Separation of Variables

Rule-3

If the D.E is of the form $\frac{dy}{dx} = f(ax+by+c)$ then put $ax+by+c=z$ and simplify finally integrating we will get the solution

Solve:

$$\frac{dy}{dx} = (4x+y+1)^2 \quad \text{---------} (1)$$

Let, $4x+y+1 = z$

differentiating with respect to x

$$4.1 + \frac{dy}{dx} + 0 = \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dz}{dx} - 4$$

putting the values in (1)

$$\frac{dz}{dx} - 4 = z^2$$

$$\frac{dz}{dx} = 4 + z^2$$

$$\frac{dz}{4+z^2} = dx$$

$$\frac{dz}{z^2+z^2} = dx$$

Integrating

$$\int \frac{dz}{z^2+z^2} = dx$$

$$\Rightarrow \frac{1}{2} \tan^{-1} \frac{z}{2} = x + C$$

Homogenous D.E

Homogenous Equation:

An equation in which the sum of the powers of x & y in every terms are equal than the equation is called homogenous equation.

Example - $ax^2 + 2hxy + by^2$ is a ~~hog~~ homogenous equation of 2nd degree in x & y

Homogenous D.E

An equation of the form

$$\frac{dy}{dx} = \frac{f(x,y)}{\varphi(x,y)}$$ in which $f(x,y)$ and $g(x,y)$ are homogenous functions of x & y of the same degree, then the

equation is called homogenous D.E

Working Rule for solving homogenous D.E:

1. Re-write the given homogenous D.E in the form $\frac{dy}{dx} = \frac{f(x,y)}{\varphi(x,y)}$ ——— (1)

2. Substitute $y = vx$.

3. Differentiate $y = vx$, the we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

4. Put the values in (1)

$$v + x\frac{dv}{dx} = \frac{f(v)}{\varphi(v)}$$

$$v + x\frac{dv}{dx} = F(v), \text{ where } F(v) = \frac{f(v)}{\varphi(v)}$$

$$x\frac{dv}{dx} = F(v) - v$$

$$\frac{dv}{F(v)-v} = \frac{dx}{x}$$ where the variables have been separated.

5. Then the general solution will be obtained by integration.

Solve: $(x^2+y^2)\,dx + 2xy\,dy = 0$

Soln: $(x^2+y^2)\,dx = 2xy\,dy$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy} \quad -(1)$$

put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Putting the values in (1)

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2x \cdot vx}$$

$$v + x\frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 + v - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$x\frac{dv}{dx} = -\frac{v^2 - 1}{2v}$$

$$2v\,dv$$