Classwork 22/11/2023

Homogenous D.E (Exam - 15 marks question)

Equation reducible to homogenous.

1. If the D.E is of the form

$$\frac{dy}{dx} = \frac{ax+by+c}{a_1x+b_1y+c_1} \quad \text{when } \frac{a}{a_1} \neq \frac{b}{b_1}$$

this type of D.E can be reduced to homogenous equation by putting $x = \alpha + h$, $y = \beta + k$, where α, β are new variables and h, k are arbitrary constants.

Therefore putting $\beta = v\alpha$ and with the help of separation of variables and then integrate we can find the solution.

Solve: $\frac{dy}{dx} = \frac{2x+y-2}{3x+y-3}$ ——— (1)

Here $\frac{2}{3} \neq \frac{1}{1}$

put, $x = \alpha + h$, $y = \beta + k$

$dx = d\alpha + 0$ $\quad dy = d\beta + 0$

$dx = d\alpha$ $\quad dy = d\beta$

$$\frac{dy}{dx} = \frac{d\beta}{d\alpha} \quad \text{——— (2)}$$

putting the values in (1)

$$\frac{d\beta}{d\alpha} = \frac{2(\alpha+h) + \beta + k - 2}{3(\alpha+h) + \beta + k - 3}$$

$$\frac{d\beta}{d\alpha} = \frac{2\alpha + \beta + 2h + k - 2}{3\alpha + \beta + 3h + k - 3} \quad \text{— (3)}$$

Now choose $2h + k - 2 = 0$

and $3h + k - 3 = 0$

$$\frac{h}{-3+2} = \frac{k}{-6+6} = \frac{1}{2-3}$$

$$= \frac{h}{-1} = \frac{k}{0} = \frac{1}{-1} \quad (h = 1,\ k = 0)$$

From (3)

$$\frac{d\beta}{d\alpha} = \frac{2\alpha+\beta}{3\alpha+\beta} \quad \text{---} \quad (4)$$

Now, put $\beta = v\alpha$

$$\frac{d\beta}{d\alpha} = v + \alpha \frac{dv}{d\alpha} \quad - (5)$$

From (4),

$$v + \alpha \frac{dv}{d\alpha} = \frac{2\alpha + v\alpha}{3\alpha + v\alpha}$$

$$v + \alpha \frac{dv}{d\alpha} = \frac{2+v}{3+v}$$

$$\alpha \frac{dv}{d\alpha} = \frac{2+v}{3+v} - v$$

$$\alpha \frac{dv}{d\alpha} = \frac{2+v-3v-v^2}{3+v}$$

or, $$\alpha \frac{dv}{d\alpha} = -\frac{(v^2+2v-2)}{v+3}$$

$$\text{or,} \quad \frac{(v+3)\,dv}{v^2+2v-2} = -\frac{d\alpha}{\alpha}$$

$$\text{or,} \quad \frac{\left\{\frac{1}{2}(2v+2)+2\right\}dv}{v^2+2v-2} + \frac{d\alpha}{\alpha} = 0$$

Integrating both sides.

$$\frac{1}{2}\int \frac{(2v+2)\,dv}{v^2+2v-2} + 2\int \frac{dv}{v^2+2v-2} + \int \frac{d\alpha}{\alpha} = 0$$

$$\text{or,} \quad \frac{1}{2}\int \frac{(2v+2)\,dv}{v^2+2v-2} + 2\int \frac{dv}{(v+1)^2-(\sqrt{3})^2} + \int \frac{d\alpha}{\alpha} = 0$$

$$\text{or,} \quad \frac{1}{2}\ln(v^2+2v-2) + \frac{2}{2\sqrt{3}}\ln\frac{v+1-\sqrt{3}}{v+1+\sqrt{3}} = c$$

$$\text{or, } \ln\left(\frac{\beta^2}{\alpha^2}+\frac{2\beta}{\alpha}-2\right)\alpha^2+\frac{2}{\sqrt{3}}\ln\frac{\frac{\beta}{\alpha}+1-\sqrt{3}}{\frac{\beta}{\alpha}+1+\sqrt{3}}=c$$

$$\text{or, } \ln\left(\beta^2+2\alpha\beta-2\alpha^2\right)+\frac{2}{\sqrt{3}}\ln\left\{\frac{\beta+(1-\sqrt{3})\alpha}{\beta+(1+\sqrt{3})\alpha}\right\}=c$$

since, $x=\alpha+1$ & $y=\beta+0$ so

putting $\alpha=x-1$ & $\beta=y$ in the above equation we get,

$$\Rightarrow \ln\left\{y^2+2y(x-1)-2(x-1)^2\right\}+\frac{2}{\sqrt{3}}\ln\left\{\frac{y+(1-\sqrt{3})(x-1)}{y+(1+\sqrt{3})(x-1)}\right\}=c$$

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(i) Defination

(ii) formation of D.E

(iii) Separation of variables.