Math-103

Matrix

25/9/23

A matrix Amxn over a field K is a rectangular array of scalar usually presented in the following form

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$A_{m\times 1}$ → কলাম ম্যাট্রিক্স

$B_{1\times n}$ → রো ম্যাট্রিক্স

a_{ij}, $i = \overline{1, m}$, $j = 1, n$

$A_{mn} = [a_{ij}]_{m\times n}$

$A_{3\times 2} = B_{3\times 2}$

$A_{3\times 2} \neq B_{2\times 3}$

Two matrices A and B are equal (ie — that's why) — A=B if they have the same size and if corresponding elements are equal.

Real Matrix $K \in R$ $\begin{bmatrix} 3 & 4 & 5 \\ 1 & 6 & 8 \end{bmatrix}$

Complex Matrix $K \in \mathbb{C}$ $\begin{bmatrix} 1-2i & i \\ 3i & -4 \end{bmatrix}$

$$\begin{bmatrix} x+y & 2x+z \\ x-y & z-t \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix} \longrightarrow \begin{aligned} x+y &= 3 \\ x-y &= 1 \\ 2x+t &= 7 \\ z-t &= 5 \end{aligned}$$

A, B, C (same size) and K, K' (scaler) হয়

তবে,

i) $(A+B)+C = A+(B+C)$

ii) $A+0 = 0+A = A$

iii) $A+(-A) = (-A)+A = 0$

iv) $A+B = B+A$

$K(A+B) = KA+KB$

$(K+K')A = KA+K'A$

$(KK')A = K(K'A)$

$1 \cdot A = A$

$(AB)C = A(BC)$

$A(B+C) = AB+AC$

$(B+C)A = BA+CA$

$K(AB) = (KA)B = A(KB)$

Transpose of a matrix: A^T

$$A = \begin{bmatrix} 5 & 1 & 3 \\ 4 & 8 & 9 \\ 11 & 13 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 4 & 11 \\ 1 & 8 & 13 \\ 3 & 9 & 2 \end{bmatrix}$$

যদি row আর column এর value same হয় তাহলে তাকে কর্ণ অতিক্রম বলে।

- $(A+B)^T = A^T + B^T$
- $(A^T)^T = A$
- $(KA)^T = KA^T$
- $(AB)^T = B^T A^T$

Diagonal and Trace:

* $a_{ij} \rightarrow i = j$ তখন Diagonal
* Diagonal element গুলোর যোগফল হল Trace
- For any square matrix A, the diagonal or main diagonal of A consists of the elements with the same subscripts that is $a_{11}, a_{22} \cdots a_{nn}$ The trace of A, written $tr(A)$ is the sum of the diagonal elements that's why $tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -4 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

Trace

$tr(AB) = tr(BA)$

$tr(A+B) = tr(A) + tr(B)$

$tr(kA) = k\, tr\, A$

$tr(A^T) = tr(A)$

যখন নির্ণায়ক 0 তখন বিপরীত ম্যাট্রিক্স নির্ণয় করা অসম্ভব নয়।

Invertiable (non-singular) Matrix:

$D \neq 0$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (Identity matrix)

$AB = BA = I$

Prove

$A \rightarrow B_1$

$A \rightarrow B_2$

ধরি,

A ম্যাট্রিক্স এর জন্য দুটি বিপরীত ম্যা
B_1 ও B_2 আছে

$AB_1 = B_1A = I$

$AB_2 = B_2A = I$

$\therefore B_1 = B_1 I = B_1(AB_2) = (B_1A)B_2 = IB_2 = B_2$

$(AB)^{-1} = B^{-1} A^{-1}$

Homework 25.9.23

Inverse 2×2 and 3×3 Matrix.

$\Rightarrow$ For 2×2 Matrix:

Suppose the 2×2 matrix is $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

determination of the squre matrix is,

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Now,

$(1,1)$ th co-ordiniate is $A_{11} = d$

$(1,2)$ th co-ordiniate is $A_{12} = -c$

$(2,1)$ th co-ordiniate is $A_{21} = -b$

$(2,2)$ th co-ordiniate is $A_{22} = a$

$$\therefore Adj(A) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ans.

For 3×3 Matrix:

Suppose the 3×3 matrix is $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

The co-ordiniates are,

$$A_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \qquad A_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$A_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \qquad A_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$A_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \qquad A_{23} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$A_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \qquad A_{32} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}$$

$$A_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

The determination of A matrix is,

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

The inverse matrix is, $A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$= \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Ans.

9.5.2023

02/10/2023

Upper triangular matrix:

$$\begin{pmatrix} a_{11} & b_{12} & c_{13} \\ 0_{21} & e_{22} & f_{23} \\ 0_{31} & 0_{32} & g_{33} \end{pmatrix}$$

$i = 2$
$j = 1$
$2 > 1$

$a_{ij} = 0$, $i > j$

Lower triangular matrix:

a_{ij}, $i < j$

$$\begin{bmatrix} 0_{11} & 0_{12} & a \\ 0_{21} & b & c \\ e & f & g \end{bmatrix}$$

Periodic matrix:

$A = [a_{ij}]_{n \times n}$ be a matrix when $A^{p+1} = A$

$A^0 = 1$
$A^2 = A \cdot A$
$A^p = A^{p-1} \cdot A$

$A^{p+1} = A^p \cdot A$
$= A^{p-1} \cdot A \cdot A$
$= A^{p-2} \cdot A \cdot A \cdot A$

If for $P \in N$, $A^{P+1} = A$, then A is called Periodic matrix and if P is the least (+)ve integer for which $A^{P+1} = A$ then A is called periodic matrix of period P

Ex: A → square matrix

$A^4 = A$ हो तो

$A \cdot A \cdot A \cdot A = A$

$A^{3+1} = A$ → (3) Ans

Q. $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 2 & 0 \\ -6 & 9 & -3 \end{bmatrix}$ check whether this matrix is periodic or not and find it's period.

$\Rightarrow A^3 = A$

Idempotent matrix: $A^2 = A$

Ex: $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$

Nilpotent matrix: $A^n = 0$ but $A^{n-1} \neq 0$

Ex: $A_{n \times n}$

$A_{5 \times 5}$

$A^5 = 0$
$A^4 \neq 0$
$A^3 \neq 0$
$A^2 \neq 0$

} nilpotent matrix

Ex: $B = \begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$

Involutory matrix; $A^2 = I$

যে বর্গাকৃতি ম্যাট্রিক্স নিজের নিজের Inverse

$A \cdot A = I$

Ex:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

If a square matrix is equal to it's own inverse, then the matrix is called involutory

Symmetric matrix: $A^T = A$

Ex:

$$\begin{pmatrix} a & h & g \\ h & b & f \\ g & f & c \end{pmatrix}$$

Skew symmetric: $A^T = -A$

Ex:

$$\begin{pmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{pmatrix}$$

Conjugate of a matrix (complex):

$$A = \begin{pmatrix} 1+i & 1-i \\ -i & 4i \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 1-i & 1+i \\ i & -4i \end{pmatrix}$$

Hermitian Matrix: $A^* = A$

Skew Hermition Matrix: $A^* = -A$

$$\begin{bmatrix} 1 & 1+i \\ -1+i & 0 \end{bmatrix}$$

$A^* = \bar{A}^T$

Ex:

$$\begin{pmatrix} 3 & 1-2i & 4+ \\ 1+2i & -4 & -2i \\ 4-7i & 2i & 5 \end{pmatrix}$$

Orthogonal matrix:

$$A A^T = A^T A = I$$

$$A^T = A^{-1}$$

Q. ... this matrix is a orthogonal matrix find it's Inverse ? (1 मार्क)

Ex:

$$\begin{bmatrix} 2/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & -1/3 \end{bmatrix}$$

Unitary Matrix:

$$A A^* = A^* A = I$$

$$A^* = A^{-1}$$

Ex:

$$A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$$

Normal matrix:

$$A \cdot A^* = A^* A$$

Ex:

$$A = \begin{pmatrix} 2-3i & -i \\ 1 & 1-2i \end{pmatrix}$$

(sub matrix)

Sub matrix:

$$\left[\begin{array}{c|cc} 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ 6 & 7 & 8 \end{array}\right] \longrightarrow \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Minors of a Matrix:

$$\left[\begin{array}{c|cc} a_{11} & a_{12} & a_{13} \\ \hline a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right]$$

Q: find minors of element a21?

