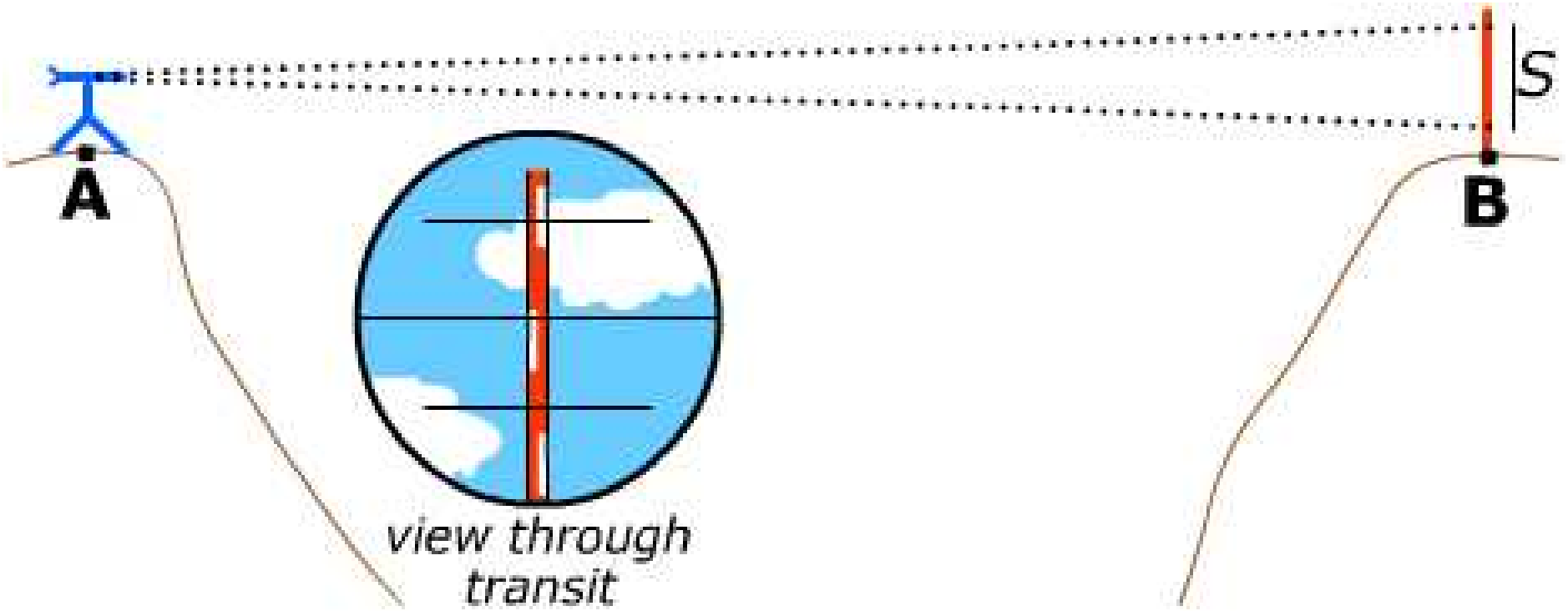


Tacheometry/Stadia Surveying

Tacheometry/Stadia Surveying

- Tacheometry is the type of surveying in which vertical and horizontal distances are computed from stadia readings without using chain or tape
- This is done by the help of a special type of transit theodolite known as tacheometer and the staff known as stadia rod

Tacheometry/Stadia Surveying

- The objectives of this type of survey is to prepare a contour map or plan as in cases of location surveys for roads, railways, reservoirs, canals and sewers
- This method is not very accurate still it is used in places where the ground is hilly with steep grades and where there is a swamp or a marshy land

Tacheometry/Stadia Surveying

- Tacheometer:
  - It is a special type of transit theodolite
  - Its telescope contains two horizontal hairs called stadia hairs in addition to the regular cross hairs
  - The extra hairs are equidistant from the central cross-hairs and they are specially termed as stadia line or webs

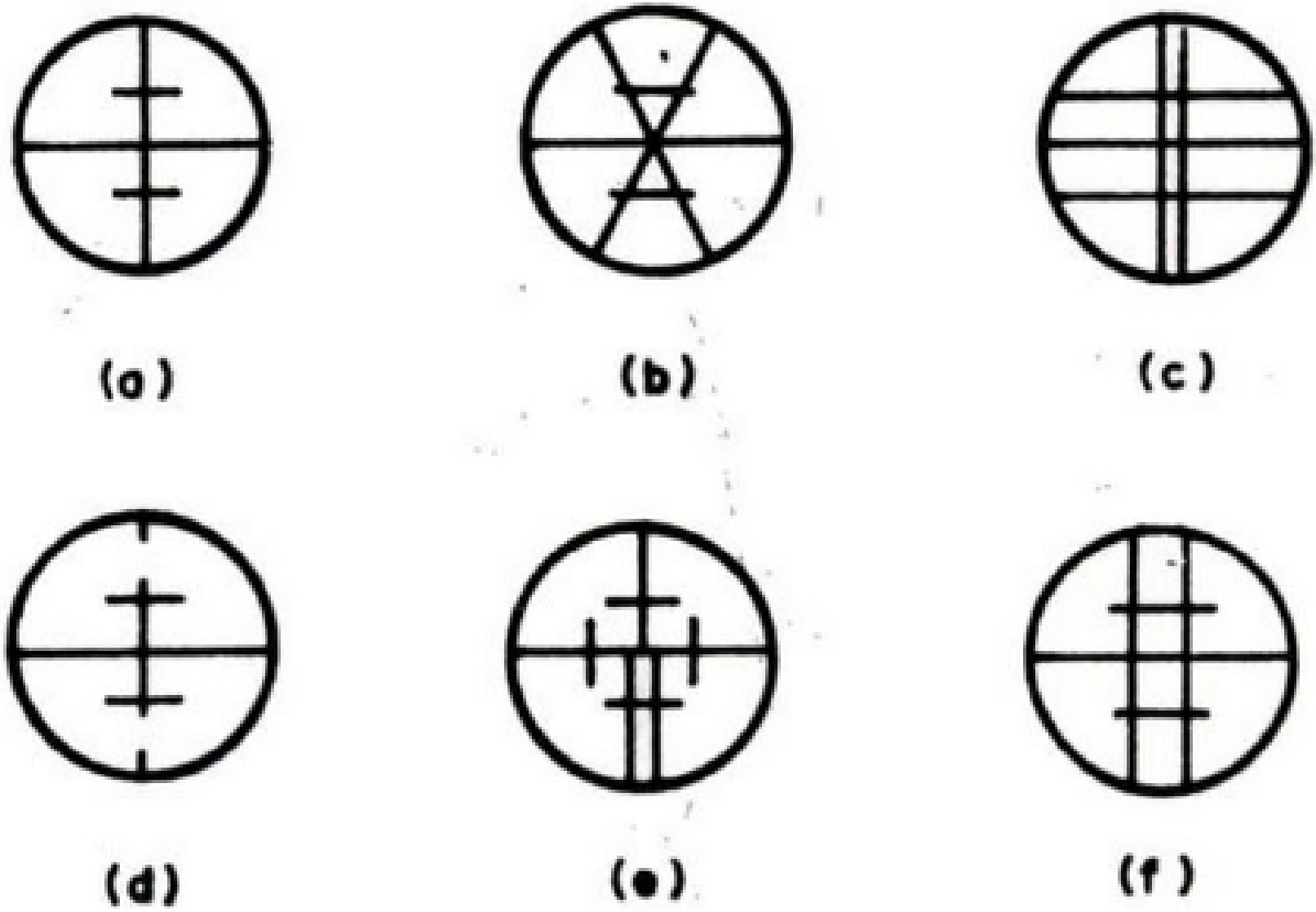


FIG. 1.1. VARIOUS PATTERNS OF STADIA DIAPHRAGM.

