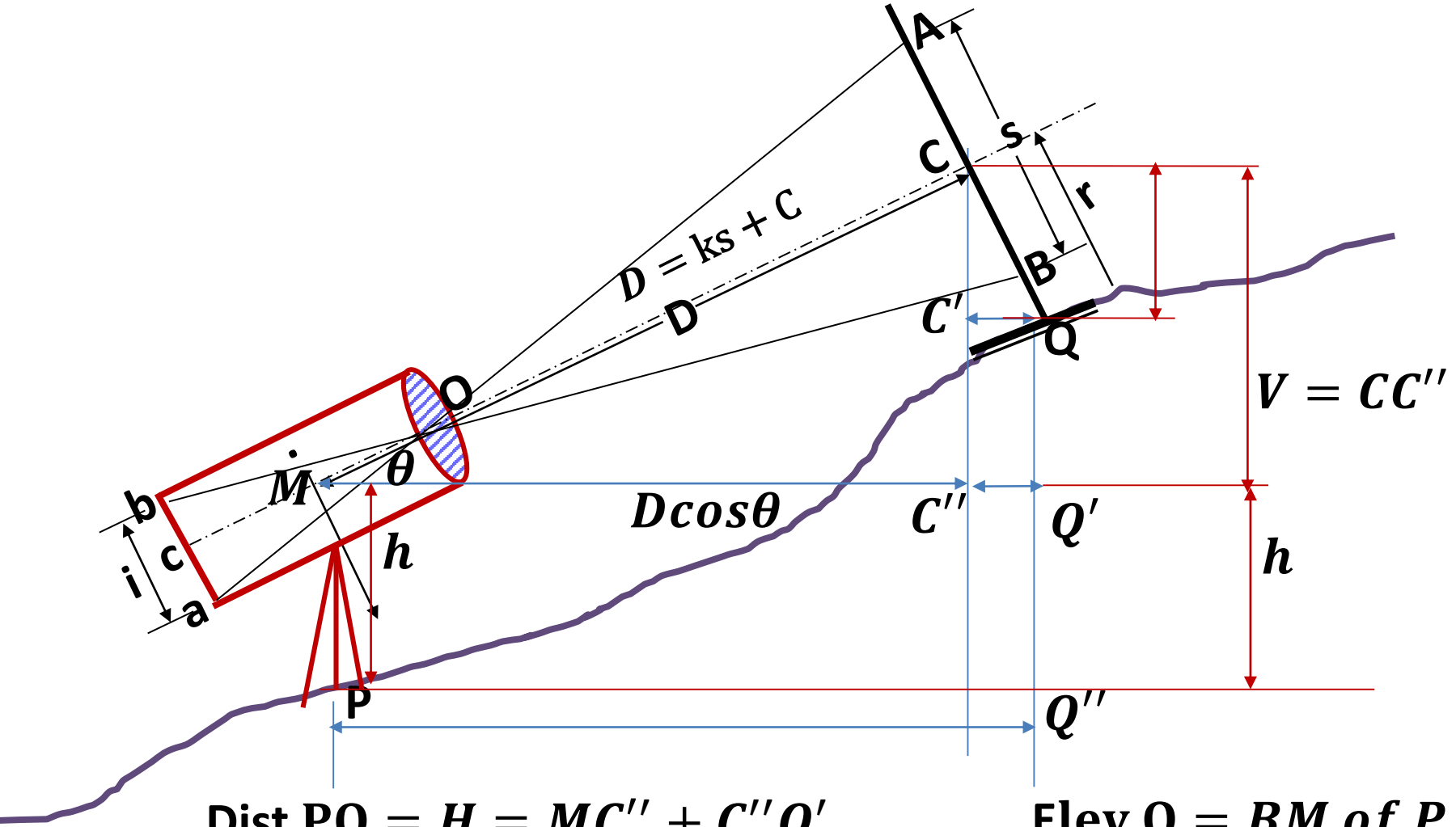


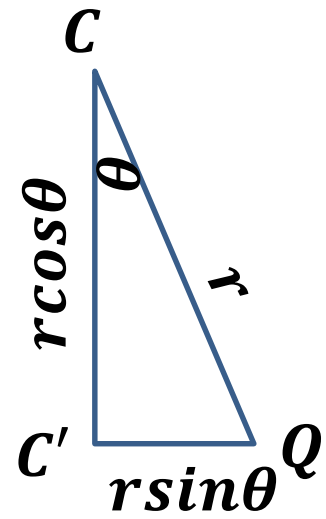
$$\textbf{Formulae for Dist, } L = \text{ks} + \text{C}$$

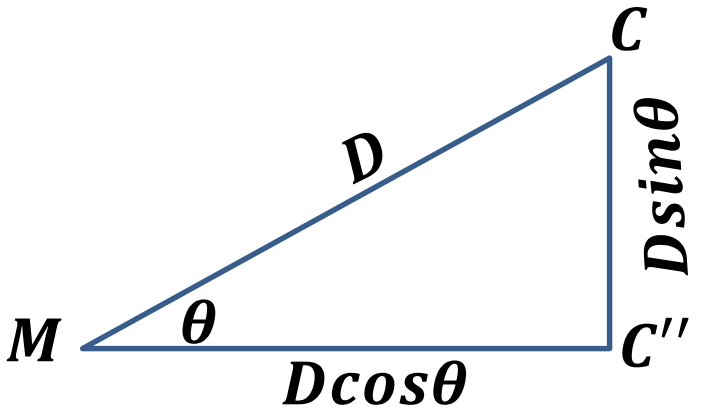
$$\textbf{Formulae for Elev.} = \text{Elev of Sta} + \text{h (HI)} - \text{r (central reading)}$$