=> $i = 2$, $j = 1$

$$M_{21} = \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix}$$

Co-factor of a matrix:

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Adjoint of a matrix:

$$A_{ij} = (-1)^{i+j} M_{ij}$$

$$\text{Adj } A = [A_{ij}]^T$$

Inverse matrix:

$$A^{-1} = \frac{1}{A} \cdot \text{Adj } A$$

Non-linier equation: $\sqrt{x} + 3y = \sin 2$

linier equation: $4x + \sqrt{3}y = \ln 2$

Class-3 04.10.23

Minors of a matrix:

$$\begin{bmatrix} a_{11} & a_{22} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow M_{21} = \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix}$$

Co-factor of a matrix:

$A_{ij} = (-1)^{i+j} M_{ij}$

Adjoint of a matrix

$\text{Adj } A = [A_{ij}]^T$

$A^{-1} = \frac{1}{|A|} \text{Adj } A$

Home work

Q. Find the inverse of the matrix,

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & 3 \end{bmatrix}$$

$|A| = 2(3-2) - 2(6+7) + 0(4+7)$

$= -24 \neq 0$

$\Rightarrow$ Since $|A| \neq 0$ the matrix A is non-singular and possesses an inverse

Co-factor of,

$A_{11} = 1$, $A_{12} = -13$, $A_{13} = 11$

$A_{21} = -6$, $A_{22} = 6$, $A_{23} = -18$

$A_{31} = 2$, $A_{32} = -2$, $A_{33} = -2$

$$Adj(A) = A^T = \begin{pmatrix} 1 & -6 & 2 \\ -13 & 6 & -2 \\ 11 & -18 & -2 \end{pmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|} = -\frac{1}{24}\begin{bmatrix} 1 & -6 & 2 \\ -13 & 6 & -2 \\ 11 & -18 & -2 \end{bmatrix}$$

Q.n Find the value of x, y, z, f

$$2\begin{bmatrix} x & z \\ y & f \end{bmatrix} - 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2x & 2z \\ 2y & 2f \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x-3 & 2z+3 \\ 2y-0 & 2f-6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$2x - 3 = 9$ | $x = 6$

$2z + 3 = 15$ | $z = 6$

$2y - 0 = 12$ | $y = 6$

$2t - 6 = 18$ | $f = 12$

$x, y, z = 6$ and $f = 12$

Home work

Q. Using Adjoint of a matrix, find the matrix A if $(4A)^{-1} = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$

⇒ Suppose, $(4A)^{-1} = X$

$\therefore X = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$ now, $|X| = 1(8-0)+1(0-0)+2(0$

$= 8 - 12 = -4$

@ factors of X matrix,

$X_{11} = \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix}$

$= 8 + 0$

$= 8$

$X_{12} = -\begin{vmatrix} 0 & 0 \\ 3 & 4 \end{vmatrix}$

$= -(0 - 0)$

$= 0$

$X_{13} = \begin{vmatrix} 0 & 2 \\ 3 & -1 \end{vmatrix}$

$= 0 - 6$

$= -6$

$X_{21} = -\begin{vmatrix} -1 & 2 \\ -1 & 4 \end{vmatrix}$

$= -(-4+2)$

$= 2$

$X_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$= 4 - 6$

$= -2$

$X_{23} = -\begin{vmatrix} 1 & -1 \\ 3 & -1 \end{vmatrix}$

$= -(-1+3)$

$= -2$

$X_{31} = \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix}$

$= 0 - 4 =$

$X_{32} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}$

$= -(0+0)$

$= 0$

$X_{33} = \begin{vmatrix} 1 & - \\ 0 & 2 \end{vmatrix}$

$= 2 + 0$

$= 2$

$$4A = \frac{Adj[A]}{|A|} \Rightarrow A = \frac{1}{4}\frac{Adj[A]}{|A|}$$

$$Adj(A) = A^T = \begin{pmatrix} 8 & 2 & -4 \\ 0 & -2 & 0 \\ -6 & -2 & 2 \end{pmatrix}$$

$$A = \frac{1}{4} \cdot \frac{1}{-4} \begin{pmatrix} 8 & 2 & -4 \\ 0 & -2 & 0 \\ -6 & -2 & 2 \end{pmatrix}$$

$$= -\frac{1}{16} \begin{pmatrix} 8 & 2 & -4 \\ 0 & -2 & 0 \\ -6 & -2 & 2 \end{pmatrix}$$ Ans.

Cramer's Rule

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$D = |A|$$

$$D_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

$$D_{x_2} = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

$$D_{x_3} = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

$$x_1 = \frac{Dx_1}{D} \qquad x_2 = \frac{Dx_2}{D} \qquad x_3 = \frac{Dx_3}{D}$$

H.W

Q. Solving the system by using cramers rule

$$x + 5y - z = 9$$
$$3x - 3y + 2z = 7$$
$$2x - 4y + 3z = 1$$

$$\Rightarrow \begin{bmatrix} 1 & 5 & -1 \\ 3 & -3 & 2 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ 1 \end{bmatrix}$$

suppose, $A = \begin{bmatrix} 1 & 5 & -1 \\ 3 & -3 & 2 \\ 2 & -4 & 3 \end{bmatrix}$

$$D = 1(-9+8) - 5(9-4) - 1(-12+6)$$
$$= -1 - 25 + 6$$
$$= -20$$

$$Dx = \begin{bmatrix} 9 & 5 & -1 \\ 7 & -3 & 2 \\ 1 & -4 & 3 \end{bmatrix}$$

$$= 9(-9+8) - 5(21-2) - 1(-28+3)$$

$$= -9 - 95 + 25 = -79$$

$$Dy = \begin{bmatrix} 1 & 9 & -1 \\ 3 & 7 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= 1(21-2) - 9(9-4) - 1(3-14)$$

$$= 19 - 45 + 11 = -15$$

$$Dz = \begin{bmatrix} 1 & 5 & 9 \\ 3 & -3 & 7 \\ 2 & -4 & 1 \end{bmatrix} = 1(-3+28) - 5(3-14) + 9(-12+6)$$

$$= 25 + 55 - 54$$

$$= 26$$

$$X = \frac{Dx}{D} = \frac{-79}{-20} = \frac{79}{20}$$

$$Y = \frac{Dy}{D} = \frac{-15}{-20} = \frac{15}{20} = \frac{3}{4}$$

$$Z = \frac{Dz}{D} = -\frac{26}{20}$$

- Block matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 7 \\ 8 & 10 & 11 & 16 \\ \hline 9 & 3 & 5 & 12 \end{array}\right]$$

- Row/Column operation:

1. Interacting any two row/column
2. Multiply each element of any row/column by a scaler.
3. Multiplied each element of any row/column by scaler then adding with another row or column

Row Echelon Form:

$$\left[\begin{array}{ccc|c} \textcircled{1} & 2 & 3 & 0 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & 2 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

non zero leading element

zero row

zero colu row

$$\begin{bmatrix} 1 & 2 & 3 & 8 \\ 0 & 4 & 5 & 9 \\ 0 & 0 & 6 & 10 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

(1) zero row and colum সবার নিচে থাকবে

(2) leading element এর নিচে সবই 0 হবে।

(3) lower row এর leading element right এ থাকবে

Condition

Q. $\begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{bmatrix}$ কে Row-echelon format এ আনো

$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & 3 & -6 & 1 \\ 0 & 0 & 5 & -12 & 2 \end{bmatrix}$$

$R_2' = R_2 - 2R_1$

$R_3' = R_3 - 3R_1$

$$\begin{bmatrix} \textcircled{1} & 2 & -1 & 2 & 1 \\ 0 & 0 & 3 & -6 & 1 \\ 0 & 0 & 0 & -6 & 1 \end{bmatrix}$$

$R_3' = 3R_3 - 5R_2$

Row reduced Echelon form

① same
② same
③ same
④ leading element must be 1
⑤ leading element এর উপরে ও নিচে শুধুই 0 থাকবে।

5.10.23

Q. #
$$\begin{bmatrix} \textcircled{1} & 2 & -1 & 2 & 1 \\ 0 & 0 & \textcircled{3} & -6 & 1 \\ 0 & 0 & 0 & \textcircled{-6} & 1 \end{bmatrix}$$
Row reduced Echelon form? নয় কি?

$$\Rightarrow \sim \begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ 0 & 0 & \textcircled{1} & -2 & 1/3 \\ 0 & 0 & 0 & 1 & -1/6 \end{bmatrix} \quad R_2' = \frac{1}{3} R_2, \quad R_3' = \frac{1}{6} R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 4/3 \\ 0 & 0 & 1 & -2 & 1/3 \\ 0 & 0 & 0 & 1 & -1/6 \end{bmatrix} \quad R_1' = R_1 + R_2$$

equivalent হচ্ছে 2টা

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 & 4/3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1/6 \end{bmatrix} \quad R_2' = R_2 + 2R_3$$

This is row Reduced echelon form.

Homework

Q. $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 6 & 0 & 1 & -2 \\ 8 & 1 & -2 & 1 \end{bmatrix}$ কে Row echelon এবং Row reduced echelon form এ convert করো?

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 6 & 0 & 1 & -2 \\ 8 & 1 & -2 & 1 \end{bmatrix}$$

conversion to row echelon form:

$$A \sim \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 6 & 1 & -2 \\ 0 & 9 & -2 & 1 \end{bmatrix} \quad \begin{aligned} R_2' &= R_2 - R_1 \\ R_3' &= R_3 - 6R_1 \\ R_4' &= R_4 - 8R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -2 & 1 \end{bmatrix} \quad \begin{aligned} R_3' &= R_3 - 2R_2 \\ R_4' &= R_4 - 3R_2 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad R_4' = R_4 + 2R_3$$

It is the row echelon from

Now, conversion to row reduced echelon form:

$$A \sim \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} R_2' &= \frac{1}{3} R_2 \\ R_4' &= -\frac{1}{3} R_4 \end{aligned}$$

H.W

1. $|A| = \begin{vmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & 3 \end{vmatrix} = -24 \neq 0$

since $A \neq 0$ the matrix A is non-singular and possesses an inverse.