Tacheometry/Stadia Surveying

- Stadia Rod:
  - This is a special type of levelling staff
  - It is graduated in feet and decimals of a foot and the smallest sub-division being 0.01 ft
  - It is usually of one piece but for convenience in carrying the rods are sometimes hinged at the middle

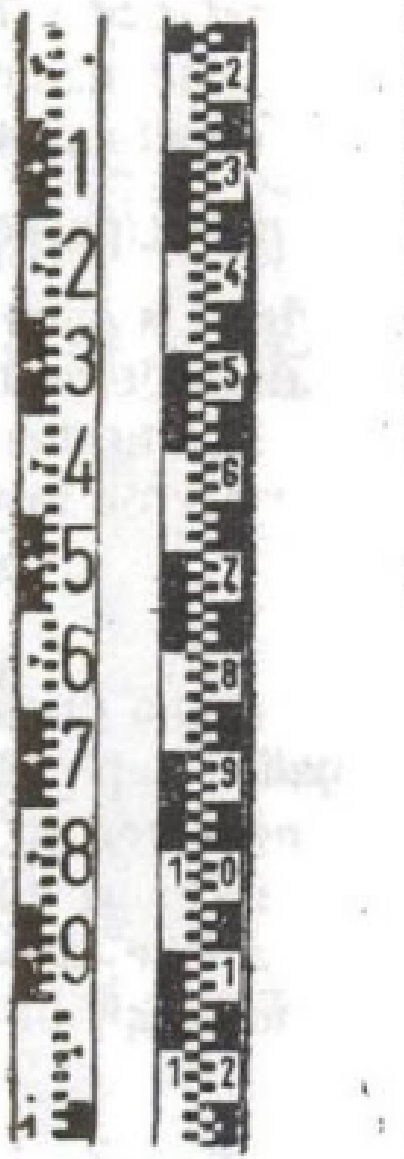
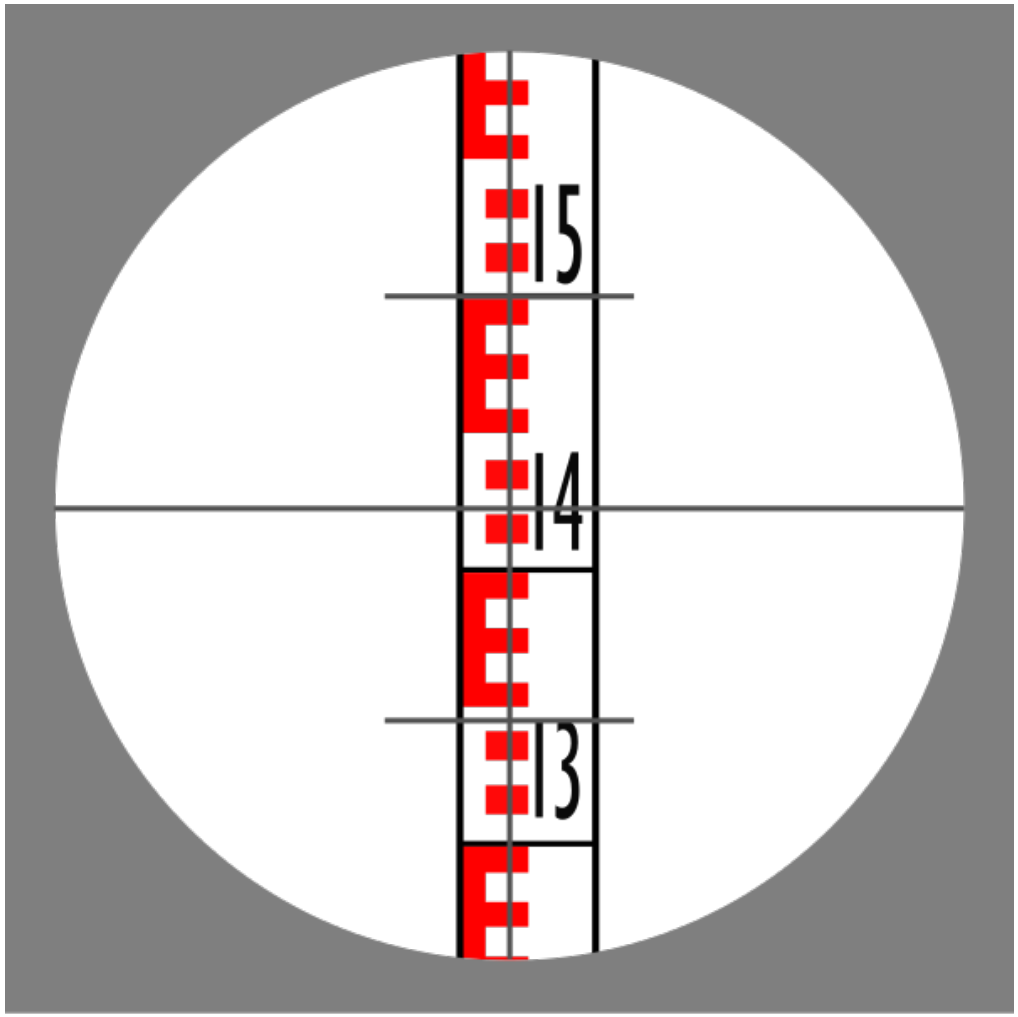
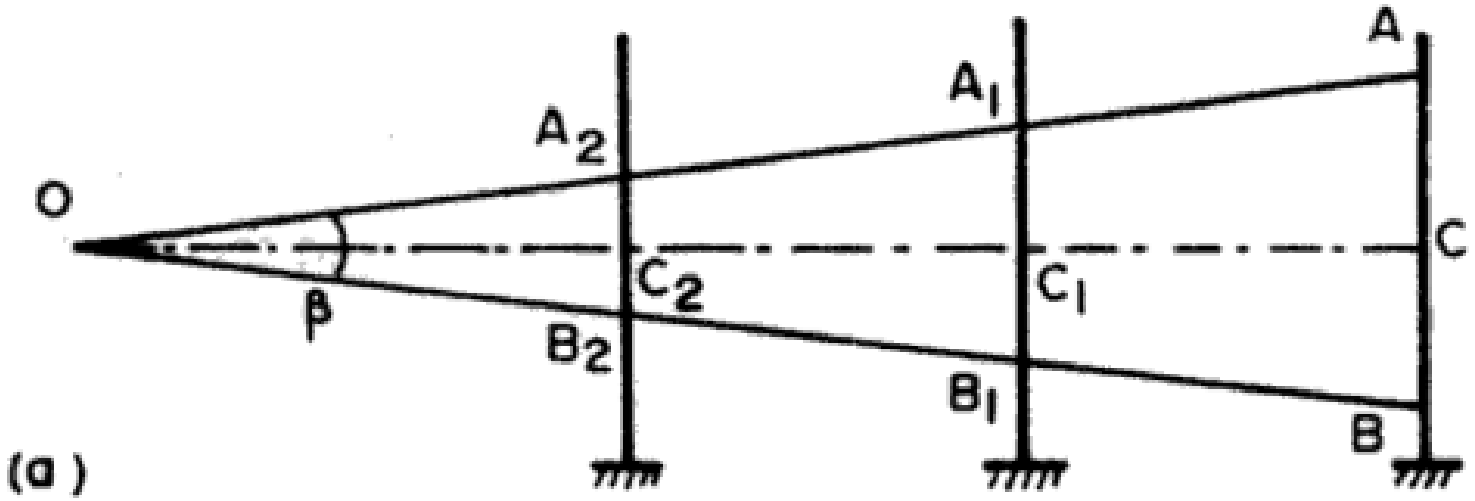