Basic Principle of Stadia Method

Inclined Sights







Dist PQ $= H = MC'' + C''Q'$

H$= Dcos\theta + rsin\theta$

H$= (ks + C)\, cos\theta + rsin\theta$

Elev Q $= BM\ of\ P + h(HI) + V(CC'') - CC'$

$= BM\ of\ P + h + Dsin\theta - rcos\theta$

$= BM\ of\ P + h + (ks + c)sin\theta - rcos\theta$

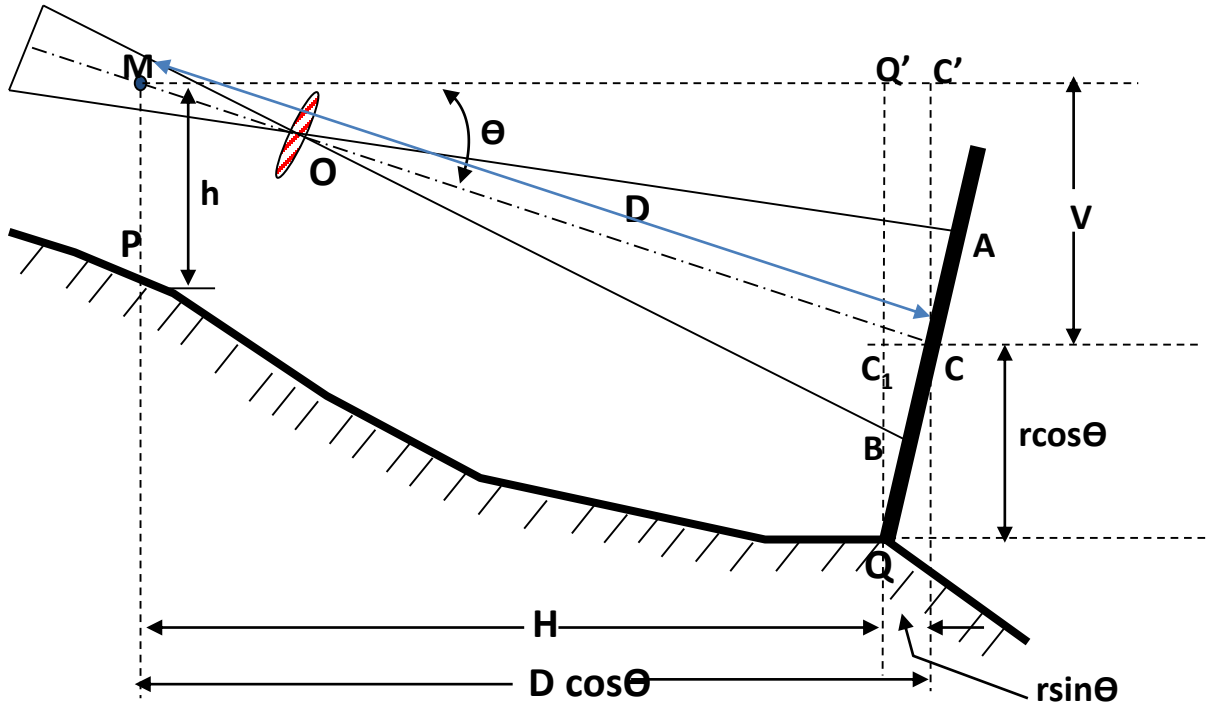
Distance and Elevation formulae for Staff Normal to Inclined L of S

Case (b): Line of Sight at an angle of depression θ

Dist of Q from P,

H = MQ' = MC' – Q'C' = $D\ cos\theta\ - r\ sin\theta$

or, $H\ =\ (k\ s\ +\ C)\ cos\theta\ - r\ sin\theta$



Elevation of Q = $Elevation\ of\ P\ +\ h - V - r\ cos\theta$

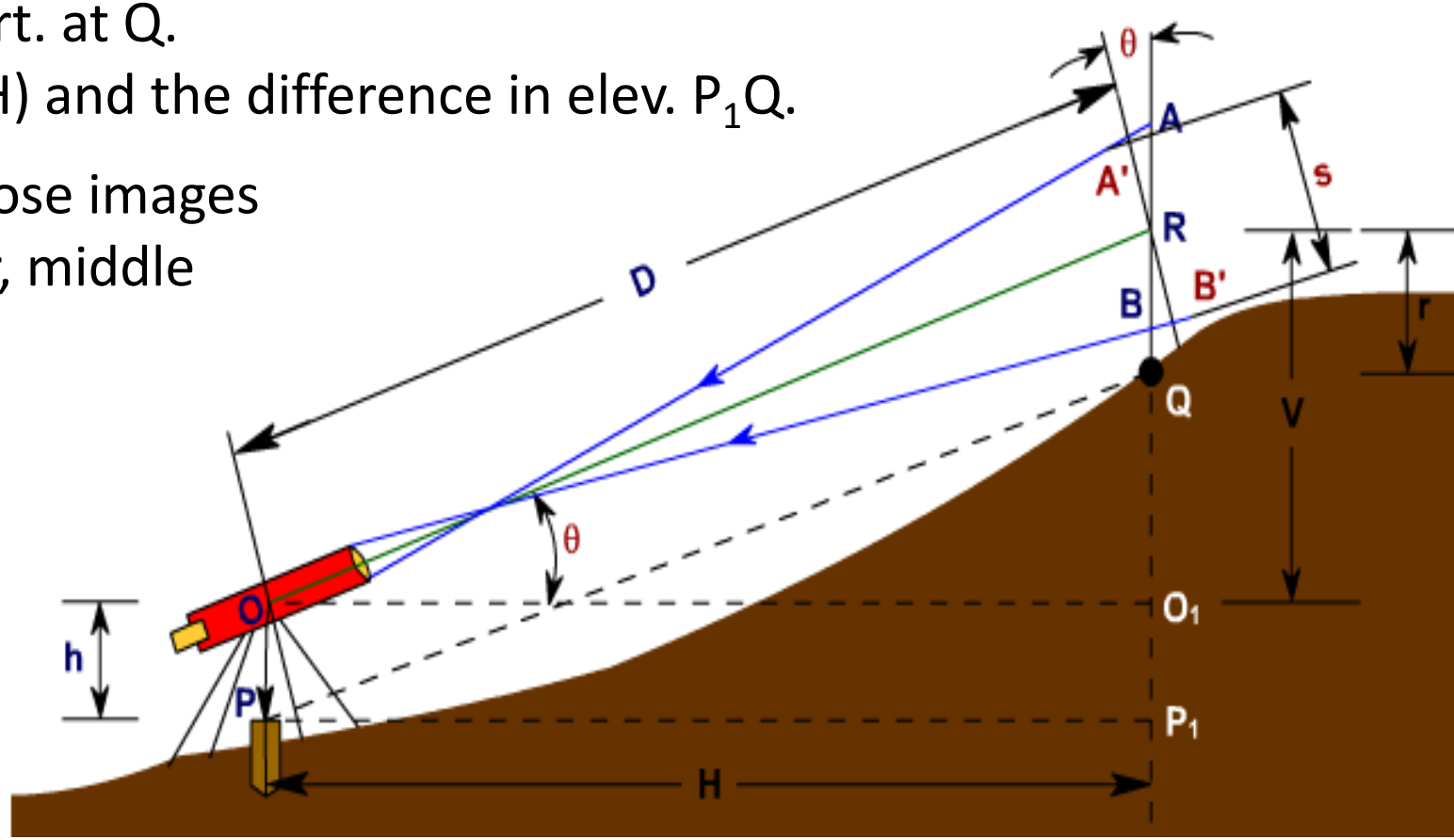
Where, $V\ = D\ sin\theta = \ (k\ s\ +\ C)\ sin\theta$

Distance and Elevation formulae for Staff Vertical to Inclined L of S

Let us consider a tacheometer (having const K and C) is placed on a sta. P.
The instr. is sighted to a staff held vert. at Q.
We need to find the hor. dist PP_1 (= H) and the difference in elev. P_1Q.

Let A, R and B be the staff points whose images are formed respectively at the upper, middle and lower cross hairs.

The line of sight, corresponding to the middle cross hair, is inclined at an angle θ.



Distance and Elevation formulae for Staff Vertical to Inclined L of S

Therefore A'B' = AB cosθ = s cosθ.

Hence, dist D (= OR) = C + K. s cosθ.

