BANGLADESH UNIVERSITY OF PROFESSIONALS
Military Institute of Science and Technology
B.Sc. in Environmental, Water Resources and Coastal Engineering
Term Final (Fall) Examination 2021: Jan-Feb 2022

Student Group: 0 < Earned Credit Hours ≤ 36

Subject: MATH-103, Differential Equation and Matrix

Time: 3.00 hours **Full Marks: 180**

INSTRUCTIONS:

a. Use **SEPARATE** answer scripts for each section.
b. **Question-1** in **Section-A** and **Question-5** in **Section-B** are compulsory
c. Answer any other **TWO** questions out of **THREE** from each section.
d. Figures in the margin indicate full **marks**.
e. Assume reasonable data if necessary.
f. **Symbols** and **abbreviations** used have their usual meanings.

SECTION-A

Question 1 (Compulsory)

a. Explain degree and order of the differential equation with example. Form a differential equation of all circles of radius $'a'$. **12**

b. If $\frac{dy}{dx} = e^{x+y}$ and it is given that for $x = 1$ and $y = 1$; find y when $x = -1$. **08**

c. In a certain city the population gets double in 2 years and after 3 years the population is 20,000. Find the number of people initially being living in the city. **10**

Question 2 **15**

a. Write down the exactness conditions. Test whether the equation $(x+y)^2dx - (y^2 - 2xy - x^2)dy = 0$ is exact and if so solve it. **08**

b. (i) Solve: $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$. **07**

(ii) Solve: $(1+y^2)dx = (tan^{-1}y - x)dy$.

Question 3 Solve by appropriate methods **12**

a. $(D^2 + a^2)y = sinax$. **10**

b. $(4D^2 + 12D + 9)y = 144e^{-3x}$. **08**

c. $(D^2 + D + 1)^2y = 0$.

Question 4

a. Formulate the method of solution of homogeneous linear equations. **10**

b. Solve: **10**

(i) $x^2y_2 + xy_1 - 4y = 0$. **10**

(ii) $x^2y_2 + y = 3x^2$.

SECTION-B

Question 5 (Compulsory)

a. Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2-z^2)$. What is the order of this partial differential equation. 10

b. Find the solution of the following partial differential equations:

(i) $py + qx = xyz^2(x^2 - y^2)$. 10

(ii) $x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2+y^2)$. 10

Question 6

a. Apply Charpit's method find the complete integral of $zpq = p + q$. 10

b. Solve the following partial differential equations:

(i) $\{D^3 - 4D^2D' + 4D(D')^2\}z = 4\sin(2x + y)$. 10

(ii) $(r + s - 6t)z = ysinx$. 10

Question 7

a. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\,sin\frac{\pi x}{l}$ from which it is released at a time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a\,sin\left(\frac{\pi x}{l}\right)cos\left(\frac{\pi ct}{l}\right)$. 18

b. Determine the value of λ so that the following linear system has (i) a unique solution (ii) infinitely many solution (iii) no solution. 12

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y10z = \lambda^2.$$

Question 8

a. Find the inverse of the matrix 10

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & -1 & 6 \\ -1 & 5 & 1 \end{bmatrix}.$$

b. Find the rank of the matrix 07

$$A = \begin{bmatrix} 4 & 2 & -1 & 3 \\ 0 & 5 & -1 & 2 \\ 12 & -4 & -1 & 5 \end{bmatrix}.$$

c. Determine the eigenvalues and one of associated eigenvectors of matrix 13

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 2 & -8 & -2 \\ 1 & 2 & 2 \end{bmatrix}.$$