FIG. 1.2. STADIA RODS



In Fig. 1.3 (*a*), let two rays *OA* and *OB* be equally inclined to the central ray *OC*. Let A_2B_2, A_1B_1 and *AB* be the staff intercepts. Evidently,

$$\frac{OC_2}{A_2B_2} = \frac{OC_1}{A_1B_1} = \frac{OC}{AB} = \text{constant } k = \frac{1}{2}\cot\frac{\beta}{2}.$$



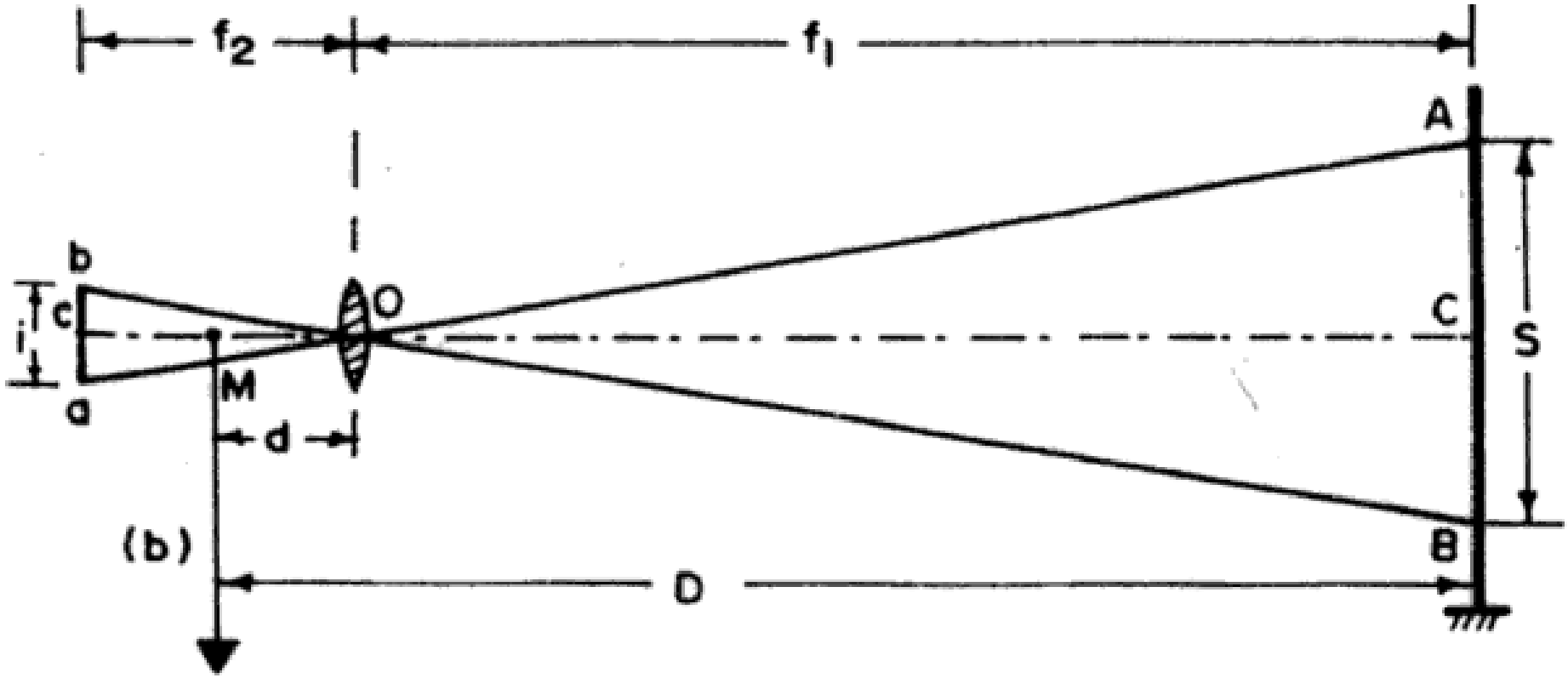


FIG. 1.3. PRINCIPLE OF STADIA METHOD.

Horizontal Sight. Consider Fig. 1.3 (*b*) in which *O* is the optical centre of the objective of an *external focusing telesope.*

Let *A*, *C* and *B*= The points cut by the three lines of sight corresponding to the three wires.

b, *c* and *a* = Top, axial and bottom hairs of the diaphragm.

ab = *i* = interval between the stadia hairs (stadia interval)

AB = *s* = staff intercept.

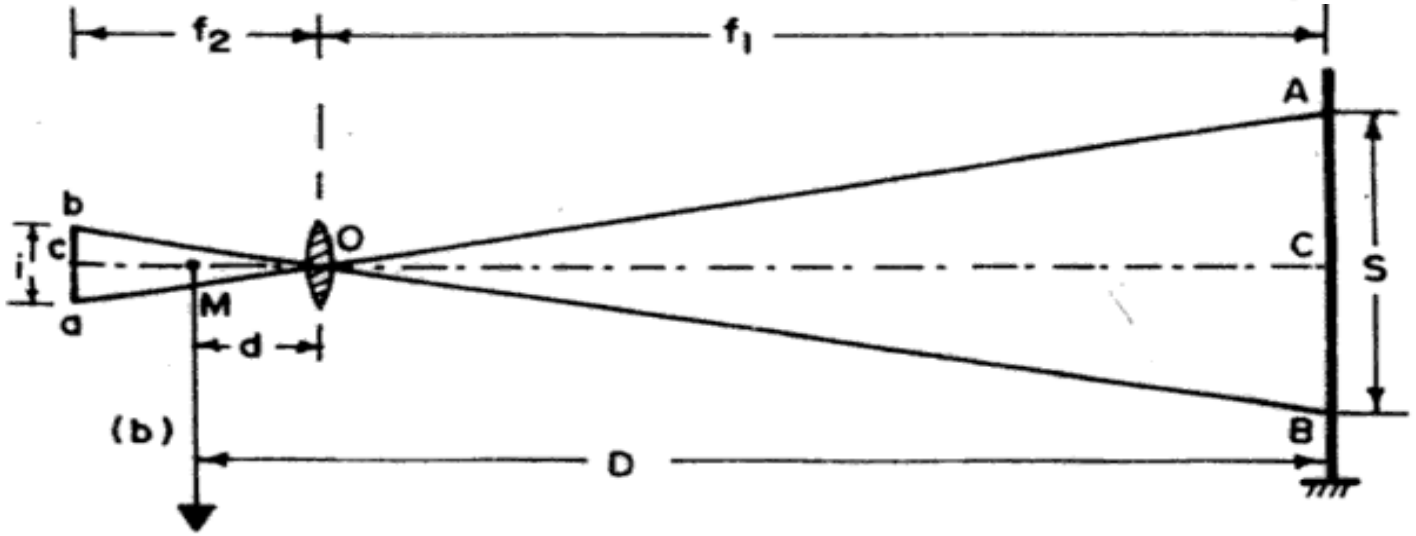
f = focal length of the objective

f_1 = Horizontal distance of the staff from the optical centre of the objective.

f_2 = Horizontal distance of the cross-wires from *O*.

d = Distance of the vertical axis of the instrument from *O*.

D = Horizontal distance of the staff from the vertical axis of the instrument.



M = Centre of the instrument, corresponding to the vertical axis.

