BANGLADESH UNIVERSITY OF PROFESSIONALS

Military Institute of Science and Technology

B.Sc. in Environmental, Water Resources and Coastal Engineering, Term Final (Spring) Examination

2021: July-Aug 2021

Student Group: 0 < Earned Credit Hours ≤ 36

Subject: Math-107, Differential, Integral Calculus and Matrices

: 2.00 hours (Online Examination for Section-A + Section-B)

: 1.00 hour (Online Examination)

Full Marks : 180
Section A : 90

INSTRUCTIONS - ONLINE EXAMINATION (IF PHYSICAL EXAMINATION IS NOT HELD)

a. **Question -1** and **Question – 4(Viva Voce)** in **Section A** are compulsory.
b. Answer any **OTHER ONE** question from **Q-2 & Q-3** of **Section-A**.
c. Figures in the margin indicate full **marks.**
d. Assume reasonable data if necessary.
e. **Symbols** and **abbreviations** used have their usual meanings.

SECTION-A

Question – 1 (Compulsory)

a. A function $f(x)$ is defined as: $f(x) = \begin{cases} x & ; x > 0 \\ 0 & ; x = 0 \\ -x & ; x < 0 \end{cases}$ **10**

Find the value of $\lim_{x\to 0} f(x)$.

b. A function $f(x)$ is defined as: $f(x) = \begin{cases} x^2 \sin(1/x) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$ **15**

Test the continuity and differentiability at $x = 0$.

c. Differentiate $e^{\sin^{-1}x}$ with respect to $cos3x$. **5**

Question – 2

a. Find $\frac{dy}{dx}$ where $(cosx)^{sinx} + (sinx)^{cosx} = 0$. **10**

b. If $y = a\cos(lnx) + b\sin(lnx)$, then show that **10**

$$x^2 y_{n+2} + (2n+1)\, x\, y_{n+1} + (n^2+1) y_n = 0$$

c. State Taylor's series. Expand $sinx$ in power of $(x - \pi/2)$. **10**

Question – 3

a. State Mean-Value theorem. Verify Mean-Value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ on $[0,4]$. **10**

b. State Euler's theorem. Hence show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tanu$; where **10**

$u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$

c. Show that, **10**

$$\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x} = -\frac{1}{2}e$$

Question – 4 (Compulsory) VIVA VOCE **30**

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Subject: Math-107, Differential, Integral Calculus and Matrices

Total :**2.00 hours (Online Examination for Section-A + Section-B)** Full Marks : 180
Section B : 1.00 hour (Online Examination) Section A : 90

INSTRUCTIONS - ONLINE EXAMINATION(IF PHYSICAL EXAMINATION IS NOT HELD)

a. **Question -1** and **Question – 4(Viva Voce)** in **Section B** are compulsory.
b. Answer any **OTHER ONE** question from **Q-6 & Q-7** of **Section-B**.
c. Figures in the margin indicate full **marks.**
d. Assume reasonable data if necessary.
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SECTION-B

Question – 5 (Compulsory)

a. Integrate the following functions:

(i) $\frac{x^2}{(x+1)(x+2)^2}$ **10**

(ii) $\frac{\sin x+\cos x}{2\cos x+3\sin x}$ **10**

(iii) $e^x\left(\frac{1-x}{1+x^2}\right)^2$ **10**

Question – 6

a. Establish Walli's formula for $\int_0^{\frac{\pi}{2}} \sin^n x dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x dx$. **12**

b. Evaluate the following definite integrals :

(i) $\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\cos x}$ **9**

(ii) $\int_0^3 \frac{dx}{(x^2+16)^{3/2}}$ **9**

Question – 7

a. Evaluate the area enclosed by $x^{2/3} + y^{2/3} = a^{2/3}$ **9**

b. Find the volume of the solid generated by revolving $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about x-axis. **9**

c. Define Beta function. Prove that $\beta(m,n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}x\cos^{2n-1}x dx$. **12**
Also evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x\cos^3 x dx$

Question – 8 (Compulsory) **30**
VIVA VOCE