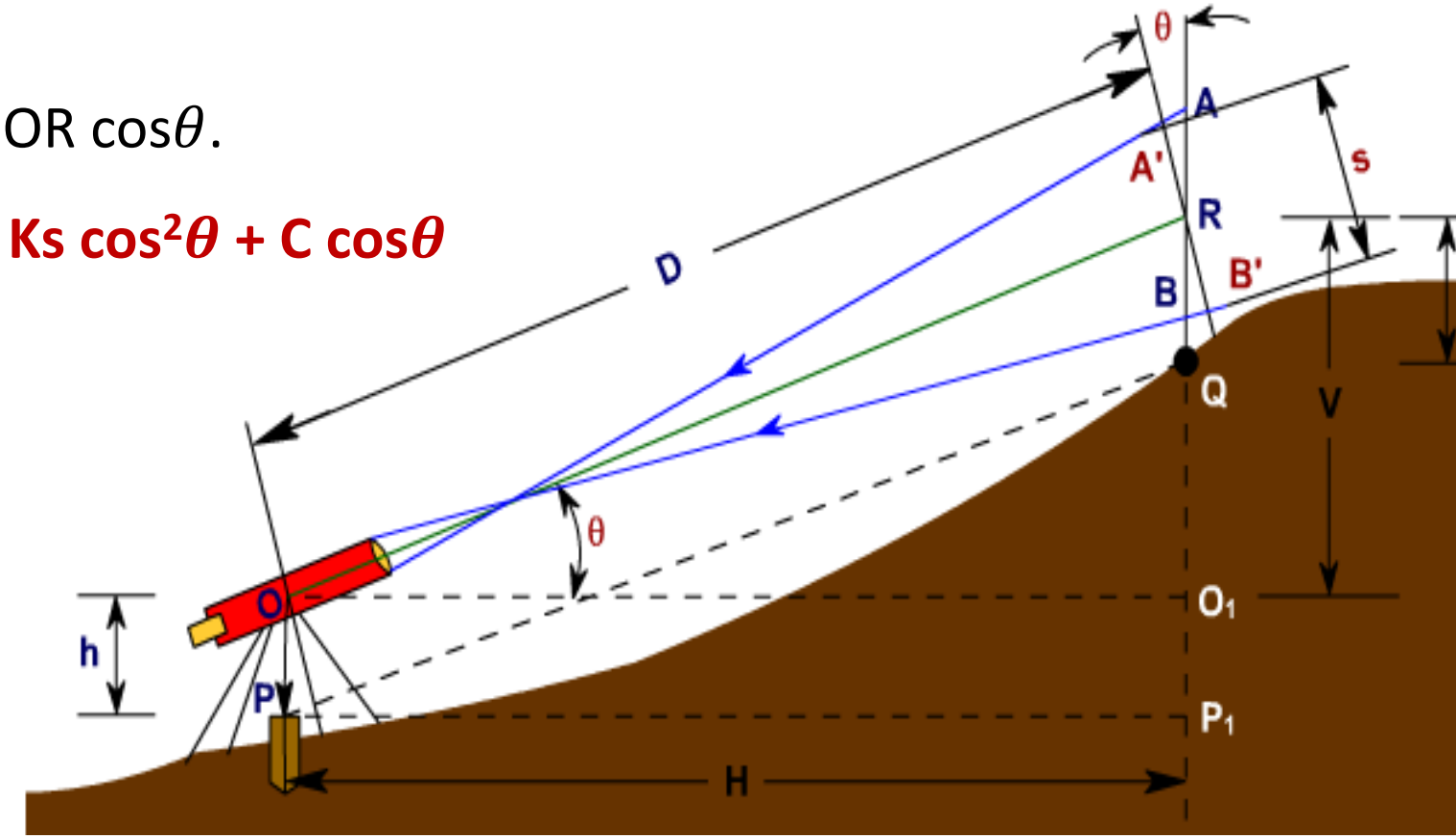
Again the horizontal dist OO_1 or H = OR cosθ.

Therefore **H = (Ks cos θ + C) cosθ = Ks cos$^2\theta$ + C cosθ**

The dist RO_1 is the vertical dist between the telescope axis and the middle cross-hair = OR sinθ.

Thus **V = (K s cosθ + c) sinθ**

= Ks sinθ cosθ + C sinθ



RL of Q = Elevation of instrument station + h + V – r.

Distance and Elevation formulae for Staff Vertical to Inclined L of S

$D\ (= OR) = C + K.\ s\cos\theta$

$H = OR\cos\theta = Ks\cos^2\theta + C\cos\theta.$

Difference in Elev. between P and Q = (h + V - r) for elevation

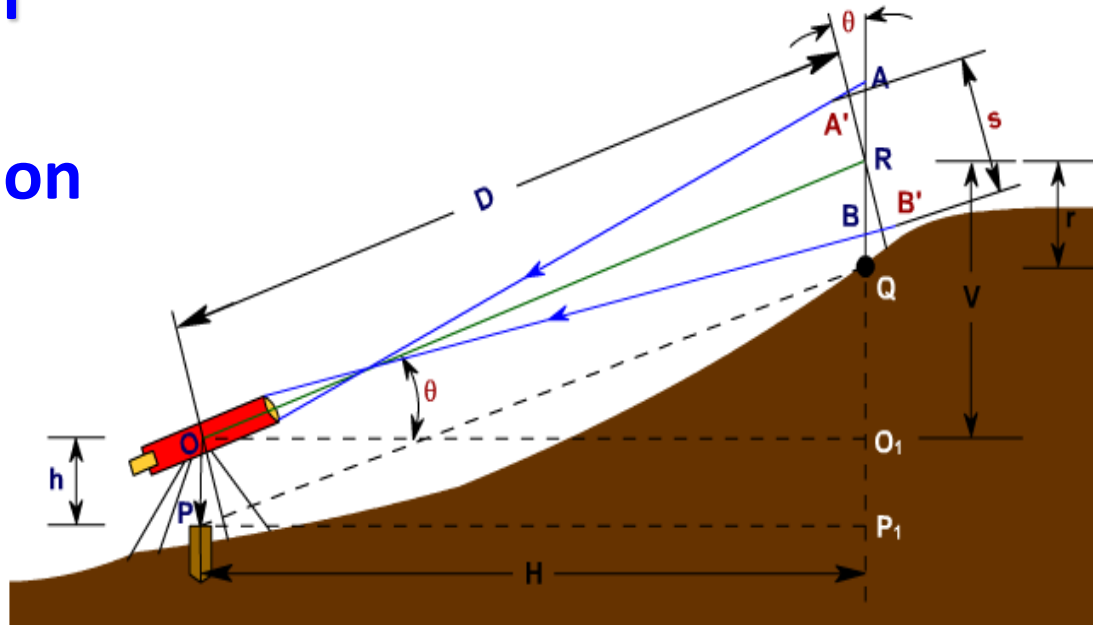
Difference in Elev. between P and Q = (h + V + r) for depression

(a) Elevation of staff sta. for angle of elevation

= Elevation of instrument station + h + V – r.