Since the rays BOb and AOa pass through the optical centre, they are straight so that $\Delta s\, AOB$ and aOb are similar. Hence

$$\frac{f_1}{f_2} = \frac{s}{i} \qquad ...(i)$$

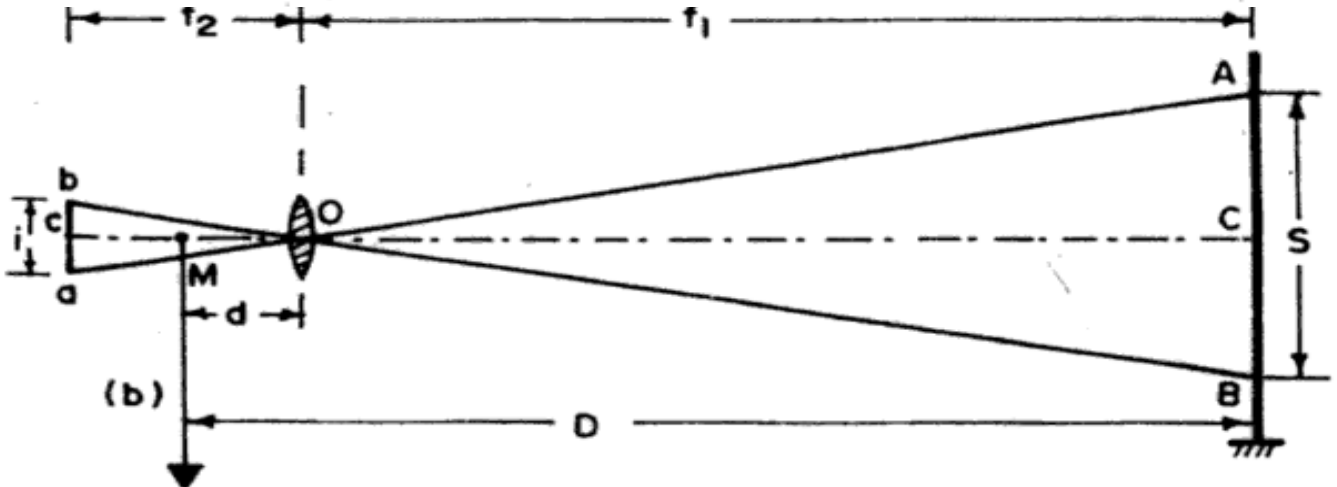
Again, since f_1 and f_2 are conjugate focal distances, we have from lens formula,

$$\frac{1}{f} = \frac{1}{f_2} + \frac{1}{f_1} \qquad ...(ii)$$

Multiplying throughout by ff_1, we get

$$f_1 = \frac{f_1}{f_2} f + f$$

Substituting the values of $\frac{f_1}{f_2} = \frac{s}{i}$ in the above, we get



$$f_1 = \frac{s}{i} f + f \qquad \text{...(iii)}$$

The horizontal distance between the axis and the staff is

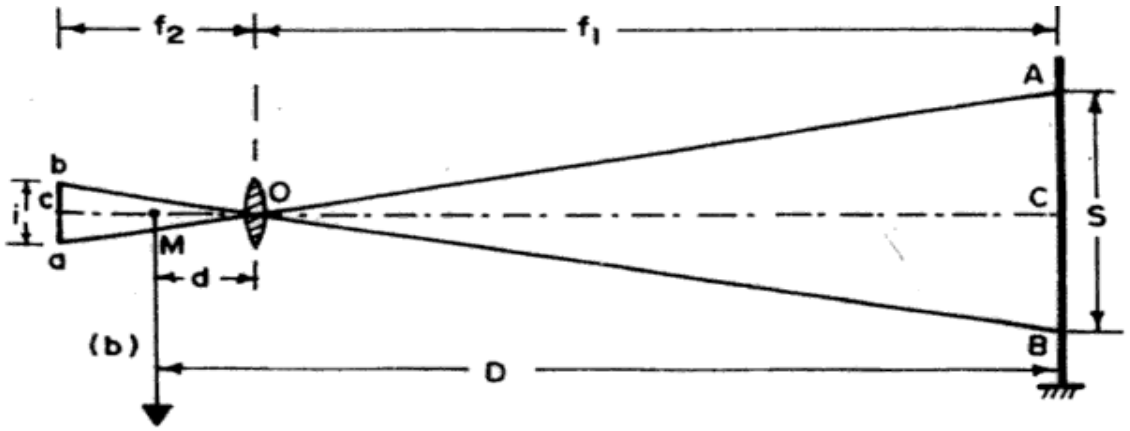
$$D = f_1 + d$$

or
$$D = \frac{f}{i} s + (f + d) \qquad \text{...[1.1 (a)]}$$

$$D = k \,.\, s + C$$

Equation 1.1 is known as the *distance equation*. In order to get the horizontal distance, therefore, the staff intercept s is to be found by subtracting the staff readings corresponding to the top and bottom stadia hairs.

The constant $k = \frac{f}{i}$ is known as the *multiplying constant* or *stadia interval factor* and the constant $(f + d) = C$ is known as the *additive constant* of the instrument.



Determination of constants k and C

The values of the multiplying constant k and the additive constant C can be computed by the following methods :

1st Method. In this method, the additive constant $C=(f+d)$ is measured from the instrument while the multiplying constant k is computed from field observations :

1. Focus the intrument to a distant object and measure along the telescope the distance between the objective and cross-hairs.

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

Since f_1 is very large in this case, f is approximately equal to f_2, *i.e.*, equal to the distance of the diaphragm from the objective.

2. The distance d between the instrument axis and the objective is variable in the case of external focusing telescope, being greater for short sights and smaller for long sights. It should, therefore be measured for average sight. Thus, the additive constant $(f+d)$ is known.

3. To calculate the multiplying constant k, measure a known distance D_1 and take the intercept s_1 on the staff kept at that point, the line of sight being horizontal. Using equation 1.1,

$$D_1 = ks_1 + C$$

or

$$k = \frac{D_1 - C}{s_1}$$

For average value, staff intercepts, s_2, s_3 etc., can be measured corresponding to distance D_2, D_3 etc., and mean value can be calculated.

2nd Method. In this method, both the constants are determined by field observations as under :

1. Measure a line, about 200 m long, on fairly level ground and drive pegs at some interval, say 50 metres.

2. Keep the staff on the pegs and observe the corresponding staff intercepts with horizontal sight.

3. Knowing the values of D and s for different points, a number of simultaneous equations can be formed by substituting the values of D and s in equation 1.1. The simultaneous solution of successive pairs of equations will give the values of k and C, and the average of these can be found.

For example, if s_1 is the staff intercept corresponding to distance D_1 and s_2 corresponding to D_2 we have

$$D_1 = ks_1 + C \qquad \ldots(i)$$

and

$$D_2 = ks_2 + C \qquad \ldots(ii)$$

Subtracting (*i*) from (*ii*), we get

$$k = \frac{D_2 - D_1}{s_2 - s_1} \qquad \ldots(1.2)$$

Substituting the values of k in (*i*), we get

$$C = D_1 - \frac{D_2 - D_1}{s_2 - s_1} s_1$$

or $$C = \frac{D_1 s_2 - D_1 s_1 - D_2 s_1 + D_1 s_1}{s_2 - s_1}$$

or $$C = \frac{D_1 s_2 - D_2 s_1}{s_2 - s_1} \qquad \ldots(1.3)$$

Thus, equations 1.2 and 1.3 give the values of k and C.

