23

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Solve: $x(z-2y) = y^2p - xy$

Soln: $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$\Rightarrow \frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)} =$

Multiply by y 1st part

$\Rightarrow \frac{y\,dx}{y^2} =$

$\frac{dx}{y^2} = \frac{dy}{-xy}$

Multiply by y

$\frac{y\,dx}{y^2} = \frac{y\,dy}{-xy}$

$\Rightarrow \frac{dx}{y} = \frac{dy}{-x}$

$\Rightarrow x\,dx + y\,dy = 0$

$\frac{x^2}{2} + \frac{y^2}{2} = a'$

$\Rightarrow x^2 + y^2 = 2a' = a$

$$\frac{dy}{-y} = \frac{dz}{z-2y}$$

$$\Rightarrow \frac{dz}{dy} = -\frac{z-2y}{y}$$

$$= -\frac{z}{y} + 2$$

$$\frac{dz}{dy} + \frac{z}{y} = 2 \quad \text{Linear ODE}$$

$$\boxed{\frac{dy}{dx} + py = q}$$

$$e^{\int p\,dx} = e^{\int \frac{1}{y} dy}$$

$$= e^{\ln y}$$

$$= y$$

Integrating factor y.

$$y\frac{dz}{dy} + z = 2y$$

$$y\,dz + z\,dy = 2y\,dy$$

$$d(zy) = 2y\,dy$$

$$zy - y^2 = y^2 + b$$

$$2y - y^2 = b$$

Solve $zp = -x$

Soln:

$$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$$

1st part

$\frac{dx}{z} \xrightarrow{\quad} \frac{dy}{0}$ → Multiply

$$\frac{dx}{z} = \frac{dz}{-x}$$

$$x^2 + z^2 = a$$

$$\frac{dy}{0} = \frac{dz}{-x}$$

$$\Rightarrow -x\,dy = 0$$

$$dy = 0$$

$$y = c$$

Find the soln of $xp + yq = z$, representing a surface meeting the parabola $y^2 = 4x$; $z = 1$

$$x/z = a \quad \text{---(1)}$$

$$y/z = b \quad \text{---(2)}$$

$$\phi\left(\frac{x}{z}, \frac{y}{z}\right) = 0$$

Putting in (ii)

$x = a$

$y = b$

$y^2 = b^2 = 4x$ $\quad b^2 = 4a$

$= 4a$ $\quad \left(\frac{y}{z}\right)^2 = 4x/z$

$y^2 = 4xz$

H.W. Find the general soln of

$yzp + zxq = xy$

Complete soln : যতগুলো I.V. থাকবে ততগুলো arbitary Constant থাকবে।

$z = ax + by$

particular soln :

$a = 2 \quad b = 5$

$z = 2x + 5y$

Singular soln :

$\frac{\delta z}{\delta a} = 0 \quad \frac{\delta z}{\delta b} = 0$

$b = \varphi(a)$

General soln : $z = ax + \varphi(a)\, y$

$az = (ut + c)^2$

$az = (x + ay + c)^2$

which is the required complete integral.

H.W. Find the complete integral of

$q^2 = z^2 p^2 (1 - p^2)$

H.W Find the ~~p, q, z~~ complete integral of ~~x+ay=u z~~

$$qz = pq$$

Soln: Ans a trial soln we assume that z is a func of $x+ay=u$ (say) where a is arbitrary constant.

$$p = \frac{\delta z}{\delta x} = \frac{dz}{du} \cdot \frac{\delta u}{\delta x} = \frac{dz}{du}$$

$$q = \frac{\delta z}{\delta y} = \frac{dz}{du} \frac{\delta u}{\delta y} = a \frac{dz}{du}$$

$$qz = \frac{dz}{du} \cdot a \frac{dz}{du}$$

$$qz = a\left(\frac{dz}{du}\right)$$

$$\frac{dz}{du} = \frac{1}{\sqrt{a}} \cdot 2\sqrt{z}$$

$$\frac{dz}{du} = 2\frac{1}{\sqrt{a}} du$$

$$\frac{dz}{\sqrt{z}} = 2\frac{1}{\sqrt{a}} du$$

$$z^{-1/2} dz = 2\frac{1}{\sqrt{a}} du$$

$$\frac{z^{1/2}}{1/2} = 2\frac{1}{\sqrt{a}} u$$

$$\Rightarrow \frac{1}{1/2}\frac{1}{\sqrt{a}} u \quad \therefore \sqrt{a}\sqrt{z} = u + c$$

$$\sqrt{z} = \sqrt{a}\, u$$