(b) Elevation of staff sta. for angle of depression

= Elevation of instr. Sta. + h – V - r

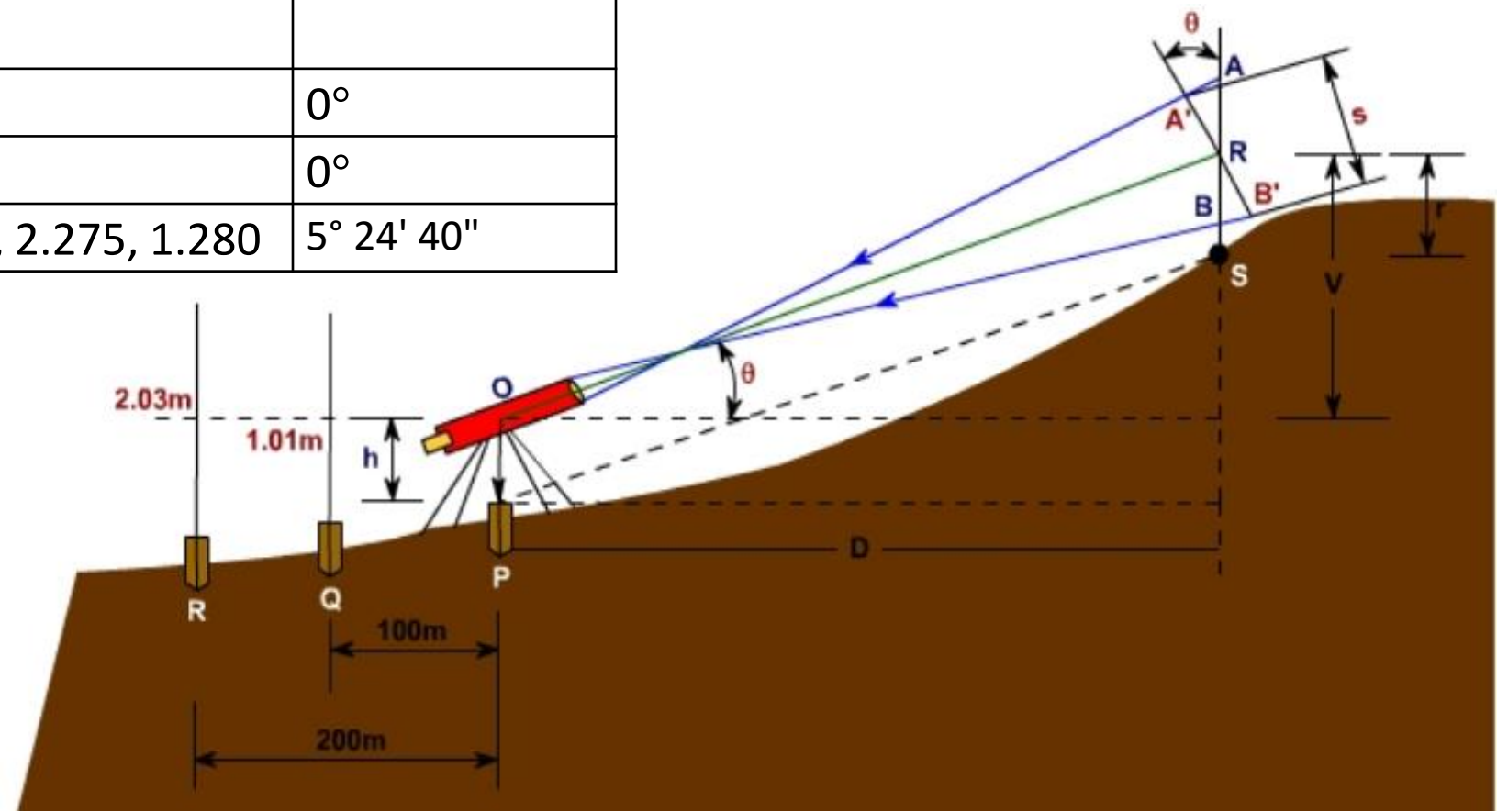


Examples 1

In order to carry out tacheometric surveying, following observations were taken through a tacheometer set up at station P at a height 1.235m.

Compute the horizontal distance of S from P and reduced level of station at S if R.L. of station P is 262.575m

Staff held Vertical at	Horizontal dist from P (m)	Staff Reading (m)	Angle of Elev
Q	100	1.01	0°
R	200	2.03	0°
S	?	3.465, 2.275, 1.280	5° 24' 40"