1.5. DISTANCE AND ELEVATION FORMULAE FOR STAFF VERTICAL : INCLINED SIGHT

Let P = Instrument station

Q = Staff station

M = Position of instruments axis

O = Optical centre of the objective

A, C, B = Points corresponding to the readings of the three hairs

$s = AB$ = Staff intercept

i = Stadia interval

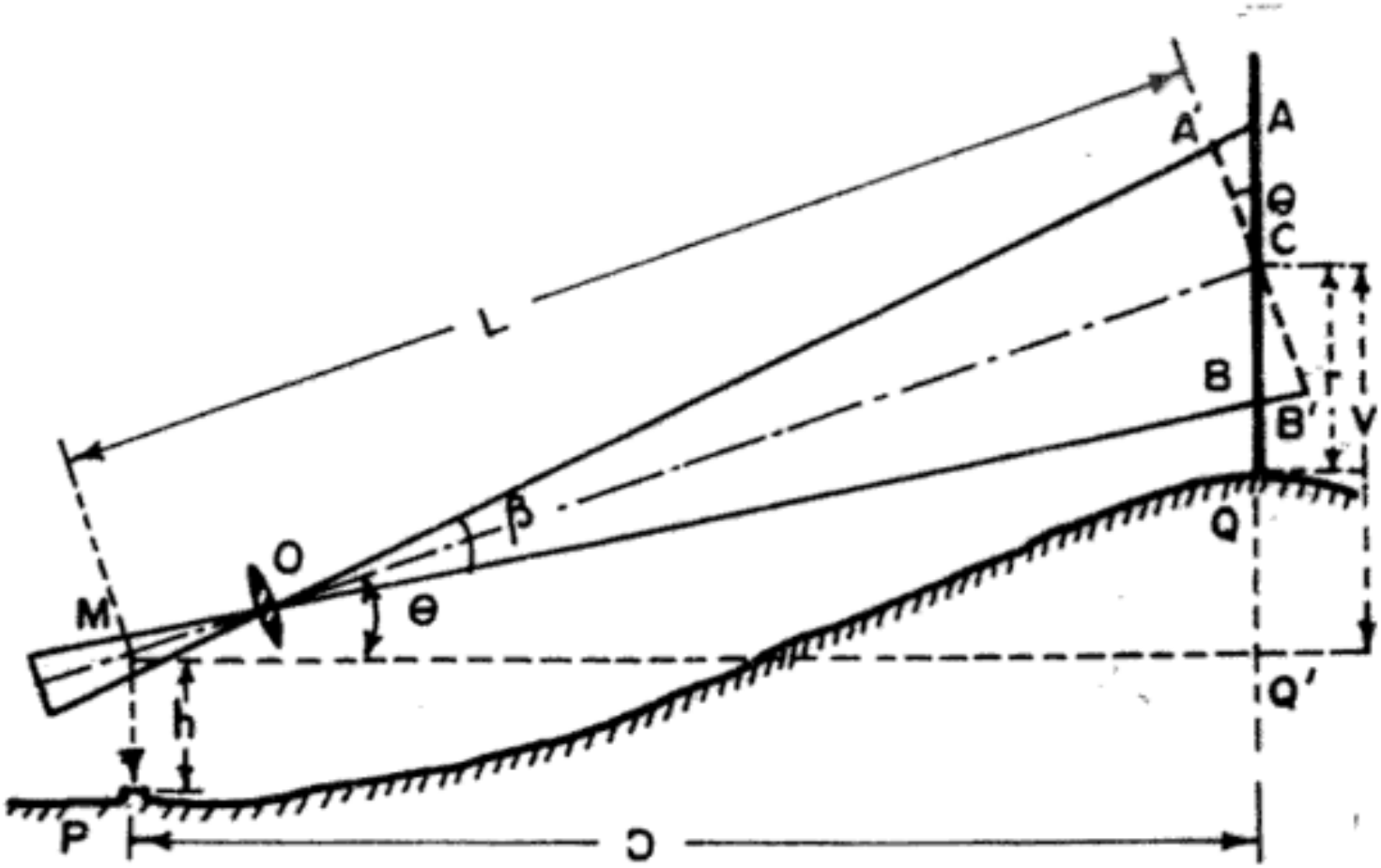


FIG. 1.5. ELEVATED SIGHT : VERTICAL HOLDING.

θ = Inclination of the line of sight from the horizontal

L = Length MC measured along the line of sight

$D = MQ'$ = Horizontal distance between the instrument and the staff

V = Vertical intercept, at Q, between the line of sight and the horizontal line.

h = Height of the instrument

r = Central hair reading

β = Angle between the two extreme rays corresponding to stadia hairs.

Draw a line $A'CB'$ (Fig. 1.5) normal to the line of sight OC.

$\angle AA'C = 90° + \frac{\beta}{2}$, being the exterior angle of the $\Delta COA'$.

Similarly, from $\Delta COB'$, $\angle OB'C = \angle BB'C = 90° - \frac{\beta}{2}$.

Since $\frac{\beta}{2}$ is very small (its value being equal to 17′ 11″ for $k = 100$), $\angle AA'C$ and $\angle BB'C$ may be approximately taken equal to 90°

$$\therefore \quad \angle AA'C = \angle BB'C \approx 90°$$

From $\Delta ACA'$, $A'C = AC \cos\theta$

or $$A'B' = AB \cos\theta = s \cos\theta \qquad ...(i)$$

Since the line $A'B'$ is perpendicular to the line of sight OC, equation 1.1 is directly applicable. Hence, we have

$$MC = L = k \,.\, A'B' + C = ks \cos\theta + C \qquad ...(ii)$$

Now, the horizontal distance

$$D = L \cos\theta = (ks \cos\theta + C) \cos\theta.$$

or $$D = ks \cos^2\theta + C\cos\theta \qquad ...(1.4)$$

Similarly, $V = L \sin\theta$

$$= (ks \cos\theta + C)\sin\theta$$

$$= ks \cos\theta . \sin\theta + C\sin\theta$$

or $$V = ks \frac{\sin 2\theta}{2} + C\sin\theta \qquad ...(1.5)$$

Thus, equations, 1.4 and 1.5 are the distance and elevation formulae for inclined line of sight.

(a) Elevation of the staff station for angle of elevation.

If the line of sight has an angle of elevation θ, as shown in Fig. 1.5, we have

Elev. of staff station = Elev. of instrument station $+ h + V - r$.

(b) Elevation of the staff station for the angle of depression

From Fig. 1.6,

Elevation of Q = Elevation of $P + h - V - r$.