$A_{11} = 1$ $\quad A_{12} = 13$ $\quad A_{13} = 11$

$A_{21} = 6$ $\quad A_{22} = 6$ $\quad A_{23} = 18$

$A_{31} = 2$ $\quad A_{32} = 2$ $\quad A_{33} = -2$

$$B = \begin{pmatrix} 1 & 13 & 11 \\ 6 & 6 & 18 \\ 2 & 2 & -2 \end{pmatrix}$$

$$adjA = B^T = \begin{pmatrix} 1 & -6 & 2 \\ -13 & 6 & -2 \\ -11 & -18 & -2 \end{pmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|} = -\frac{1}{24}\begin{pmatrix} 1 & -6 & 2 \\ -13 & 6 & -2 \\ 11 & -18 & -2 \end{pmatrix}$$

CamScanner

H.W

4. $x + 5y - z = 9$

$3x - 3y + 2z = 7$

$2x - 7y + 3z = 1$

$$D = \begin{vmatrix} 1 & 5 & 1 \\ 3 & -3 & 2 \\ 2 & -7 & 3 \end{vmatrix} = 25 \neq 0$$

$$Dx = \begin{vmatrix} 9 & 5 & -1 \\ 7 & -3 & 2 \\ 1 & -7 & 3 \end{vmatrix} = -79$$

$$Dy = \begin{vmatrix} 1 & 9 & -1 \\ 3 & 7 & 2 \\ 2 & 1 & 3 \end{vmatrix} = -15$$

$$Dz = \begin{vmatrix} 1 & 5 & 9 \\ 3 & -3 & 7 \\ 2 & -4 & 1 \end{vmatrix} = 26$$

$x = \frac{-79}{-20}$ $\quad y = \frac{-15}{-20}$ $\quad z = \frac{26}{25}$

$= \frac{79}{20}$ $\quad = \frac{3}{4}$

CamScanner

Class-4 05.10.23

System of Linear eqn:

$$ax + by = c$$

$$a, b \in R$$

Leading Constant → a_1, Leading variable → x_1

$$a_1x_1 + a_2x_2 + \cdots a_nx_n = b$$

[if $b = 0$ → Homogenious eqn

[$b \neq 0$ → non-Homogeneous

$$\sum_{i=1}^{n} a_i x_i = b$$

$\{s_1, s_2 \ldots s_n\}$ → Soln set

Qⁿ: Find the soln set of $x_1 + 2x_2 = 4$

⇒ Solve for x_1

$x_1 = 4 - 2x_2$ → free variable

Soln set:

$x_2 = t \in R$

$x_1 = 4 - 2t$, t = parametric

Q. Find the sol^{n} set $x_1 - 2x_2 + x_3 = 5$

$\Rightarrow$ Solve for x_1,

$x_1 = 5 + 2x_2 - x_3$

x_2, x_3 → free variable

Solution set:

$x_2 = t \quad , t \in \mathbb{R}$

$x_3 = s \quad , s \in \mathbb{R}$

$x_1 = 5 + 2t - s$

Number of free variable = number variable − number of equation

Q. Find the solution set $x + 2y - z = 2$

$y - z = 1$

$\Rightarrow y = 1 + z$

$z = t$

$y = 1 + t$

$x = 2 - 2y + z$

$= 2 - 2(1+t) + t$

$= -t$

solution set $\{ x = -t, \; y = 1 - t, \; z = t, \; t \in \mathbb{R} \}$ → General solution

$t = 2$ হলে,

$x = -2$, $y = 3$, $z = 2$

$(-2, 3, 2)$ ⟶ particular solution

A system of two linear equation with two variable :

$L_1 : ax + by + z = h$ (Linear)
$L_2 : cx + dy = k$ (") } System ①

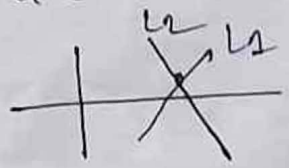
২ টি চলক solution হতে পারে

① unique soln
② particular বা General
③ no soln

Case 1: System ① has exactly one soln so it has unique soln.

If L_1 and L_2 intersect at exactly one point that's why when the lines have distinct slopes or co-efficents of x and y are not proportional.

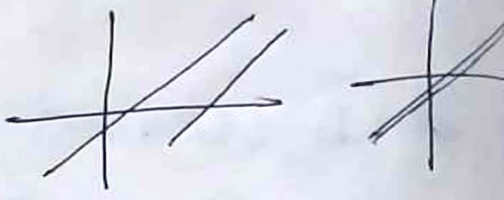
$$\frac{a}{c} \neq \frac{b}{d}$$



consistent. [soln exist করে]

Case-2: System ① has infinitely many sol's L_1 & L_2 and coincident, that's why lines have the same slope and same y intercepts.

$$\frac{a}{c} = \frac{b}{d} = \frac{h}{k}$$

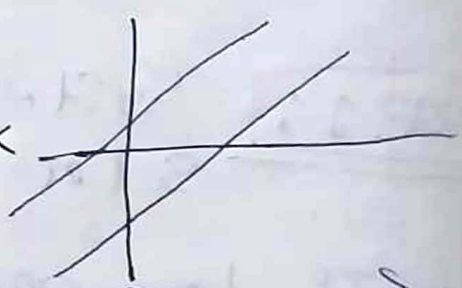


Consistent

Case-3: System ① has no solution $L_1 \parallel L_2$ and distinct, that's why lines have same slope but distinct y intercept

that's why

$$\frac{a}{c} = \frac{b}{d} \neq \frac{h}{k}$$



We can it inconsistent [Soln হবে]

No sol^n → inconsistent (0 = a) False

consistent : 0 = 0 (True)

A system of linear equ^n

$$2x - y = 1$$
$$3x + 2y = 12$$

$\Rightarrow \frac{2}{3} \neq -\frac{1}{2}$ $\left(\frac{a}{b} \neq \frac{c}{d}\right)$ → unique sol^n

$y = 2x - 1$

$3x + 4x - 2 = 12$

$x = 2$

$y = 3$

08/10/2023

#

$$2x - y = 1$$
$$6x - 3y = 3$$

$\Rightarrow \frac{2}{6} = \frac{1}{3} = \frac{1}{3}$ Infinite sol^n

$y = 2x - 1$ ⟶ $x = t$, $t \in \mathbb{R}$

$y = 2t - 1$

$6x - 6x + 3 = 3$

$0 = 0$

Number of equation 1টা (2টা equ same)

No. varible → 2টা

free varible → 1টা

$2x - y = 1$

$6x - 3y = 12$

$\Rightarrow \frac{2}{6} = \frac{1}{3} \neq \frac{1}{12}$

No Soln

$y = 2x - 1$

$6x - 6x + 3 = 12$

$0 = 9$

∴ Inconsistent system

$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n = b_m$

$\Rightarrow$ Matrix form,

$$\begin{bmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \vdots & \\ a_{m1} & a_{m2} \cdots a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$Ax = b$

Augmented matrix

$[A \mid b]$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$$

Gaussian Elimination Row Echelon Form :

Consider 3×3 matrix.

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] \sim \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1' \\ 0 & a_{22} & a_{23} & b_2' \\ 0 & 0 & a_{33} & b_3' \end{array}\right]$$

row echelon form

$$\sim \begin{aligned} & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1' \\ & a_{22}x_2 + a_{23}x_3 = b_2' \\ & + a_{33}x_3 = b_3' \end{aligned}$$

$$\Rightarrow x_3 = \frac{b_3'}{a_{33}}$$

Gauss Jordan Elimination Row reduced Echelon form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & b_1' \\ 0 & 1 & 0 & b_2' \\ 0 & 0 & 1 & b_3' \end{array}\right] \Rightarrow \begin{aligned} x_1 &= b_1' \\ x_2 &= b_2' \\ x_3 &= b_3' \end{aligned}$$

4. $3x - 2y + 8z = 9$
$-2x + 2y + z = 3$
$x + 2y - 3z = 8$

2) The augmented matrix is

$$\left[\begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ -2 & 2 & 1 & 3 \\ 1 & 2 & -3 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ 0 & 2 & 19 & 27 \\ 0 & 8 & -17 & 15 \end{array}\right] \quad \begin{array}{l} R_2' = 3R_2 + 2R_1 \\ R_3' = 3R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 3 & -2 & 8 & 9 \\ 0 & 2 & 19 & 27 \\ 0 & 0 & -93 & -93 \end{array}\right] \quad R_3' = R_3 - 4R_2$$

system eqn is,

$3x - 2y + 8z = 9$
$2y + 19z = 27$
$-93z = -93$
$z = 1$

$2y + 19 = 27$
$2y = 27 - 19$
$2y = 8$
$y = 4$

$3x - 8 + 8 = 9$
$x = 3$

row reduced echelon form,

$$\sim \begin{bmatrix} 1 & -2/3 & 8/3 & 3 \\ 0 & 1 & 29/2 & 27/2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1' = \frac{1}{8} R_1$

$R_2' = \frac{1}{2} R_2$

$R_3' = \frac{1}{29} R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 29/2 & 27/2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1' = R_1 + 2/3 R_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x = 3$

$y = 4$

$z = 2$

Home work

\# $2x - 3y + 5z = 1$

$3x + y - z = 2$

$x + 4y - 6z = 1$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & -3 & 5 & 1 \\ 3 & 1 & -1 & 2 \\ 1 & 4 & -6 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 2 & -3 & 5 & 1 \\ 0 & 11 & -17 & 1 \\ 0 & 11 & -17 & 1 \end{array}\right] \quad \begin{aligned} R_2' &= 2R_2 - 3R_3 \\ R_3' &= 2R_3 - R_1 \end{aligned}$$

$$\sim \left[\begin{array}{ccc|c} 2 & -3 & 5 & 1 \\ 0 & 11 & -17 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3' = R_3 - R_2$$