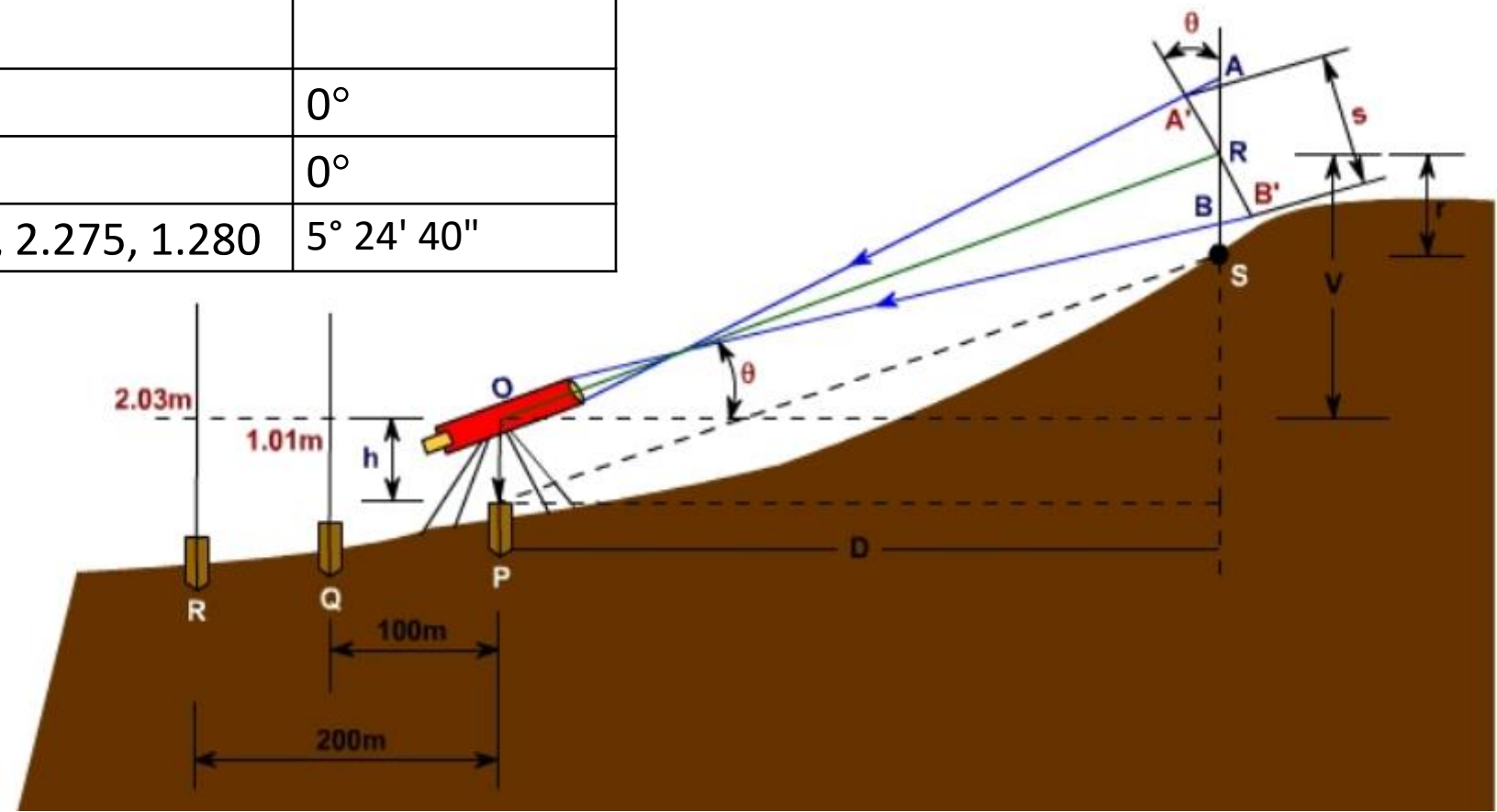


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Solution

We know, H = K.s + C,

100 = K. 1.01 + C ---------- Eqn 1

200 = K. 2.03 + C ---------- Eqn 2

Solving eqn 1 and 2 , value of K = 1 and C = 0.98

Horizontal distance of S from P, $H = Kscos^2\theta + Ccos\theta$

H = 98.04 x 2.185 x $\cos^2$ 5° 24' 40" + 0.98 cos 5° 24' 40"

= 212.312 + 0.9756 = 213.288 m

For observation at staff sta S, the staff intercept s = 3.465 - 1.280 = 2.185 m

The angle of elev. at S, θ = 5° 24' 40"

V = Ks sin θ cos θ + C sin θ

= (20.11 + 0.0924)m = 20.203 m

Thus R.L. of station S = R.L. of P + h + V - r

= 262.575 + 1.235 + 20.203 - 2.275 = 281.738 m

Examples 2

A tacheometer is placed at a station A and readings on a staff held vertical upon a B.M. (RL =100 m) and at station B are 0.640, 2.200, 3.760 and 0.010, 2.120, 4.230 m respectively. The angle of depression of the telescope in the first case is 6° 19' and in the second case is 7° 42'. Find the horizontal distance from A to B and the RL of station B, if the instrument has constants 100 and 0.5.

Examples 2

An anallactic tacheometer is placed at an intermediate point on traverse course PQ following observation were made on a vertically held staff:

Staff station	Vertical angle	Staff intercept	Axial hair reading	
P	+ 8° 36'	2.350	2.105	RL of P 321.50, k = 100
Q	+ 6° 6'	2.055	1.895	

Determine distance PQ and RO of Q.

For P, $H = Ks\cos^2\theta = 229.75\ m$

$V = Ks\sin\theta\cos\theta = 34.745\ \mathrm{m}$

$P\ is\ huger\ from\ intr\ axix$
$= 34.745 - 2.105 = 32.640\ m$

For Q, $H = Ks\cos^2\theta = 203.18m$

$V = Ks\sin\theta\cos\theta = 21.713\ \mathrm{m}$

$Q\ is\ huger\ from\ intr\ axix$
$= 21.713 - 1.895 = 19.818\ m$

$$Dist\ PQ = 229.75 + 203.18 = 432.93\ m$$

$$Difference\ in\ elevation\ of\ P\ and\ Q = 32.640 - 19.818 = 12.822\ m\ (P\ higher)$$

$$Therefore, RL\ of\ Q = RL\ of\ P - 12.822\ m = 321.50 - 12.822 = 308.678\ m)$$

Examples 3

To determine the distance between two points P and Q and elevation of Q, following observation were made:

Height of tacheometer at P = 1.280 m

Vertical angle at P = + 4° 25' 20 "

Staff reading (with staff vertical) = 1.545, 1.905, 2.265

R.L. of P = 123.265 m.

Calculate the unknown parameters after establishing the fundamental relations.

Examples 4

Stadia Interval Factor A theodolite is set up at one end of a level base line 150.0m long. The line is marked by stakes at every 30.0m and a stadia rod is held at each stake. The stadia intercept at each location is observed as follows: 0.302, 0.600, 0.899, 1.207, and 1.506 meters, respectively. Compute the stadia interval factor (K) for each distance and also determine the average value of K.

k=D/s

k1 = 30/ 0.302 = 99.3

k2 = 60/ 0.600 = 100.0

k3 = 90/ 0.899 = 100.1

k4 = 120/1.207 = 99.4

k5 = 150/1.506 = 99.6

kave = 99.7

Examples 5

Horizontal Stadia Sights. An automatic level with an internal focusing telescope was set up somewhere at mid-length of a long-span steel bridge. The rod readings tabulated below were observed on a stadia rod held successively at the vicinity of the concrete abutments in the southern and northern approaches of the bridge. If the stadia interval factor of the instrument is 98.5, determine the length of the bridge.

Rod Position	Hair Readings		
	Upper (a)	Middle (c)	Lower (b)
Rod at Southern Approach	2.98 m	1.68 m	0.38 m
Rod at Northern Approach	3.54 m	2.02 m	0.49 m

Solution

ss= 2.98 - 0.38 =2.60

Ds= 98.5 * 2.60 = 256.1 m

sn= 3.54 - 0.49 = 3.05

Dn= 98.5 * 3.05 = 300.4 m

D = 256.1 + 300.4 = 556.5 m

Stadia Systems - Tangential Method

This method used when staff is held much away from the instrument making it difficult to read it. This is useful when the diaphragm does not have stadia hairs.

Depending upon the angles (i.e., angles of elevation or depression), there can be three case.