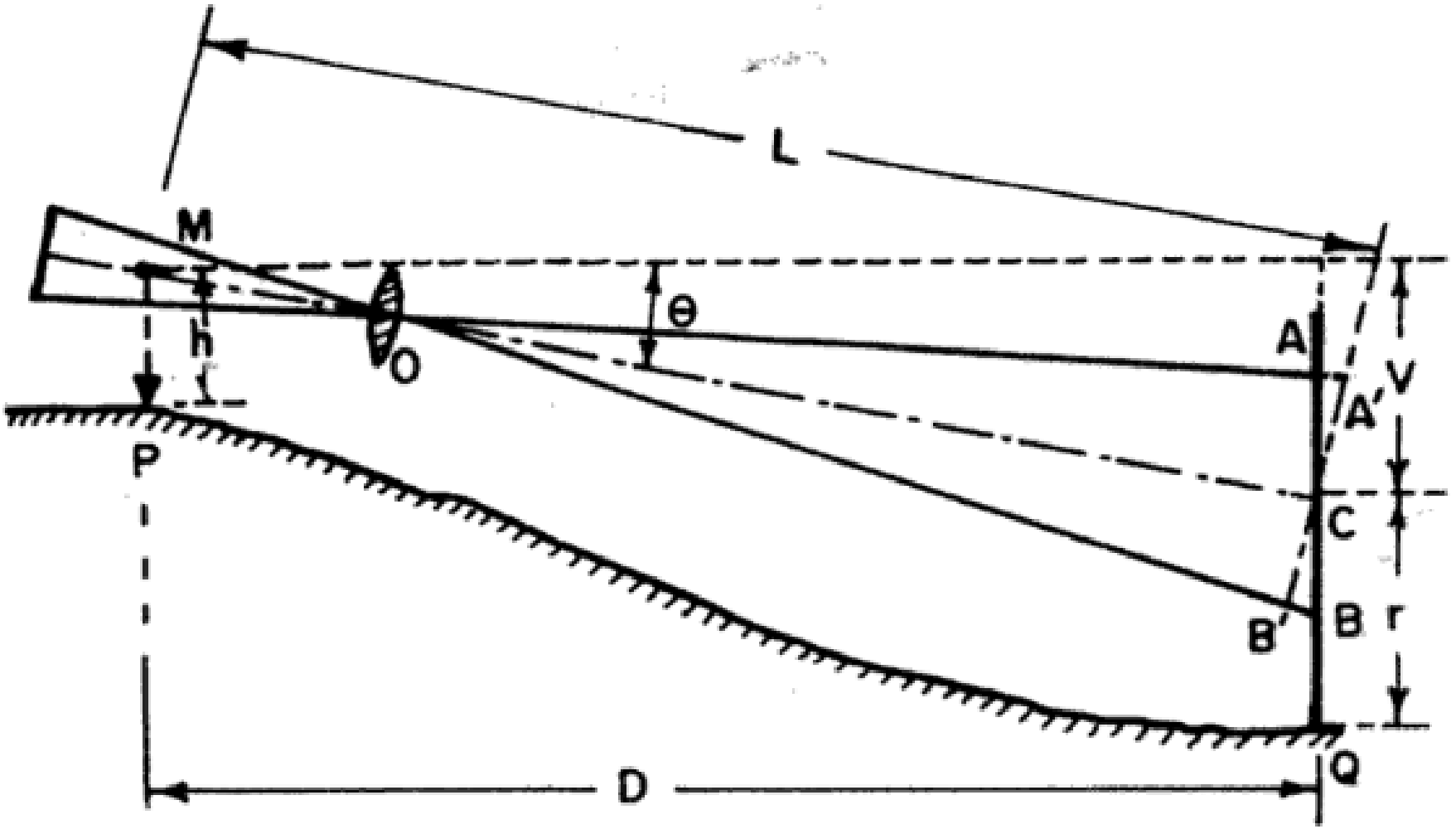
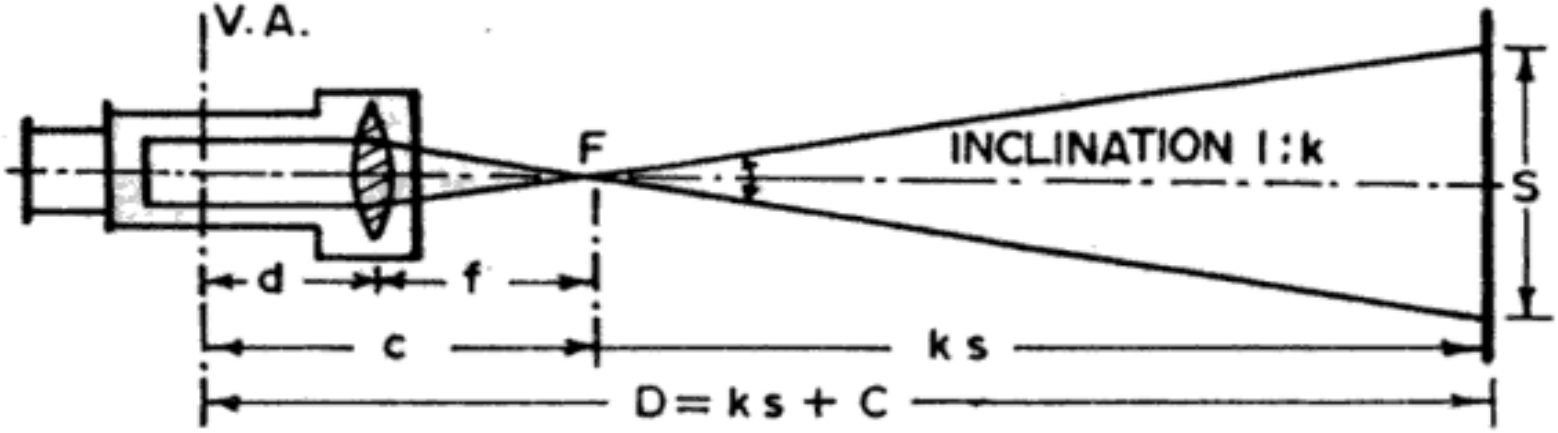


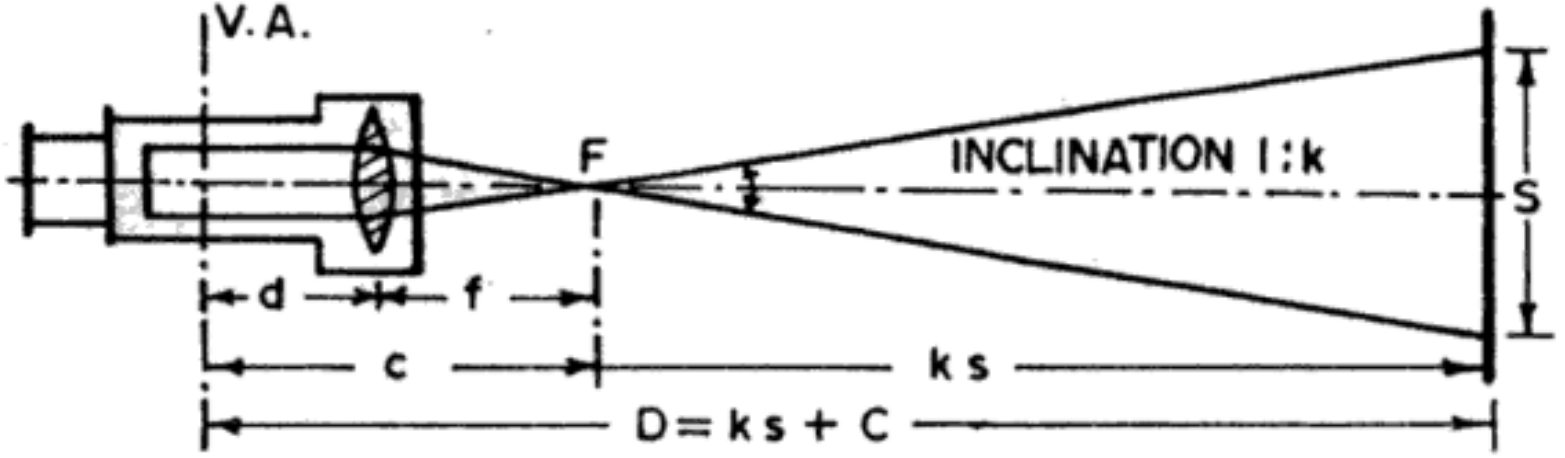
FIG. 1.6. DEPRESSED SIGHT : VERTICAL HOLDING.

1.7. THE ANALLACTIC LENS

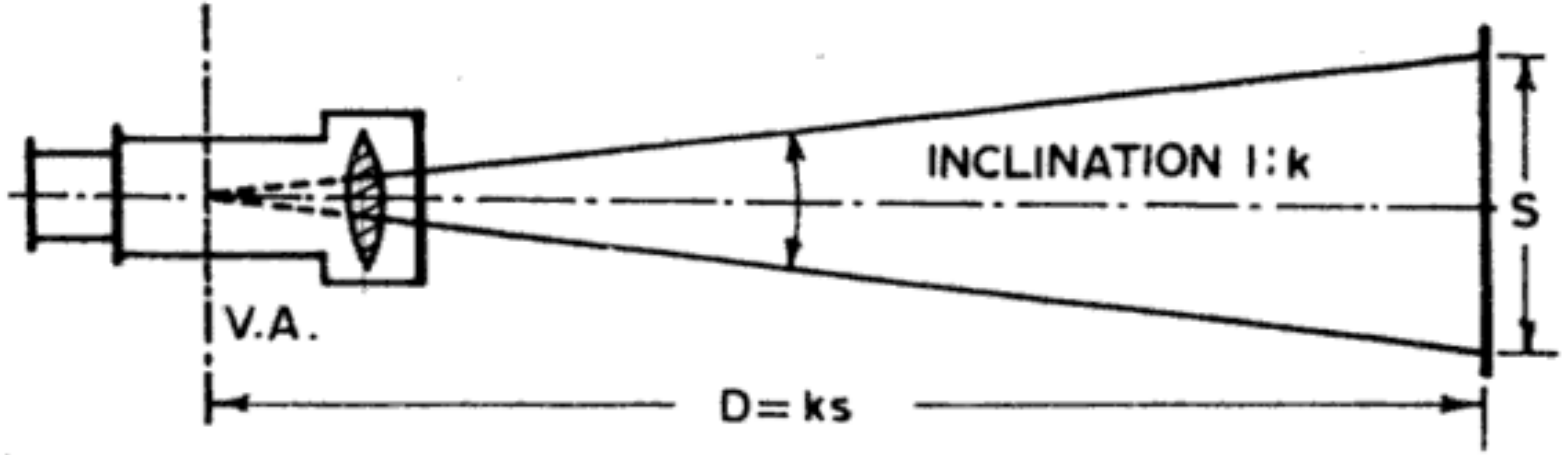
In the distance formula $D = ks + C$, the staff intercept s is proportional to $(D - C)$ which is the distance between the staff and the exterior principal focus of the objective (see Fig. 1.4). This is because the vertex of the measuring triangle (or anallactic point) falls at the exterior principal focus of the objective and *not* at the vertical axis of the instrument. In 1840, Porro devised the *external focusing anallactic telescope*, the special feature of which is an additional (convex) lens, called an *anallactic lens* (or anallatic lens), placed between the diaphragm and the objective at a fixed distance from the latter. Fig. 1.9 (*a*) shows the lines of sight with an ordinary telescope, and Fig. 1.9 (*b*) shows the lines of sight with an anallactic lens.