$0x + 0y + 0z = 0$

Infinite solution

$x - 4y + 5z = 8$

$3x + 7y - z = 3$

$x + 15y - 11z = -14$

The augmented matrix,

$$\left[\begin{array}{ccc|c} 1 & -4 & 5 & 8 \\ 3 & -7 & -1 & 3 \\ 1 & 15 & -11 & -14 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -4 & & 8 \\ 0 & -19 & & -21 \\ 0 & 38 & & -45 \end{array}\right] \quad R_2' = -3R_1 + R_2$$

$x - 4y + 5z = 8$

$-19y - 16z = -2$

$38y - 32z = -45$

12/10/2022

Find the values of $\mathcal{J}$ and M such that the following system of linear eqn s has (i) no soln (ii) more than one soln (iii) a unique soln.

$$\Rightarrow \left.\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 10 \\ x+2y+\mathcal{J}z &= M \end{aligned}\right\} \quad \text{— (1)}$$

Augmented

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & J & M \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & J-3 & M-10 \end{array}\right] \quad \begin{aligned} R_2' &= R_2 - R_1 \\ R_3' &= R_3 - R_2 \end{aligned}$$

Case-1: $0 = a$

$J = 3$, $M \neq 10$

NO soln

Case-2: $0 = 0$

$J = 3$, $M = 10$

more than 1 soln

Case-3:

$J \neq 3$, unique soln

Case-1 : If $J=3$ and $M \neq 10$ then 3rd equⁿ is of the form $0=a$ where $a=M-10 \neq 0$ which is not true. So the system is inconsistent. Thus (1) has no solⁿ for $J=3$ and $M \neq 10$

Case-2 : If $J=3$ and $M=10$ the 3rd equⁿ vanishes and having 2 eqⁿs in 3 variable. Hence (i) has more than one solⁿ for $J=3, M=10$

Case-3 : For a unique solⁿ the co-efficient of z in 3rd equⁿ must be non-zero that's why $J \neq 3$ and M may have any value. Therefore (i) has unique solⁿ for $J \neq 3$ and arbitrary values of M.

CT Syllabus

Linear combination : Let, V be a vector space over a field F, then $\vec{V} \in V(F)$ is said to be linear combination of the vectors $\vec{V_1}$, $\vec{V_2}$, -- $\vec{V_n} \in V(F)$ if there exists scalars $\alpha_1, \alpha_2 -- \alpha_n \in F$ such that

$$\vec{V} = \alpha_1 \vec{V_1} + \alpha_2 \vec{V_2} + \cdots \alpha_n \vec{V_n}$$

(vector space) জন্য

$V \to \{\vec{V_1}, \vec{V_2} -- \vec{V_n}\}$ + identity $\to a + 0 = a$

$a + (-a) = 0$

$\{\vec{V_1}, \vec{V_2} -- \vec{V_R}\} \to$ sub space

multiplicative identity $a \cdot 1 = a$

$a \cdot a^{-1} = 1$

$\{\vec{V_1}, \vec{V_2} -- \vec{V_R}\}$ sub ~~sup~~ space

$V = v_1, v_2, v_3 -- v_n$

$v = \alpha_1 v_1 + \alpha_2 v_2 + -- \alpha_n v_n$

Exam

Show that $\vec{v} = (1, -2, 5)$ is a linear combination of $\vec{v_1} = (1, 1, 1)$, $\vec{v_2} = (1, 2, 3)$ and $\vec{v_3} = (2, -1, 1)$

Soln: In order to show that $\vec{v}$ is a linear combination of $\vec{v_1}$, $\vec{v_2}$, $\vec{v_3}$ there exist ($\exists$) scalars $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ such that

$$\vec{v} = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$$

$$\Rightarrow (1, -2, 5) = \alpha_1(1, 1, 1) + \alpha_2(1, 2, 3) + \alpha_3 (2, -1, 1)$$

$$= (\alpha_1, \alpha_1, \alpha_1) + (\alpha_2, 2\alpha_2, 3\alpha_2) + (2\alpha_3, -\alpha_3, \alpha_3)$$

$$= \{\alpha_1 + \alpha_2 + 2\alpha_3, \alpha_1 + 2\alpha_2 - \alpha_3, \alpha_1 + 3\alpha_2 + \alpha_3\}$$

Equating corresponding components from both sides and forms system of eq^{n}s

$$\alpha_1 + \alpha_2 + 2\alpha_3 = 1$$
$$\alpha_1 + 2\alpha_2 - \alpha_3 = -2$$
$$\alpha_1 + 3\alpha_2 + \alpha_3 = 5$$

$$\sim \quad \alpha_1 + \alpha_2 + 2\alpha_3 = 1$$
$$\alpha_2 - 3\alpha_3 = -3$$
$$2\alpha_2 - \alpha_3 = 4$$

$R_2' = R_2 - R_1$
$R_3' = R_3 - R_1$

$$\sim \quad \alpha_1 + \alpha_2 + 2\alpha_3 = 1$$
$$\alpha_2 - 3\alpha_3 = -3$$
$$5\alpha_3 = 10$$

By back substitution method from above system,

$\alpha_3 = 2$, $\alpha_2 = 3$, $\alpha_1 = -6$

Therefore $\vec{V} = -6\vec{v_1} + 3v_2 + 2\vec{v_3}$

Thus $\vec{V}$ is a linear combination of $\vec{v_1}$, $\vec{v_2}$, $\vec{v_3}$

Condition ① linear 2ता 2टा

↑ ②

Generator or Span: Let, $S = \{\vec{v_1} -- \vec{v_n}\}$ is a finite subset of a vector space $V(F)$. If $\vec{u} = \alpha_1 \vec{v_1} + -- \alpha_n v_n$ $\vec{u} \in V$ and $\alpha_1, \alpha_2 -- \alpha_n \in F$. Then S is called generator or span of $V(F)$

Linear Independent : $\alpha_1 v_1 + \alpha_2 v_2 + -- \alpha_n v_n = 0$

" dependent : $\alpha_1, \alpha_2, -- \alpha_n = 0$

Linear Dependence: Let, $V(F)$ be a vector space over the field F. Then the vectors $\vec{v_1}, -- \vec{v_n} \in V(F)$ is said to be linearly dependent or simply dependent if for any scalar $\alpha_1, -- \alpha_n \in F$, such that

$\alpha_1 \vec{v_1} + -- + \alpha_n \vec{v_n} = 0$

where all scalar of the set $\{\alpha_1 -- \alpha_n\}$ is zero.

16/10/23

Linear independence: Let, $V(F)$ be a vector space over the field F. Then the vectors $\vec{v_1}, \cdots \vec{v_n} \in V(F)$ is said to be linearly independent or simply independent if for any scalar $\alpha_1 \cdots \alpha_n \in F$, such that $\alpha_1 \vec{v_1} + \cdots + \alpha_n \vec{v_n} = 0$ where, all scalar of the set $\{\alpha_1, \cdots \alpha_n\}$ is zero.

Q. Test whether the vector are linearly independent or dependent?

$(1, 2, 1)$, $(2, 1, 0)$, $(1, -1, 2)$

Sol^n: There exists scalars $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ such that

$\alpha_1(1, 2, 1) + \alpha_2(2, 1, 0) + \alpha_3(1, -1, 2) = 0$

$(\alpha_1 + 2\alpha_2 + \alpha_3, 2\alpha_1 + \alpha_2 - \alpha_3, \alpha_1 + 2\alpha_3) = (0, 0, 0)$

Equating corresponding components and forming a system of eq^n $\cdots$

$$\alpha_1 + 2\alpha_2 + \alpha_3 = 0$$
$$2\alpha_1 + \alpha_2 - \alpha_3 = 0$$
$$\alpha_1 + 2\alpha_3 = 0$$

$$\sim \begin{cases} \alpha_1 + 2\alpha_2 + \alpha_3 = 0 \\ -3\alpha_2 - 3\alpha_3 = 0 \\ -2\alpha_2 + \alpha_3 = 0 \end{cases} \quad \begin{aligned} R_2' &= R_2 - 2R_1 \\ R_3' &= R_3 - R_1 \end{aligned}$$

$$\sim \begin{cases} \alpha_1 + 2\alpha_2 + \alpha_3 = 0 \\ -3\alpha_2 + \alpha_3 = 0 \\ 9\alpha_3 = 0 \end{cases} \quad R_3' = 3R_3 - 2R_2$$

By back substitution,

$$\alpha_3 = 0, \quad \alpha_2 = 0, \quad \alpha_1 = 0$$

Home work

Test whether the vector are linearly independent or dependent ?