- **Both the Angles being Angles of Elevation**
- **Both the Angles being Angles of Depression**
- **One Angles being Angles of Elevation and one being angle of depression**

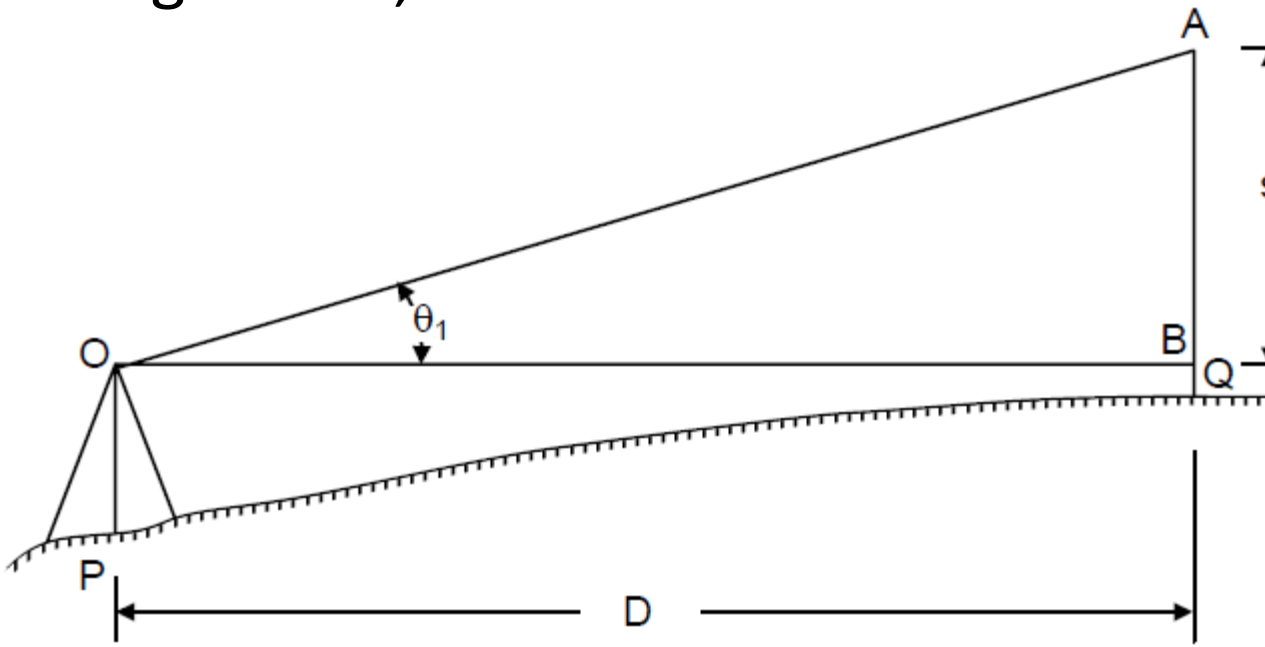
Tangential Method

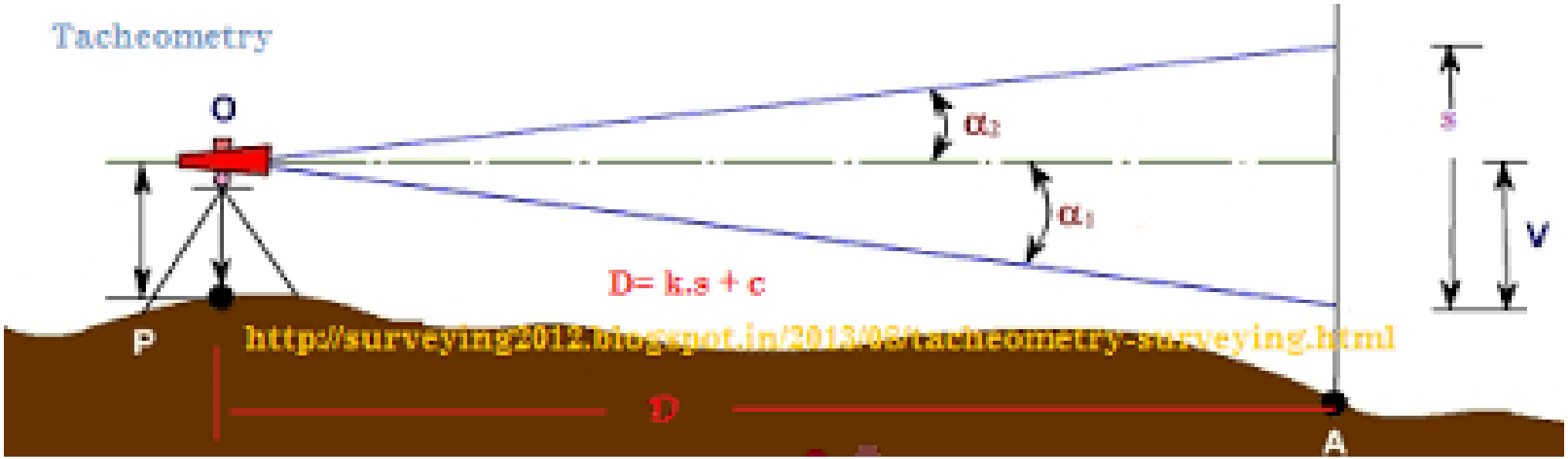
Both the Angles being Angles of Elevation

Here the ground is such that the horizontal line of sight is possible. The reading at *B* is observed by keeping the telescope level at *A*. The reading of *A* is noted and the angle θ^1 is also measured. By knowing the staff intercept, the horizontal distance *D* can be given as,

$$D = s.\cot\theta_1 .$$

The elevation of *Q* is determined from the reading obtained at *B* as in ordinary levelling.




Tacheometry
O
α2
α1
s
V
D= k.s + c
P
http://surveying2012.blogspot.in/2013/06/tacheometry-surveying.html
D
A

Tangential Method

Both the Angles being Angles of Elevation

When the ground does not permit a horizontal sight, two vertical angles θ^1 and θ^2 are measured

