BANGLADESH UNIVERSITY OF PROFESSIONALS
Military Institute of Science and Technology
B.Sc. in Environmental, Water Resources, and Coastal Engineering,
Term Final (Fall) Examination 2023: Dec 2023- Feb 2024

Student Group: 0<Earned Credit Hours ≤ 36

Subject: Math 103, Differential Equations and Matrix

Time: 3.00 hours **Full Marks: 180**

INSTRUCTIONS:

a. Use **SEPARATE** answer scripts for each section.
b. **Question-1** in **Section-A** and **Question-5** in **Section-B** are compulsory
c. Answer any other **TWO** rest of the **THREE** questions from each section.
d. Figures in the margin indicate full **marks**.
e. Assume reasonable data if necessary.
f. **Symbols** and **abbreviations** used have their usual meanings.

SECTION-A

Question 1 (Compulsory)

a. Define the followings with example **3+4+3**
(i) Ordinary differential equation (ODE)
(ii) Partial differential equation (PDE)
(iii) Order of a differential equation (DE)

b. Form the DE of the curve represented by $y^2 - 2ax + x^2 = a^2$ where "a" is an arbitrary constant. **10**

c. Define clairauits equations. Solve $y = px + \frac{a^2}{p}$ and find the singular solution (if exists). **2+8**

Question 2

a. Solve the following first order DEs: **10+10**
(i) $x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$
(ii) $(x^2 + y^2)dx - 2xydy = 0$

b. Solve: $p^2 + 2xp - 3x^2 = 0$ where $p = \frac{dy}{dx}$. **10**

Question 3

Solve the following higher order DEs **3 × 10 = 30**

a. $(D^3 - D^2 - 8D + 12)y = 0$
b. $(D^3 + 3D^2 + 3D + 1)y = e^{-x}$
c. $(D^2 - 9)y = 2xe^{3x}$

Question 4

a. Solve the following DE **15**

$$(x^2D^2 - xD + 1)\, y = lnx$$

b. Use the power series method to find the general solution of the DE **15**

$$y'' + xy' + (x^2 + 2)y = 0$$

SECTION-B

Question 5 **(Compulsory)**

a. Show that $u(x,y,z) = a$ and $v(x,y,z) = b$ are two independent solutions of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ where P, Q, R are functions of x, y, and z. **10**

b. Find the differential equation of all spheres of radius λ, having centre in the xy-plane. **10**

c. Find the general integral of **10**

$$x(z-2y) = y^2p - xyq$$

Question 6

a. Find complete integral of $px + qy - pq = 0$ where p and q bears the usual meaning. **10**

b. Classify the Partial Differential Equation **10**

$$x\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + y\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} = 0$$

c. Solve $(D^2 + 2DD' + D'^2)z = x^2 + xy + y^2$ **10**

Question 7

a. Define orthogonal and symmetric matrix. **2+8**
Check whether the following system of linear equations is consistent or not and if consistent find the solution

$$3x + 3y + 2z = 5$$
$$2x + 4y - 6z = -4$$
$$x + 5y + 3z = 10$$

b. Define linearly dependence for vector space. **2+8**
Examine whether the vectors $(1,-1,3), (1,4,5), (2,-3,-7)$ are in $\mathbb{R}^3$ space linearly dependent or independent.

c. Use Cayley-Hamilton theorem to find the inverse of the matrix A, **7+3**
where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$
Also find A^4.

Question 8

a. Define Basis. Find the rank and nullity of A **1+7**
Where, $A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 2 & 5 & 1 \\ -4 & -8 & 1 & -3 & 1 \end{bmatrix}$

b. Define linear combination of vectors. **1+9**
For which value of τ, $\vec{V} = (1,\tau,5)$ is a linear combination of $\overrightarrow{V_1} = (1,-3,2)$ and $\overrightarrow{V_2} = (2,-1,1)$.

c. Define eigenvalues and eigenvectors. **2+10**
Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$