$(1, 1, 2)$, $(1, 0, 1)$, $(2, 1, 3)$

$\Rightarrow (\alpha_1 + \alpha_2 + 2\alpha_3, \alpha_1 + \alpha_3, 2\alpha_1 + \alpha_2 + 3\alpha_3) = 0$

Equating corresponding components and forming a system of eq^n

$$\alpha_1 + \alpha_2 + 2\alpha_3 = 0$$
$$\alpha_2 + \alpha_3 = 0$$
$$2\alpha_1 + \alpha_2 + \alpha_3 = 0$$

$$\alpha_1 + \alpha_2 + 2\alpha_3 = 0$$
$$\alpha_2 + \alpha_3 = 0 \qquad R_3' = R_3 - R_2$$
$$\alpha_2 + \alpha_2 = 0$$

by back substitution $\alpha_3 = 0$

$\alpha_2 = 0$, $\alpha_1 = 0$

Basis :

let V be a vector space over the field F and $\{\vec{v_1}, \vec{v_2}, \cdots \vec{v_n}\}$ be a finite set of vectors in V then the set $\{\vec{v_1}, \cdots \vec{v_n}\}$ is called basis of V if and only

(i) $\{\vec{v_1}, \vec{v_2} \cdots \vec{v_n}\}$ is linearly independent

ii) $\{\vec{v_1}, \vec{v_2}, \cdots \vec{v_n}\}$ span $\vec{V}$

Eigen value & ~~Let, A~~ and Eigen vector

Let, A be a square matrix of order n. A non zero vector $\vec{x} \in \mathbb{R}^n$ is called an eigen vector of A if $A\vec{x}$ is a scalar multiple of $\vec{x}$ that's why

$A\vec{x} = \gamma\vec{x}$ for some scalar γ the scalar γ is called an eigenvalue of A and $\vec{x}$ is called an eigenvector of A corresponding to eigenvalue γ.

Example

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

A $\quad$ $\vec{x}$ $\quad$ $= \gamma\vec{x}$

$A\vec{x} = \gamma\vec{x}$

$A\vec{x} - \gamma\vec{x} = 0$

$(A - \gamma)\vec{x} = 0$

$(A - \gamma I)\vec{x} = 0$

$A - \gamma I$ characteristic matrix

Ex $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} -$

Example

characteristic matrix: $A - \gamma I$

characteristic polynomial: $|A - \gamma I|$

characteristic eqn: $|A - \gamma I| = 0$

$A - \gamma I$

$\Rightarrow \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} \gamma & 0 \\ 0 & \gamma \end{pmatrix}$

$\Rightarrow \begin{vmatrix} 1-\gamma & 2 \\ 3 & 4-\gamma \end{vmatrix}$

$\Rightarrow \gamma^2 - 5\gamma - 2$

short cut →

$$\lambda^3 - (\text{sum of diagonal elements})\lambda^2 + (\text{sum of minor of diagonal elements})\lambda - |A| = 0$$

Cayley - Hamilton theorem: Every square matrix satisfies it's characteristic equation

A

Ex: $\gamma^2 - 6\gamma + 8 = 0$

$A^2 - 6A = -8$

Q. $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ Find A^{-1} using Cayley - Hamilton theorem

determination

$$\Rightarrow |A - \gamma I| = 0$$

$$\begin{vmatrix} 3-\gamma & 2 & 4 \\ 2 & -\gamma & 2 \\ 4 & 2 & 3-\gamma \end{vmatrix} = 0$$

$$\Rightarrow (3-\gamma)(-3\gamma+\gamma^2) - 2(6-2\gamma) + 4(4+4\gamma) = 0$$

$$\Rightarrow \gamma^3 - 6\gamma^2 - 15\gamma - 8 = 0$$

$$\Rightarrow A^3 - 6A^2 - 15A - 8 = 0$$

Multiplied by A^{-1}

$$A^{-1}A^3 - 6A^{-1}A^2 - 15 - 8A^{-1} = 0$$

$$A^2 - 6A - 15 - 8A^{-1} = 0$$

$$8A^{-1} = A^2 - 6A - 15$$

---

* verify cayley - Hamilton theorem 19/10/23

for $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find A^{-2}, A^4

$\Rightarrow$ $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - \lambda^2 - 5 - 1 + 5 = 0$ ⟶ λ এর মান বসে

$A^3 - A^2 - 5A + 5I = 0$

Multiply by A^{-1}

$A^2 - A - 5 + 5A^{-1} = 0$

$5A^{-1} = -A^2 + A + 5I$

$A^{-1} = \frac{1}{5}(-A^2 + A + 5I)$

A^{-1}/A

$A^4 - A^3 - 5A^2 + 5A =$

$A^4 = A^3 + 5A^2 - 5A$

λ → λ Lamda

Find the eigen value and eigenvectors o
A.

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Sol^n ⇒ The characteristic eq^n of A

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \begin{bmatrix} \text{sum of} \\ \text{diagonal} \\ \text{elements} \end{bmatrix} \lambda^2 + \begin{bmatrix} \text{Sum of Minor} \\ \text{of diagonal} \\ \text{element} \end{bmatrix} \lambda$$

$$-|A| = 0 \quad \text{(determinate)}$$

or, $\lambda^3 + 6\lambda^2 - 15\lambda - 8 = 0$

or $-1 - 6 + 15 - 8 = 0$

$\lambda = -1$

$$\Rightarrow \lambda^3 + \lambda^2 - 7\lambda^2 - 7\lambda - 8\lambda - 8 = 0$$

$$\Rightarrow \lambda^2(\lambda+1) - 7\lambda(\lambda+1) - 8(\lambda+1) = 0$$

$$\Rightarrow (\lambda+1)(\lambda^2 - 7\lambda - 8) = 0$$

$\lambda + 1 = 0$

$\lambda = -1$

$\lambda^2 - 7\lambda - 8 = 0$

$\lambda^2 - 8\lambda + \lambda - 8 = 0$

$\lambda(\lambda-8) + 1(\lambda-8)$

$(\lambda-8)(\lambda+1)$

$\lambda = 8, \lambda = -1$

दूसरी vector का

$\lambda = \textcircled{-1}, -1, \textcircled{8}$ → एक्वल vector का

$$A\vec{x} = \lambda\vec{x}$$

$$(A - \lambda I)\vec{x} = 0$$

$$\begin{bmatrix} 3-\lambda & 2 & 4 \\ 2 & -\lambda & 2 \\ 4 & 2 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

For $\lambda = -1$

$$\begin{bmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Row echelon form

$$\Rightarrow \begin{bmatrix} 4 & 2 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \qquad R_2' = 2R_2 - R_1 \quad R_3' = R_3 - R_1$$

augmented matrix

$4x_1 + 2x_2 + 4x_3 = 0$

$2x_1 + x_2 + 2x_3 = 0$ → [2 টা free variable]

$x_1 = \frac{1}{2}(-x_2 - 2x_3)$

$x_2 = 2t$
$x_3 = s$ $\Big\}$ $t, s \in \mathbb{R}$

$x_1 = \frac{1}{2}(-2t - 2s) = -t - s$

For $\lambda = -1$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t-s \\ 2t \\ s \end{bmatrix} = \begin{bmatrix} -t \\ 2t \\ 0 \end{bmatrix} + \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix}$$

$$= t\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

-1 এর জন্য eigen vector

Now, $\lambda = 8$ এর জন্য →

$$\begin{bmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 2 & 4 \\ 0 & -36 & 18 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2' = 2R_1 + 5R_2$

$R_3' = 4R_1 + 5R_3$

$$\begin{bmatrix} -5x_1 + 2x_2 + 4x_3 \\ 0 - 2x_2 + x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

⟶ 2 हरी का

$x_3 = 2t, \quad t \in \mathbb{R}$

$x_2 = \frac{x_3}{2} = \frac{2t}{2} = t$

$-5x_1 = -2x_2 - 4x_3$

$= -2t - 8t$

$= \frac{2}{5}t + \frac{8}{5}t$

$= \frac{10}{5}t = 2t$

$x_3 = 2t, \quad t \in \mathbb{R}$

$x_2 = \frac{x_3}{2} = \frac{2t}{2} = t$

$-5x_1 =$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ 2t \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

✓ 1 R vector

Home work

Find the eigen value and eigen vectors of A

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow |A - \lambda I| = 0$$

$$\Rightarrow \left| \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)\{(4-\lambda)(3-\lambda)-2\} - 1(6-2\lambda-2) + 1(2-4+\lambda) = 0$$

$$\Rightarrow (3-\lambda)(12-7\lambda+\lambda^2-2) - 1(4-2\lambda) + 1(-2+\lambda) = 0$$

$$\Rightarrow (3-\lambda)(\lambda^2-7\lambda+10) - 4 + 2\lambda + \lambda - 2 = 0$$

$$\Rightarrow 3\lambda^2 - 21\lambda + 30 - \lambda^3 + 7\lambda^2 - 10\lambda - 4 + 3\lambda - 2 = 0$$

$$\Rightarrow -\lambda^3 + 10\lambda^2 - 28\lambda + 24 = 0$$

$$\Rightarrow \lambda^3 - 10\lambda^2 + 28\lambda - 24 = 0$$

$$\Rightarrow \lambda^3 - 2\lambda^2 - 8\lambda^2 + 16\lambda + 12\lambda - 24 = 0$$

$$\Rightarrow \lambda^2(\lambda-2) - 8\lambda(\lambda-2) + 12(\lambda-2) = 0$$

$$\Rightarrow (\lambda-2)(\lambda^2-8\lambda+12) = 0$$

$$\lambda = 2, \quad \lambda = 2, \quad \lambda = 6$$

$$(A - \lambda I)\vec{x} = 0$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & 1 & 1 \\ 2 & 4-\lambda & 2 \\ 1 & 1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

for $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2' = R_2 - 2R_1$

$R_3' = R_3 - R_1$

$x_1 + x_2 + x_3 = 0$

or, $x_1 = -x_2 - x_3$

$x_1 = -t - s$

$x_2 = t$

$x_3 = s$

$t, s \in \mathbb{R}$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t-s \\ t \\ s \end{bmatrix} = \begin{bmatrix} -t \\ t \\ 0 \end{bmatrix} + \begin{bmatrix} -s \\ 0 \\ s \end{bmatrix}$$

$$= t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Ans.

for $\lambda = 6$,

$$\begin{bmatrix} -3 & 1 & 1 \\ 2 & -2 & 2 \\ 1 & 1 & -3 \end{bmatrix} \begin{bmatrix} x_2 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_3' = R_1 + 3R_3$

$R_2' = 2R_1 + 3R_2$

$$\Rightarrow \begin{bmatrix} -3 & 1 & 1 \\ 0 & -4 & 8 \\ 0 & 4 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -3 & 1 & 1 \\ 0 & -4 & 8 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \qquad R_3' = R_2 + R_3$$

$$\Rightarrow -3x_1 + x_2 + x_3 = 0$$
$$-4x_2 + 8x_3 = 0$$

$$x_2 = 2x_3$$
$$-3x_1 = x_2 - x_3$$
$$3x_1 = x_2 + x_3 = 3x_3$$
$$x_1 = x_3$$

Let, $x_3 = t$, $t \in \mathbb{R}$

$$x_1 = t$$
$$x_2 = 2t$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 2t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$ Ans.

