Classwork

First Order but higher Degree D.E.

A. D.E involving first order but higher than one is called first order but higher degree D.E i.e $P^n + A_1 P^{n-1} + A_2 P^{n-2} + \ldots\ldots + A_n = 0$

is a first order and n.th degree D.E where $P = \frac{dy}{dx}$ and $A_1, A_2, A\ldots, A_n$ are functions of $x, y$ the equation is solved by five methods.

(I) Solvable for P

(II) Solvable for y

(III) Solvable for x

(IV) Clairauts eqn

(V) Lagrangers eqn.

$\left(\frac{dy}{dx}\right)^2$ , $\left(\frac{dy}{dx}\right)^3$ , ..... $\left(\frac{dy}{dx}\right)^n$

($p^2$ over the first, $p^3$ over the second, $p^n$ over the last)

$O=1, D=2$ , $O=1, D=3$ $O=1, D=n$

$$p = \frac{dy}{dx}$$

* Solvable for p:

If the equation $p^n + A_1 p^{n-1} + A_2 p^{n-2} + \ldots + A_n$ can be expressed into n-factors of first order & first degree. $(p-\alpha_1)(p-\alpha_2)\ldots(p-\alpha_n) = 0$ then equating to zero each factor n-equation of first order and first degree are obtained, then the solutions are obtained by integration.

Solve: $p^2 + 2xp - 3x^2 = 0$ where $p = \frac{dy}{dx}$.

Soln → $p^2 + 2xp - 3x^2 = 0$

$\Rightarrow p^2 + 3xp - xp - 3x^2 = 0$

$p(p+3x) - x(p+3x) = 0$

$(p-x)(p+3x) = 0$

$p - x = 0$ ___ (2)

$p + 3x = 0$ ___ (3)

From (2)

$p - x = 0$

$\frac{dy}{dx} - x = 0$

$dy - x\,dx = 0$

Integrating,

$y = \frac{x^2}{2} - \frac{c}{2} = 0$

$2y - x^2 - c = 0$

From (3)

$p + 3x = 0$

$\frac{dy}{dx} + 3x = 0$

$dy + 3x\,dx = 0$

Integrating

$$y + \frac{3x^2}{2} - \frac{c}{2} = 0$$

$$2y + 3x^2 - c = 0$$