29.11.23 (Wednesday)

Math-103 (Wing Cdr. Monir)

Exact D.E: Let M and N both are functions of x, y.

If $Mdx + Ndy = 0$ is obtained by differentiating $f(x, y) = 0$ then $Mdx + Ndy = 0$ is called exact differential.

Example:

$$xy = c \quad \text{——} (1)$$

$$d(xy) = 0$$

$$ydx + xdy = 0 \quad \text{——} (2)$$

Since (2) is called exact d.e.

Rules from finding the sol^n of an exact eqn:

Step 1: Comparing the given d.e with $Mdx+Ndy=0$ and find M, N then find $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, we conclude that the given eqn is exact

Step 2: The soln of an exact eqn is given by

$$\int M\,dx + \int (\text{Term of } N \text{ not containing } x)\,dy = \text{constant}$$

y is constant

Solve: $(4x+3y+1)dx + (3y+2y+1)dy = 0$ ——— (1)

Which is of the form $Mdx+Ndy=0$

where $M = 4x+3y+1$

$N = 3x+2y+1$

$\frac{\partial M}{\partial y} = 3$ $\qquad$ $\frac{\partial N}{\partial x} = 3$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So given diff. eqn is exact.

The soln of the eqn is,

$$\int (4x+3y+1)\,dx + \int (2y+1)\,dy = c$$

y is constant

$$\Rightarrow \frac{4x^2}{2} + 3xy + x + \frac{2y^2}{2} + y = c$$

$$\Rightarrow 2x^2 + 3xy + x + y^2 + y = c$$

$$\Rightarrow 2x^2 + y^2 + 3xy + x + y = 0$$

② $(x^3+3xy^2)dx + (y^3+3x^2y)dy = 0$

③ $(2x+3y-5)dy + (3x+2y-5)dx = 0$

②

$(x^3+3xy^2)dx + (y^3+3x^2y)dy$

$M = x^3+3xy^2$

$N = y^3+3x^2y$

$\frac{\partial M}{\partial x} = 3x^2 + 3y^2$

$\frac{\partial N}{\partial y} = 3y^2 + 3x^2$

$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

Soln: $\int (x^3+3xy^2)dx + \int y^3 dy = c$

$\Rightarrow \frac{x^4}{4} + 3y^2\frac{x^2}{2} + \frac{y^4}{4} = c$

$\Rightarrow \frac{x^4}{4} + \frac{3}{2}x^2y^2 + \frac{y^4}{4} = c$

$\Rightarrow x^4+y^4+6x^2y^2 = c$

③

$(2x+3y-5)dy + (3x+2y-5)dx = 0$

$M = 3x+2y-5 \qquad \frac{\partial M}{\partial x} = 3$

$N = 2x+3y-5 \qquad \frac{\partial N}{\partial y} = 3$

$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

Soln,

$\int (3x+2y-5)dx + \int (3y-5)dy = c$

$\Rightarrow \frac{3x^2}{2} + 2xy - 5x + \frac{3y^2}{2} - 5y = c$

$\Rightarrow 3x^2 + y^2 + 4xy - 10x - 10y = c$