28/10/23

Dimension

If a vector space V has a basis consisting of n vectors then the number n is called the dimension of V, denoted by dim(V).

Row space

The subspace of $\mathbb{R}^n$ generated or spanned by the row vectors of A is called the row space of A.

Column Space

The subspace of $\mathbb{R}^n$ generated or spanned by the column vectors of A is called the column space of A.

Solution Space

The sol^n set of a homogeneous or non-homogeneous system in n-unknowns is actually a subspace of $\mathbb{R}^n$, is called sol^n space.

Null space

The sol^n space of the homogeneous syste
of eq^{n}s, which is a subspace of R^n is
called the null space of A.

Rank

The common dimension of the row spa
and column space of a matrix A is
called rank of A. denoted by rank (A

Nullity:

The demension of null space of A is called
the nullity of A denoted by nullity (A).

Rank + nullity = num of column

Exam
Q. Find rank or nullity of a matrix?

Find the rank and nullity of A

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

row echelon from

$$= \begin{bmatrix} 1 & 0 & -4 & -28 & -37 & 13 \\ 0 & 1 & -2 & -12 & -6 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basis of row space = Basis of colour space = 2

leading variable

Rank (A) = 2

$$\begin{bmatrix} 1 & 0 & -4 & -28 & -37 & 13 \\ 0 & 1 & -2 & -12 & -6 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - 4x_3 - 28x_4 - 37x_5 + 13x_6 = 0$$

$$x_2 - 2x_3 - 12x_4 - 6x_5 + 5x_6 = 0$$

Rank + nullity = num of cols

$2 + 3 = 5$

Rank(A) = The num of leading variables in the general sol^n of $Ax = 0$

nullity(A) = The num of parameter in the general sol^n of $Ax = 0$

Home work

(1) # Find the rank and nullity of A

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 2 & 4 & 1 & 3 & 0 \\ 3 & 6 & 2 & 5 & 1 \\ -4 & -8 & 1 & -3 & 2 \end{bmatrix}$$

(2) # Find the num of parameters in the general sol^n of $Ax = 0$, if A is a 5×7 matrix of rank 3

nullity

$7 - 3 = 4$

(1)

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 1 & 0 \\ 2 & 4 & 1 & 3 & 0 & 0 \\ 3 & 6 & 2 & 5 & 1 & 0 \\ -4 & -8 & 1 & -3 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 2 & 2 & -2 & 0 \\ 0 & 0 & 1 & 1 & 5 & 0 \end{bmatrix}$$

$R_2' = R_2 - 2R_1$
$R_3' = R_3 - 3R_1$
$R_4' = R_4 + 4R_1$

$$= \begin{bmatrix} 1 & 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 7 & 0 \end{bmatrix}$$

$R_4' = R_4 - R_2$
$R_3' = R_3 - 2R_2$

$\therefore$ rank $(1+2) = 3$

nullity = no. of colum - rank
$= 6 - 3 = 3$

Solving linear system of eqn using inverse

$$x + y + z = 6$$
$$3x + 3y + 4z = 20$$
$$2x + y + 3z = 13$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ 13 \end{pmatrix}$$

$$A\vec{x} = \vec{b}$$

$$A^{-1} A\vec{x} = A^{-1}\vec{b}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 20 \\ 13 \end{pmatrix}$$

Ans ↓

$$= \begin{pmatrix} 5 & -2 & 1 \\ -1 & 1 & -1 \\ -3 & 2 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ 20 \\ 13 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

26/10/23

ODE

$\frac{dy}{dx}$

PDE

$\frac{\partial y}{\partial x}$

Independent variable more than one.

$$\frac{\left\{1+\left(\frac{\partial u}{dx}\right)^2\right\}^{3/2}}{\frac{\partial^2 u}{\partial x^2}} = c$$

$$= \left\{1+\left(\frac{\partial u}{dx}\right)^2\right\}^{3/2}$$

$$= c \frac{\partial^2 u}{\partial x^2} \Rightarrow \left\{1+\frac{\partial u}{\partial x}\right)^2\right\}^3$$

$$= c^2 \left(\frac{\partial^2 u}{\partial x^2}\right)^2$$

PDE: A diff eqn involving derivatives of dependent variable with respect to (w.r.to) more than one independent variable is called partial differential eqn.

dependent

$$\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = 5$$

Degree: The degree of PDE is the degree of the highest order derivative occuring in it after the diff eqn has been made free from radicals and fractions.

P.V $\$ \times$ D.V

D.D.V $\times$ D.V

D.D.V $\times$ D.D.V

$$v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = 3$$

$$\left(\frac{\partial v}{\partial s}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 = 0$$

$$v^2 + \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} = 0$$

Dependent variable and its differe
tial co-efficients occur only in the
first degree and are not multip
to gether.

1st order, 2nd degree Linear PDE

3টির বেশি

Linear হলে

PDE

$$\frac{\partial^3 y}{\partial s^3} + \frac{\partial^2 y}{\partial z^2} - \frac{\partial y}{\partial u} + y = (\ln s)^3 \longrightarrow (L)$$

$$\frac{\partial^2 y}{\partial s^2} + \frac{\partial y}{\partial z} + y^2 = \ln t \longrightarrow (NL)$$

$$y\frac{\partial y}{\partial s} + \frac{\partial y}{\partial z} = \sin t \longrightarrow (NL)$$

Homogeneous হলে

$$\frac{\partial^3 y}{\partial s^3} + \frac{\partial^2 y}{\partial z^2} - \frac{\partial y}{\partial u} + y = t \longrightarrow NH$$

$$\frac{\partial^2 y}{\partial s^2} + \frac{\partial y}{\partial z} + y^2 = 0 \longrightarrow H$$

$$y\frac{\partial y}{\partial s} + \frac{\partial y}{\partial z} - y = 0 \longrightarrow H$$

General Soln
↑ complete primitive

$y = cx + d$

$c, d \in R$

$c = 2$

$d = 2$

$\frac{dy}{dx} = 5$

$y = 5x + c$

$\frac{dy}{dx} = k$

$y = 2x + 3$ → Particular soln

PDE. $z = f(x, y)$

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{dy}$, $r = \frac{\partial^2 z}{\partial x^2}$,

$s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$

$z(x_1, \ldots \ldots x_n)$

$p_1 = \frac{\partial z}{\partial x_1}$ ——

$p_n = \frac{\partial z}{\partial x_n}$

$u_x = \frac{\partial u}{\partial x}$

$u_y = \frac{\partial u}{\partial y}$

$u(x, y)$

$u_{xx} = \frac{\partial^2 u}{\partial x^2}$

$u_{xy} = \frac{\partial^2 u}{\partial x \partial y}$

$u_{yy} = \frac{\partial^2 u}{\partial y^2}$

Math

$z = ax + by + a^2 + b^2$ ——— (1)

$a, b \rightarrow$ constants

$x, y \rightarrow$ variable

Differentiating (1) partially w.r.to x

$$\frac{\partial z}{\partial x} = a$$

Diff --- (1) " " y

$$\frac{\partial z}{\partial y} = b$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = z$$

H.W $z = ax + (1-a)y + b$

20/10/23

Find PDE $z = axe^y + \frac{1}{2}a^2e^{2y} + b$

$$\frac{\partial z}{\partial x} = ae^y$$

$$\frac{\partial z}{\partial y} = axe^y + a^2e^{2y}$$

$$= x(ae^y) + (ae^y)^2$$

$$\frac{\partial z}{\partial y} = x\frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^2$$

which is the required PDE

From PDE by eliminating A and P from

$$z = Ae^{pt}\sin px$$

$$\frac{\partial z}{\partial x} = Ae^{pt}\cos px$$

$$\frac{\partial^2 z}{\partial x^2} = -Ae^{pt}p^2\sin px$$

$$\frac{\partial^2 z}{\partial x^2} = -p^2 z$$

$$\frac{\partial z}{\partial t} = \sin px\, APe^{pt}$$

$$\frac{\partial^2 z}{\partial t} = AP^2\sin px\, e^{pt}$$

$$\frac{\partial^2 z}{\partial A} = p^2 z$$

$$\frac{d^2 z}{dx^2} + \frac{\partial^2 z}{\partial t^2} = 0$$

Find the differential eqn of all spheres of radius λ, having centre in the xy-plane.