(a) LINES OF SIGHT WITH ORDINARY TELESCOPE



(a) LINES OF SIGHT WITH ORDINARY TELESCOPE



(b) LINES OF SIGHT WITH ANALLACTIC LENS

The word *'anallactic'* means 'unalterable' or 'invariable'; by the provision of anallactic lens, the vertex is formed at the vertical axis and its position is always fixed irrespective of the staff position. The anallactic lens is generally provided in external focusing telescope only and not in internal focusing telescope since the latter is virtually anallactic due to very small additive constant.

Theory of Anallactic Lens : Horizontal Sights

Fig. 1.10 shows the optical diagram of an external focusing anallactic telescope.

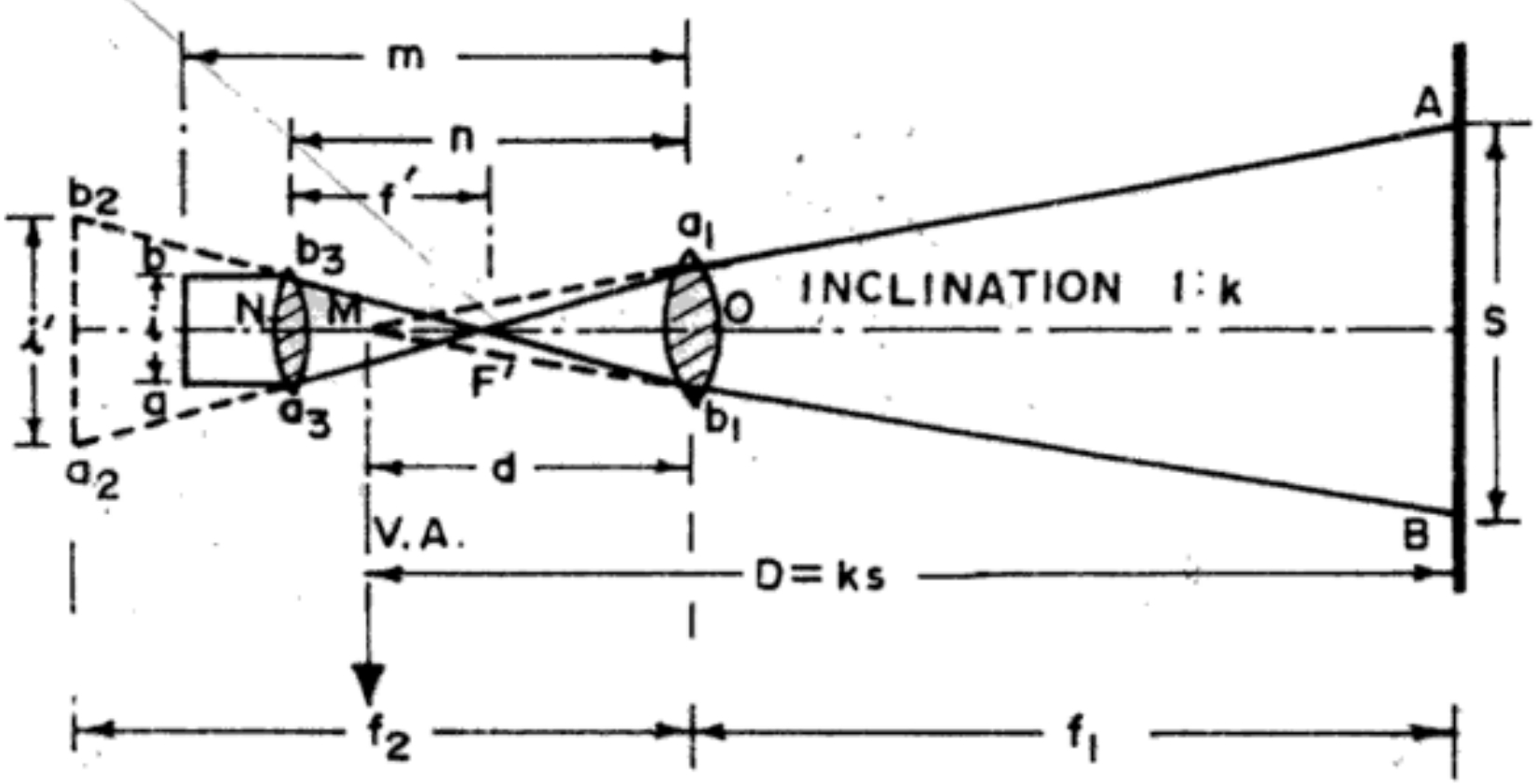


FIG. 1.10. THEORY OF ANALLACTIC LENS.

