BANGLADESH ...

Military Institute of Science and Technology

B.Sc. in Environment, Water Resource and Coastal Engineering

Student Group : 0 < Earned Credit Hour <= 36, Final Examination (Spring) : Feb 2021

Subject: MATH-107, Differential, Integral Calculus and Matrix

Total: 2.00 hours
Section A : 1.00 hour

Full Marks: 180
Section A : 90

INSTRUCTIONS:

a. Use **SEPARATE** answer scripts for each section.
b. **Question – 1 and Question – 4 (Viva Voce)** in **Section A** are compulsory.
c. Answer any **OTHER ONE** question from this section **(From Q - 2 & Q - 3)**.
d. Figures in the margin indicate full **marks**.
e. Assume reasonable data if necessary.
f. **Symbols** used have their usual meanings.

SECTION-A

Question – 1 (Compulsory)

a. Define differentiability. Test the continuity and differentiability of $f(x)$ at $x=\frac{3}{2}$, where, **15**

$$f(x)=\begin{cases} 3+2x \text{ when } -\frac{3}{2}<x<0 \\ 3-2x \text{ when } 0<x<\frac{3}{2} \\ -3-2x \text{ when } x\geq\frac{3}{2} \end{cases}$$

b. Evaluate $\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}$ **07**

c. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1-x^2}\right)$. **08**

Question – 2

a. State Leibnitz's theorem. If $y=\sin(m\sin^{-1}x)$ then show that, **10**

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2-m^2)y_n=0$$

b. Expand $\sin x$ in the power of $x-\frac{\pi}{2}$ by using Taylor's series. **10**

c. Find the minimum and maximum value of $y=4e^{2x}+9e^{-2x}$. **10**

Question – 3

a. Define homogeneous function. If u is a homogeneous function of x and y of degree n, prove that, **15**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$$

b. Define radius of curvature. Find the radius of curvature at the point θ on the cycloid $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$. **15**

Question – 4 (Compulsory) **30**

Marks allotted for Viva 30

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Total: 2.00 hours
Section B : 1.00 hour

Full Marks: 180
Section B : 90

INSTRUCTIONS:

a. Use **SEPARATE** answer scripts for each section.
b. **Question – 5 and Question – 8 (Viva Voce)** in **Section B** are compulsory.
c. Answer any **OTHER ONE** question from this section **(From Q - 6 & Q - 7)**.
d. Figures in the margin indicate full **marks**.
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SECTION-B

Question – 5 (Compulsory)

a. Reduce the system of linear equations **10**

$$\left.\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}\right\}$$

into echelon form and hence solve it.

b. Define rank of a matrix. Hence find the rank of the matrix: **10**

$$A = \begin{bmatrix} 2 & -3 & 5 & -3 & -2 \\ 3 & 4 & 3 & -1 & -3 \\ 5 & 6 & -1 & 3 & -5 \end{bmatrix}$$

c. Show that the vectors $v_1 = (1, -2, -1), v_2 = (1, -4, 3)$ and $v_3 = (0, 2, 4)$ are linearly independent. **10**

Question – 6

a. Evaluate the followings: **08**

(i) $\int \frac{\sin 2x}{a\sin^2 x + b\cos^2 x} dx$

(ii) $\int_0^{\frac{\pi}{2}} \frac{1}{1-\sin x} dx$

b. Find the reduction formula for $\int \cos^n x dx$ and hence evaluate $\int \cos^3 x dx$. **12**

c. Evaluate: $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{1}{\sqrt{n^2-1}} + \frac{1}{\sqrt{n^2-2^2}} + \cdots + \frac{1}{\sqrt{n^2-(n-1)^2}}\right]$ **10**

Question – 7

a. Prove that, $\int_0^{\frac{\pi}{2}} \sin^p\theta\cos^q\theta d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)}$ **10**

b. Show that the volume of a sphere of radius "a" is $\frac{4}{3}\pi a^3$. **10**

c. Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **10**

Question – 8 (Compulsory) **30**

Marks allotted for Viva 30