2> eqn of sphere:

$$x^2 + y^2 + z^2 = \lambda^2$$

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

$$x^2 + y^2 = \lambda^2$$

Sol^n → From 3D-ordinate geometry of three dimensions the eqn of any sphere of radius λ, having centre $(a, b, 0)$ in the xy-plane is given by

$$(x-a)^2 + (y-b)^2 + (z-0)^2 = \lambda^2 \quad \text{---} (1)$$

a, b are arbitrary constant differentiating (1) w.r to x and y we get

$$2(x-a) + 2z\frac{\partial z}{\partial x} = 0$$

$$(x-a) + z\frac{\partial z}{\partial x} = 0$$

$$\Rightarrow x - a = -z\frac{\partial z}{\partial x}$$

and, $(y-b) + z\frac{\partial z}{\partial y} = 0$

$$y - b = -z\frac{\partial z}{\partial y}$$

from (1)

$$z^2\left(\frac{\partial z}{\partial x}\right)^2 + z^2\left(\frac{\partial z}{\partial y}\right)^2 + z^2 = r^2$$

$$\Rightarrow z^2\left\{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1\right\} = r^2$$

Find a partial D.E by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$\Rightarrow$ ~~from~~. diff^n w.r to x

$$c^2 x + a^2 z\frac{\partial z}{\partial x} = 0 \quad \text{---(1)}$$

w.r. to y

$$c^2 y + z b^2 \frac{\partial z}{\partial x} = 0 \quad \text{---(2)}$$

From (1) ——

$$c^2 + a^2 z\frac{\partial^2 z}{\partial x^2} + a^2\left(\frac{\partial z}{\partial x}\right)^2 = 0$$

$$\frac{2x}{a^2} + \frac{2z}{c^2}\frac{\partial z}{\partial x} = 0$$

$$0 = 0 - a^2 \frac{z}{x} \frac{\partial z}{\partial x}$$

$$-a^2 \frac{z}{x} \frac{\partial z}{\partial x} + a^2 z \frac{\partial^2 z}{\partial x^2} + a^2 \left(\frac{\partial z}{\partial x}\right)^2 = 0$$

$$\left(\frac{\partial z}{\partial x}\right)^2 + z \frac{\partial^2 z}{\partial x^2} - \frac{z}{x} \frac{\partial z}{\partial x} = 0$$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ का derivative लाओ।

$$\frac{1}{a^2} \cdot 2x + \frac{1}{c^2} 2z \frac{\partial z}{\partial x} = 0$$

$$x + \frac{a^2}{c^2} z \frac{\partial z}{\partial x} = 0$$

$$c^2 x + a^2 z \frac{\partial z}{\partial x} = 0$$

$$c^2 + a^2 z \frac{\partial^2 z}{\partial x^2} + a^2 \left(\frac{\partial z}{\partial x}\right)^2 = 0$$

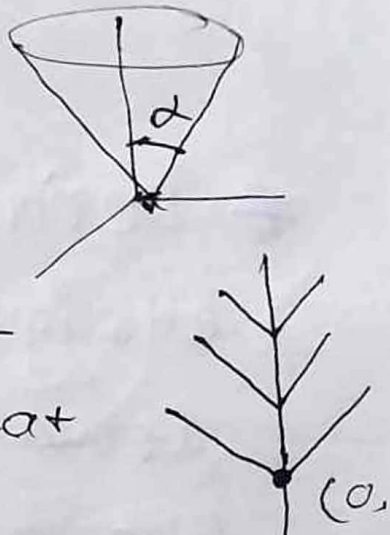
$$c^2 x = -a^2 z \frac{\partial z}{\partial x}$$

$$c^2 = -a^2 \frac{z}{x} \frac{\partial z}{\partial x}$$

exam 2 2022

Find the diff eqⁿ of the set of all right circular cones whose axis coincide with z-axis

Soln: The general eqⁿ of the set of all right circular cones whose axis coincide with z-axis having semi-vertical angle α and vertex at $(0,0,c)$ is given by



$$x^2 + y^2 = (z-c)^2 \tan^2\alpha \quad \text{—— (1)}$$

Diff (1) w.r to x and y partially

$$2x = 2(z-c)\frac{\partial z}{\partial x}\tan^2\alpha$$

$$x = (z-c)\frac{\partial z}{\partial x}\tan^2\alpha \quad \text{—— (2)}$$ y ছাড়া

$$y = (z-c)\frac{\partial z}{\partial y}\tan^2\alpha \quad \text{—— (3)}$$ x ছাড়া

$$y(z-c)\frac{\partial z}{\partial x}\tan^2\alpha = x(z-c)\frac{\partial z}{\partial y}\tan^2\alpha$$

$y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

required PDE

1/11/23

Derivation of partial deff eqn by the elimination of arbitary func ϕ from the eqn $\phi(u,v) = 0$.
where u and v are func of x, y, z

or, Show that, $\phi(u,v) = 0$ is the general sol^n of $Pp + Qq = R$

sol^n $\Rightarrow$ we have $\phi(u,v) = 0$

$z \rightarrow$ dependent and $x, y \rightarrow$ independent

$\frac{\partial z}{\partial x} = p$, $\frac{\partial z}{\partial y} = q$, $\frac{\partial x}{\partial y} = 0$, $\frac{\partial y}{\partial x} = 0$

diff partially (1) with respect to x

$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}p\right) + \frac{\partial}{\partial}$

$= 0\left(\frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial x} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial x} + \frac{\partial v}{\partial z}\cdot\right.$

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x}+p\frac{\partial u}{\partial z}\right)=-\frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial x}+p\frac{\partial v}{\partial z}\right)$$

——— (2)

Diff partially (2) with respect to y

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x}\frac{\partial x}{\partial y}+\frac{\partial u}{\partial u}\frac{\partial y}{\partial y}+\frac{\partial u}{\partial z}\frac{\partial z}{\partial y}\right)+\frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial v}{\partial y}\right.$$

$$\left.+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right)=0$$

$$\Rightarrow \frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial y}+q\frac{\partial u}{\partial z}\right)=-\frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial y}+q\frac{\partial v}{\partial z}\right)$$ ——— (3)

From (3)

left side same $\dfrac{\partial \phi}{\partial u}\Big/\dfrac{\partial \phi}{\partial v}=-\dfrac{\frac{\partial v}{\partial y}+q\frac{\partial v}{\partial z}}{\frac{\partial u}{\partial y}+q\frac{\partial u}{\partial z}}$

From (2)

left side same $\dfrac{\partial \phi}{\partial u}\Big/\dfrac{\partial \phi}{\partial v}=-\dfrac{\frac{\partial v}{\partial x}+p\frac{\partial v}{\partial z}}{\frac{\partial u}{\partial x}+p\frac{\partial u}{\partial z}}$

$$\therefore -\frac{\frac{\partial v}{\partial x}+p\frac{\partial v}{\partial z}}{\frac{\partial u}{\partial x}+p\frac{\partial u}{\partial z}}=-\frac{\frac{\partial v}{\partial y}+q\frac{\partial v}{\partial z}}{\frac{\partial u}{\partial y}+q\frac{\partial u}{\partial z}}$$ (Proved)

$$\Rightarrow \frac{\partial v}{\partial u} + \frac{\partial u}{\partial y} + p\frac{\partial v}{\partial z}\frac{\partial u}{\partial y} + q\frac{\partial v}{\partial x}\frac{\partial u}{\partial z} + pq\frac{\partial v}{\partial z}\frac{\partial u}{\partial z}$$

$$= \frac{\partial v}{\partial y}\frac{\partial u}{\partial x} + p\frac{\partial u}{\partial z}\frac{\partial v}{\partial y} + q\frac{\partial v}{\partial z}\frac{\partial u}{\partial x} + pq\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}$$

P Q

$$\Rightarrow p\left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial u}{\partial z}\frac{\partial v}{\partial y}\right) + q\left(\frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}\right)$$

$$= \frac{\partial v}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}$$

R

$$\therefore \boxed{Pp + Qq = R}$$ proved

Thus, we obtain a linear partial differential eq^n of first order and first degree in p and q

Show that $u(x,y,z) = a$ and $v(x,y,z) = b$ are two independent sol^n of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ where P, Q, R are funcs of x, y and z

→ $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

Sol^n = using total diff

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz = 0$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz = 0$$

By ~~Apply~~ cross multiplication ——

$$\frac{dx}{\frac{\partial u}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial v}{\partial z}\frac{\partial v}{\partial y}} = \frac{dy}{\frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}} = \frac{dz}{\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}}$$

2> $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

where, $P = \frac{\partial u}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial v}{\partial z}\frac{\partial v}{\partial y}$

$Q = \frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}$

$R = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$

The eqn of the type $Pp + Qq = R$ where, P, Q, R are the func of x, y, z

$$p = \frac{\partial z}{\partial x} \qquad q = \frac{\partial z}{\partial y}$$

⇒ Lagrange's Linear PDE : $Pp + Qq = R$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \rightarrow \text{auxilary / subsidary eqn}$$

$u = a, \quad v = b$

u, v fun of x, y, z $\quad \phi(u, v) = 0$

Grouping method.

$$\frac{\frac{dx}{P} = \frac{dy}{Q}}{u = a} \qquad \frac{\frac{dy}{Q} = \frac{dz}{R}}{v = b}$$