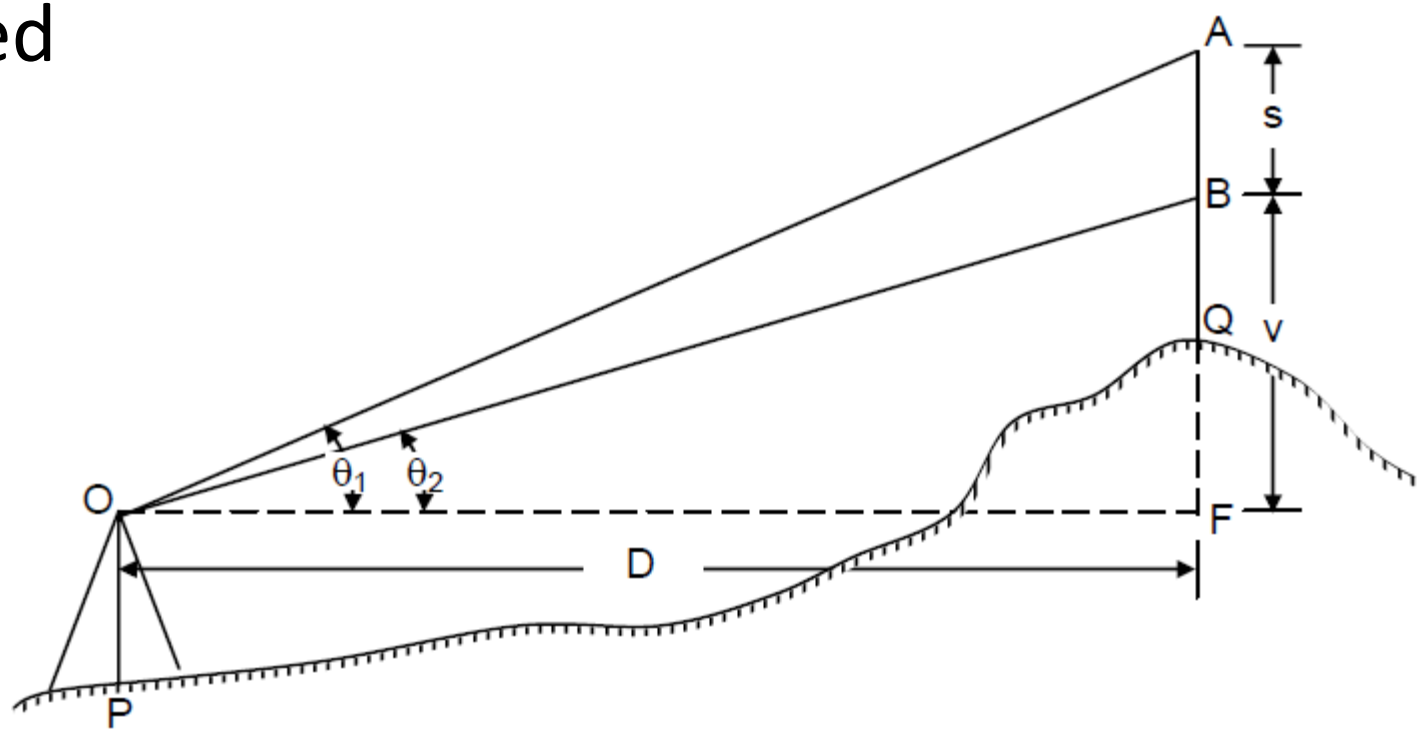
$AF = D \tan \theta_1$

and $BF = D \tan \theta_2$

$\therefore s = AF - BF = D (\tan\theta_1 - \tan\theta_2)$

$$\mathbf{D = \frac{s}{(\tan \theta_1 - \tan \theta_2)}}$$

$$V = D. \tan \theta_2 = \frac{s.\tan \theta_2}{(\tan \theta_1 - \tan \theta_2)}$$



RL of Q = HI + FB – QB
= HI + D.tan θ_2 -QB

Tangential Method

Both the Angles being Angles of Depression

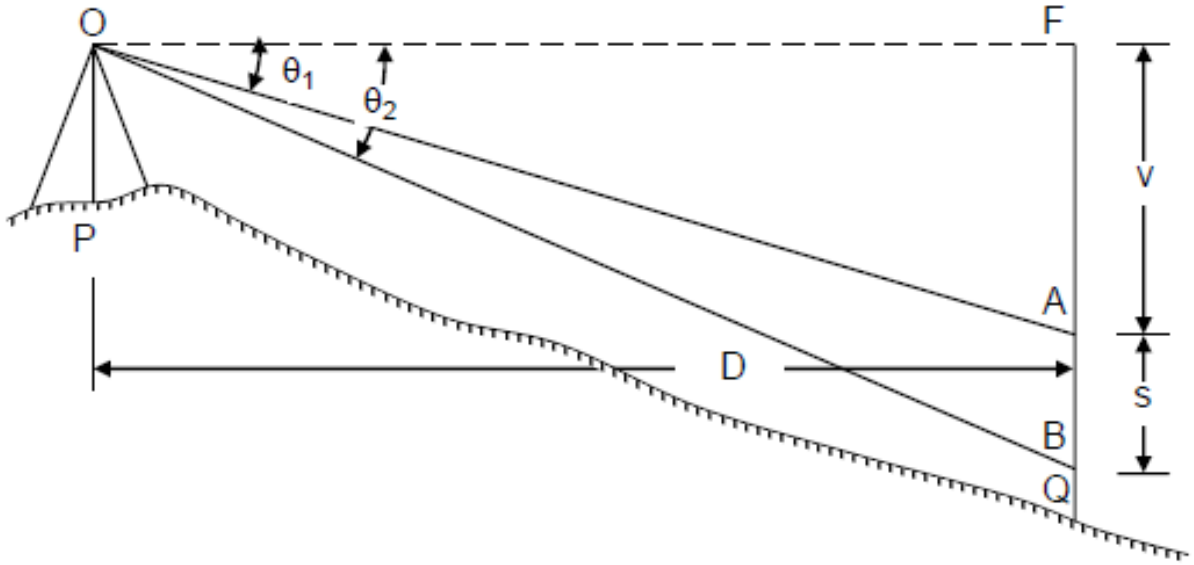
$AF = D \tan \theta_1$

and $BF = D \tan \theta_2$

$\therefore s = BF - AF = D (\tan\theta_2 - \tan\theta_1)$

$$\mathbf{D = \frac{s}{(\tan \theta_2 - \tan \theta_1)}}$$

$$V = D. \tan \theta_1 = \frac{s.\tan \theta_1}{(\tan \theta_2 - \tan \theta_1)}$$



RL of Q = RL of P + HI – D.tanθ_1 – s - QB

Tangential Method

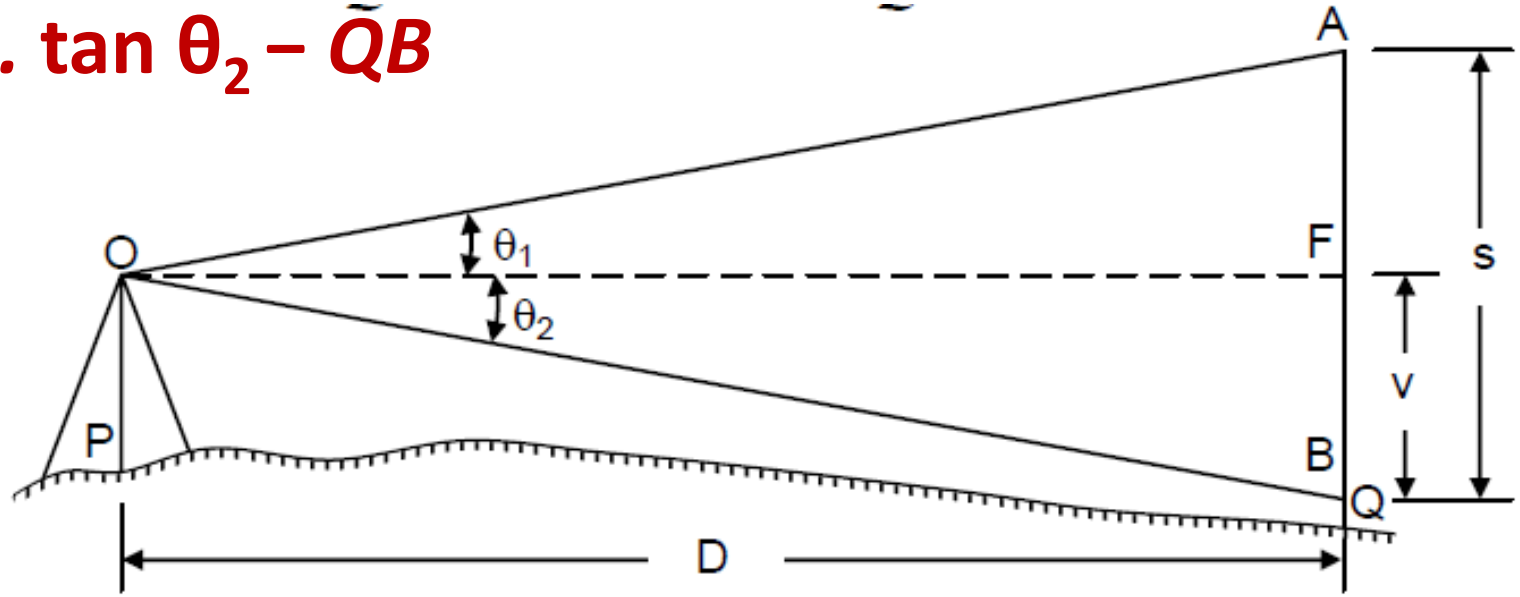
One Angle of Elevation and Another Angle of Depression