Let

O = Optical centre of the objective

N = Optical centre of the anallactic lens

M = Position of the vertical axis of the instrument

F' = Exterior principal focus of the anallactic lens

A, B = Points on the staff corresponding to the stadia wires

a_1, b_1 = corresponding points on objective

a_3, b_3 = Corresponding points on anallatic lens

a, b = Position of stadia wires

a_2, b_2 = Corresponding points if there were no anallactic lens

f_1 and f_2 = The conjugate focal length of the objective

D = distance of the staff from the vertical axis

d = distance of the vertical axis from the objective

m = distance of the diaphragm from the objective

n = distance of the anallactic lens from the objective

a_3, b_3 = Corresponding points on anallatic lens

a, b = Position of stadia wires

a_2, b_2 = Corresponding points if there were no anallactic lens

f_1 and f_2 = The conjugate focal length of the objective

D = distance of the staff from the vertical axis

d = distance of the vertical axis from the objective

m = distance of the diaphragm from the objective

n = distance of the anallactic lens from the objective

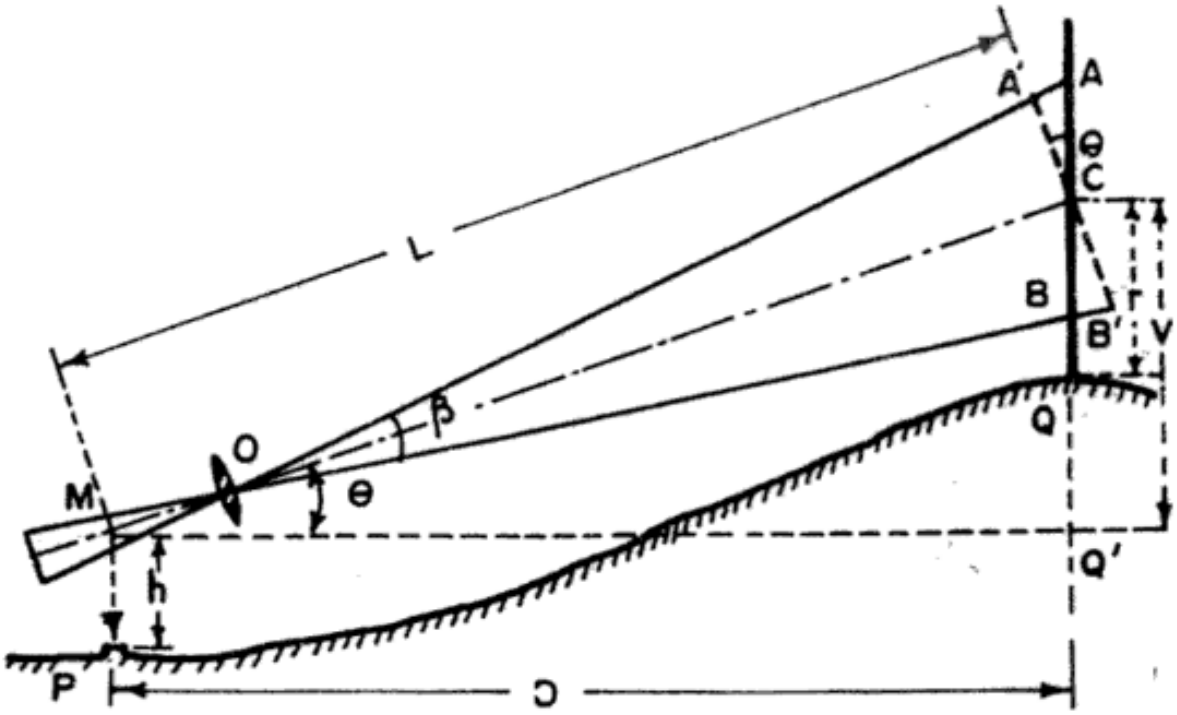
f = focal length of object glass

f' = focal length of the anallactic lens

i = Stadia interval

$s = AB$ = staff intercept.

Example 1.1. *A tacheometer was set up at a station A and the readings on a vertically held staff at B were 2.255, 2.605 and 2.955, the line of sight being at an inclination of + 8° 24'. Another observation on the vertically held staff at B.M. gave the readings 1.640, 1.920 and 2.200, the inclination of the line of sight being + 1° 6'. Calculate the horizontal distance between A and B , and the elevation of B if the R.L. of B.M. is 418.685 metres. The constants of the instruments were 100 and 0.3.*



Solution.

(a) Observation to B.M. :

$$V = ks\frac{\sin 2\theta}{2} + C\sin\theta$$

Here, $k = 100$; $s = 2.200 - 1.640 = 0.560$ m ;

$C = 0.3$ m

$\therefore$ $V = \frac{1}{2} \times 100 \times 0.56 \sin 2° 12' + 0.3 \sin 1° 6'$

$= 1.075 + 0.006 = 1.081$ m

Elevation of collimation at the instrument

$= 418.685 + 1.920 - 1.081$

$=$ **419.524 m**

(b) Observation to B :

$s = 2.955 - 2.255 = 0.700$ m ; $\theta = 8° 24'$

$D = ks\cos^2\theta + C\cos\theta$

$= 100 \times 0.7 \cos^2 8° 24' + 0.3 \times \cos 8° 24'$

$= 68.506 + 0.2968 \approx$ **68.80 m**

$V = ks\frac{1}{2}\sin 2\theta + C\sin\theta$

$= \frac{1}{2} \times 100 \times 0.7 \sin 16° 48' + 0.3 \sin 8° 24'$

$= 10.116 + 0.044 = 10.160$

R.L. of $B = 419.524 + 10.160 - 2.605$

$=$ **427.079 m**

Example 1.2. *The elevation of a point P is to be determined by observations from two adjacent stations of a tacheometric survey. The staff was held vertically upon the point, and the instrument is fitted within an anallactic lens, the constant of the instrument being 100. Compute the elevation of the point P from the following data, taking both the observations as equally trustworthy :*

Inst. station	*Height of axis*	*Staff point*	*Vertical angle*	*Staff readings*	*Elevation of station*
A	*1.42*	*P*	*+ 2° 24′*	*1.230, 2.055, 2.880*	*77.750 m*
B	*1.40*	*P*	*– 3° 36′*	*0.785, 1.800, 2.815*	*97.135 m*

Also, calculate the distance of A and B from P.

Solution.

(*a*) ***Observation from A to P :***

$$s = 2.880 - 1.230 = 1.65 \text{ m}$$

$$D = ks\cos^2\theta$$

$$= 100 \times 1.65 \cos^2 2° 24' = \mathbf{164.7\ m}$$

$$V = ks\frac{\sin 2\theta}{2}$$

$$= \tfrac{1}{2} \times 100 \times 1.65 \sin 4° 48' = 6.903$$

$$\text{R.L. of } P = 77.750 + 1.420 + 6.903 - 2.055$$

$$= 84.018 \text{ m}$$

(*b*) ***Observation from B to P :***

$$s = 2.815 - 0.785 = 2.03 \text{ m}$$

$$D = ks\cos^2\theta = 100 \times 2.03 \cos^2 3° 36' = \mathbf{202.2\ m.}$$

$$V = ks\tfrac{1}{2}\sin 2\theta = \tfrac{1}{2} \times 100 \times 2.03 \sin 7° 12' = 12.721 \text{ m}$$

$$\text{R.L. of } P = 97.135 + 1.40 - 12.721 - 1.800 = 84.014$$

Average elevation of P

$$= \tfrac{1}{2}(84.018 + 84.014) = \mathbf{84.016\ m}$$

Example 1.3. *Determine the gradient from a point A to a point B from the following observations made with a tacheometer fitted with an anallactic lens. The constant of the instrument was 100 and the staff was held vertically :*

Inst. station	*Staff point*	*Bearing*	*Vertical angle*	*Staff readings*
P	*A*	*134°*	*+ 10° 32′*	*1.360, 1.915, 2.470*
	B	*224°*	*+ 5° 6′*	*1.065, 1.885, 2.705*

Solution.

(*a*) ***Observation from P to A :***

$$s = 2.470 - 1.360 = 1.11 \text{ m}$$

$$D = ks\cos^2\theta = 100 \times 1.11 \cos^2 10° 32'$$

$$= 107.3 \text{ m}$$

$$V = ks\,\tfrac{1}{2}\sin 2\theta = \tfrac{1}{2} \times 100 \times 1.11 \sin 21° 4' = 19.95 \text{ m}$$

Difference in elevation between *A* and instrument axis

$$= 19.95 - 1.915$$

$$= 18.035 \text{ m} \qquad (A \text{ being higher})$$

(*b*) ***Observation from P to B :***

$$s = 2.705 - 1.065 = 1.64 \text{ m}$$

$$D = ks\cos^2\theta = 100 \times 1.64 \times \cos^2 5° 6' = 162.7 \text{ m}$$

$$V = ks\,\tfrac{1}{2}\sin 2\theta = \tfrac{1}{2} \times 100 \times 1.64 \sin 10° 12'$$

$$= 14.521 \text{ m}$$

Difference in elevation between B and instrument axis
$= 14.521 - 1.885 = 12.636$ m (B being higher)

(c) *Gradient from A to B :*

Distance $AP = 107.3$ m

Distance $BP = 162.7$ m

$\angle APB = 224° - 134° = 90°$

$$AB = \sqrt{AP^2 + BP^2} = \sqrt{(107.3)^2 + (162.7)^2} = 194.9 \text{ m}$$

Difference in elevation between A and B
$= 18.035 - 12.636 = 5.399$ (A being higher)

Gradient from A to B

$$= \frac{5.399}{194.9} = 1 \text{ in } 36.1 \text{ (falling).}$$

Thank you