$s = AF + BF$

$= D \tan \theta_1 + D \tan \theta_2$

and $V = D \tan \theta_2$

RL* of *Q* = RL of P + *HI* – *D.* tan θ_2 – *QB



Stadia Systems – Subtense Bar Method

In the subtense bar method, the horizontal angle subtended by two targets fixed on a horizontal bar at a known distance apart is measured at instrument station by theodolite.

The two targets are at a distance *s* apart, and each at *s*/2 from the centre, i.e. vertical axis.

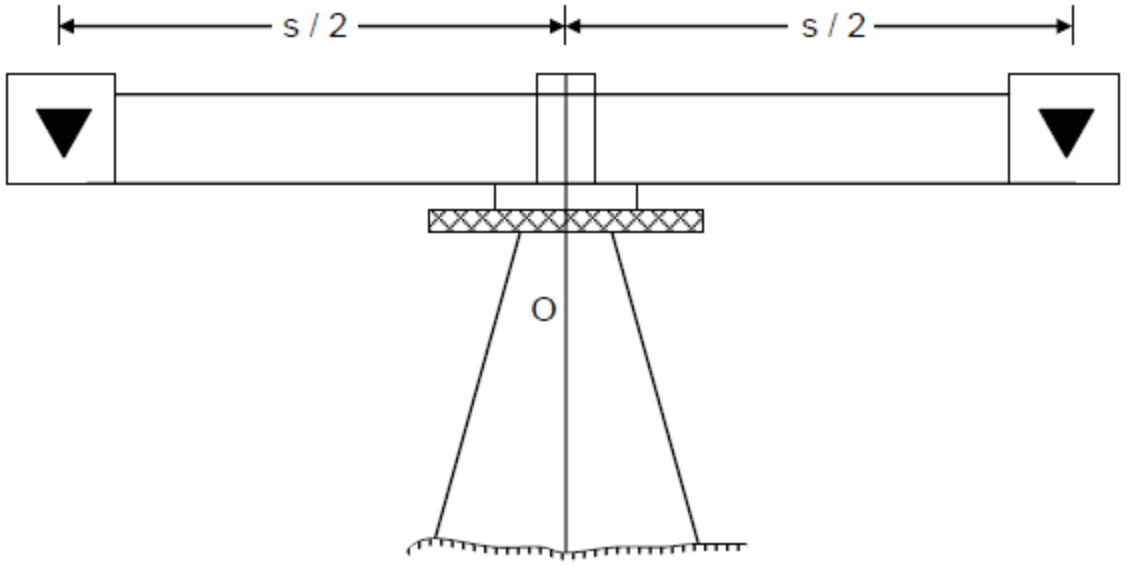
The bar can be mounted on a tripod stand and can be rotated about its vertical axis.

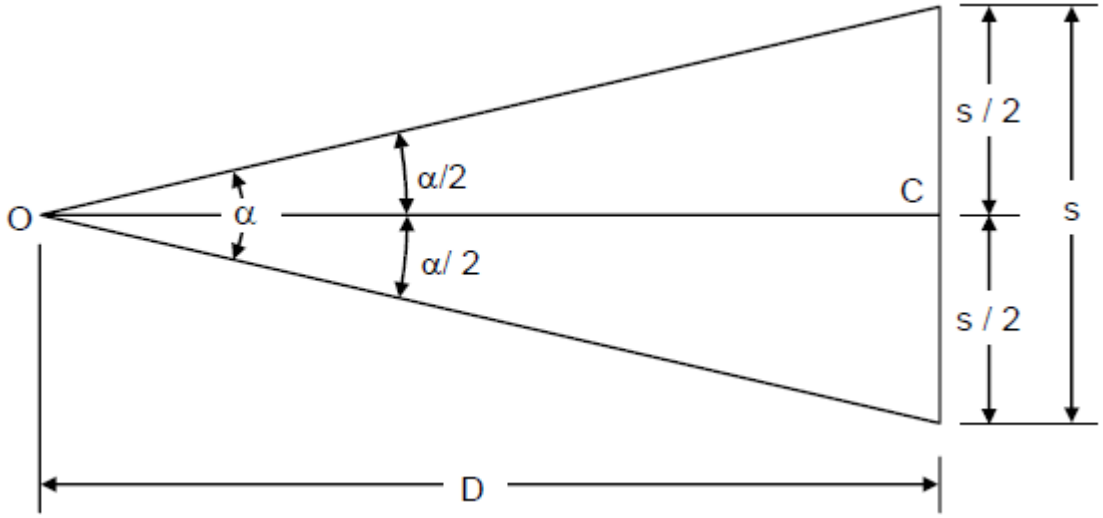
The subtense bar should be kept perpendicular to the line of sight, which is set through a sight rule or a small telescope fitted at the centre of the subtense bar.

The subtense bar method can be used for measuring the length of traverse lines in rough country. This system can also be used for contouring.

The horizontal angle α is measured carefully by means of a theodolite.

Stadia Systems – Subtense Bar Method





From the geometry, $D = \frac{1}{2} s . \cot \frac{\alpha}{2}$ where *s* = the distance between the targets of subtense bar, and α = apex angle subtended by targets at *O*.

As α is small $\tan \frac{\alpha}{2} = \frac{\alpha}{2}$

so, ***D = s/α, (α is in radians), Or D =*** $\frac{206265.s}{\alpha''}$ **(*α* in seconds)**

If the vert. angle to the centre of the bar is measured, elev. differences can be determined.

Electronic Tacheometry
Total Station Theodolite / EDM



The stadia procedure is used less and less often these days, more commonly geomatic engineers use a combination theodolite - **EDM known as a total station**.
Often these instruments are connected to a field computer which stores readings and facilitates the processing of the data electronically.
This instrumentation has facilitated the development of this method of detail and contour surveying into a very slick operation.
It is now possible to produce plans of large areas that previously would have taken weeks, in a matter of days.

The early EDM instruments could measure long distances with an accuracy of about 5 parts per million (i.e. 5 mm for every km or to 150 mm over a 30 km line), but later versions were more accurate, able to measure with an accuracy of about 1 part per million (1 mm per kilometre or 30 mm over a 30 km line)

Present days EDM (well known as Total Station) instruments can also measure with an accuracy of about 1 part per million, but generally only for shorter lines of about one kilometre.