$\phi(u, v) = 0$

Find the general soln of $\underset{P}{yz} + \underset{Q}{zx} = \underset{R}{xy}$ soln

Subsidary are $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$

=> method of multipliers

choose multipliers l, m, n such that

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{pl + Qm + Rn}$$

$$\frac{dz}{R} = \frac{l\,dx + m\,dy + n\,dz}{0}$$

$$R(l\,dx + m\,dy + n\,dz) = 0 \quad \text{as } R \neq 0$$

So, $l\,dx + m\,dy + n\,dz = 0$

=> $\phi(x^2 - y^2, y^2 - z^2) = 0$

required general soln

Grouping method,

$$\frac{dx}{yz} = \frac{dy}{zx}$$

$$\frac{dx}{y} = \frac{dy}{x}$$

$$\Rightarrow x\,dx = y\,dy$$

$$\Rightarrow \frac{x^2}{2} = \frac{y^2}{2} + a'$$

$$\Rightarrow x^2 - y^2 = 2a' = a$$

Let, $u = a \Rightarrow x^2 - y^2 = a$

$$\frac{dy}{zx} = \frac{dz}{xy}$$

$$y\,dy = z\,dz$$

$$\frac{y^2}{2} = \frac{z^2}{2} + b'$$

$$y^2 - z^2 = 2b' = b$$

Let, $v = b \Rightarrow y^2 - z^2 = b$

$$\phi(u, v) = 0$$

$$\Rightarrow \phi(x^2 - y^2, y^2 - z^2) = 0$$

required general soln

Soln subsidary eqns are -

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Grouping method,

$\frac{dx}{yz} = \frac{dy}{zx}$ | $\frac{dy}{zx} = \frac{dz}{xy}$

$\frac{dx}{y} = \frac{dy}{x}$

$\Rightarrow x\,dx = y\,dy$

Find general sol^n of $\underset{P}{yz}\,\underset{\frac{\partial z}{\partial x}}{p} + \underset{Q}{zx}\,\underset{q}{} = \underset{R}{xy}$

sol^n subsidary eq^n are

$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy} \Rightarrow \left| \frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} \right.$

Find the general integral of

$(mz - ny)p + (nx - lz)q = ly - mx$

$\Rightarrow$ subsidary eq^{n}s are; $\frac{dx}{mz-ny} = \frac{dy}{nx-lx} = \frac{dz}{ly-m}$

choosing x, y, z as multipliers

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} = \frac{x\,dx + y\,dy + z\,dz}{mxz - nxy + nxy - lyz + lyz - mxz}$$

$$\frac{dz}{ly-mx} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$(ly - mx)(x\,dx + y\,dy + z\,dz) = 0$$

$$\downarrow$$

$$x\,dx + y\,dy + z\,dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c'$$

$$\Rightarrow x^2 + y^2 + z^2 = 2c' = a \quad \text{— u}$$

2nd part — choosing multipliers

l, m, n

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx} = \frac{l\,dx + m\,dy + n\,dz}{lmz - lny + mnx - mlz + nly - mnx}$$

$$(ly - mx)(l\,dx + m\,dy + n\,dz) = 0$$

$$l\,dx + m\,dy + n\,dz = 0 \qquad [ly - mx \neq 0]$$

Integrating, $lx + my + nz = b \longrightarrow v$

$$\phi(x^2 + y^2 + z^2,\ lx + my + nz) = 0$$

Find the general integral of $(y^2+z^2-x^2)p - 2xyq + 2xz = 0$

Sol^n subsidary eq^n are, $\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz}$

1st part $\Rightarrow$ $\frac{dy}{-2xy} = \frac{dz}{-2xz}$

$$\frac{dy}{y} = \frac{dz}{z} \Rightarrow \ln y = \ln z + \ln a$$

$$\Rightarrow \ln y - \ln z = \ln a$$

$$\Rightarrow \ln \frac{y}{z} = \ln a$$

$$\Rightarrow \frac{y}{z} = a$$

2nd part choosing x, y, z as multipliers

$$\frac{dx}{xy^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz} = \frac{x\,dx+y\,dy+z\,dz}{xy^2+xz^2-x^3-2xy^2-2xz^2}$$

$$= \frac{dz}{-2xz} = \frac{x\,dx+y\,dy+z\,dz}{-x^3-xy^2-xz^2}$$

$$= \frac{dz}{-2xz} = \frac{x\,dx+y\,dy+z\,dz}{-x(x^2+y^2+z^2)}$$

$$\frac{dz}{2z} = \frac{x\,dx + y\,dy + z\,dz}{(x^2+y^2+z^2)}$$

$$\frac{dz}{z} = \frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2}$$

$$\ln z = \ln(x^2+y^2+z^2) + \ln b$$

$$\ln(x^2+y^2+z^2) - \ln z = -\ln b$$

$$\ln \frac{x^2+y^2+z^2}{z} = \ln \frac{1}{b}$$

$$\frac{x^2+y^2+z^2}{z} = c \longrightarrow V$$

$$\phi\left(y/z,\ \frac{x^2+y^2+z^2}{z}\right) = 0$$

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* $x(z-2y) = y^2 p - xyz$

Soln: $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$\Rightarrow \frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$

$\therefore \frac{dx}{y} = \frac{dy}{-x}$

$\Rightarrow -x\,dx = y\,dy$

$\Rightarrow x\,dx + y\,dy = 0$

$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = a'$

$\Rightarrow x^2 + y^2 = 2a' = a$

$\frac{dy}{x-y} = \frac{dz}{x(z-2y)}$

$\Rightarrow \frac{dz}{dy} = -\frac{z-2y}{y}$

$= -\frac{z}{y} + 2$

$\Rightarrow \frac{dz}{dy} + \frac{z}{y} = 2$

$\boxed{\frac{dy}{dx} + Py = Q} \rightarrow e^{\int P dx}$

$= e^{\int 1/y\, dy}$

$= e^{\ln y}$

$= y$ → integrating factore

If $= y$

$y\frac{dz}{dy} + z = 2y$

$\Rightarrow y\,dz + z\,dy = 2y\,dy$

$\Rightarrow d(zy) = y^2 + b'$

$\Rightarrow zy - y^2 = b$

H.W $zp = -x$ → solve

Soln: $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$

$x^2 + y^2 = a$

$\frac{dy}{0} = \frac{dz}{-x}$

$\Rightarrow -x\,dy = 0$

$\Rightarrow dy = 0$

$y = b$

Find the soln of $xp + yq = z$ representing a surface meeting the parcabola $y^2 = 4x$, $z = 1$

Soln:

$x/z = a$ — ① $\quad \phi\left(\frac{x}{z}, \frac{y}{z}\right) = 0$

$y/z = b$ — ②

$z = 1$ in eqn (1)(2) →

$x = a$

$y = b$

$\Rightarrow y^2 = b^2$

$\Rightarrow y^2 = 4x$

$\Rightarrow y^2 = 4a = b^2$

$\rightarrow y^2 = 4xz$

$b^2 = 4a$

$\left(\frac{y}{z}\right)^2 = 4\frac{x}{z}$

//

H.W general sol^n $yzp + zxq = xy$

Non linear P.D.E:

Complete sol^n:

$$z = ax + by$$

particular sol^n:

$$a = 2, \; b = 5$$

$$z = 2x + 5y$$

Singular sol^n: $\frac{\partial z}{\partial a} = 0, \quad \frac{\partial z}{\partial b} = 0$

general sol^n: $z = ax + \psi(a)y \qquad b = \psi(a)$

~~# Standard form p, q, z~~

H.W complete sol^n:

$$q^2 = z^2 p^2 (1 - p^2)$$

Find the complete integral of

$$4z = pq$$

Soln: As a trial soln we assume that z is a fun^c of $x + ay = u$ (say) where a is arbitary constant,

$$p = \frac{\partial z}{\partial x} = \frac{dz}{du}\frac{\partial u}{\partial x} = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{dz}{du}\frac{\partial u}{\partial y} = a\frac{dz}{du}$$

$$4z = \frac{dz}{du}\, a\, \frac{dz}{du}$$

$$= a\left(\frac{dz}{du}\right)^2$$

$$\frac{dz}{du} = \frac{1}{\sqrt{a}} \cdot 2\sqrt{z}$$

$$\Rightarrow \frac{dz}{\sqrt{z}} = \frac{2}{\sqrt{a}}\, du$$

$$\Rightarrow z^{-1/2}\, dz = 2\frac{1}{\sqrt{a}}\, du$$

$$\Rightarrow \frac{z^{1/2}}{1/2} = 2\frac{1}{\sqrt{a}}u + c$$

$$\Rightarrow \sqrt{z} = \frac{1}{\sqrt{a}}u + c$$

$$\Rightarrow \sqrt{a}\sqrt{z} = u + c$$

$$\Rightarrow a\, z = (u+c)^2 = u^2 + 2uc + c^2$$

* general soln $yzp + zxq = xy$.

Soln:

$yzp + zxq = xy$

$\Rightarrow yzp - xy = -zxq$

$\Rightarrow yz$

$yzp + zxq = xy$

subsidiary eqn $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$

$\frac{dx}{yz} = \frac{dy}{zx}$

$\Rightarrow \frac{dx}{y} = \frac{dy}{x}$

$\Rightarrow x\,dx = y\,dy$

$\Rightarrow \int x\,dx = \int y\,dz$

$\Rightarrow \frac{1}{2}x^2 - \frac{1}{2}y^2 = \frac{1}{2}a'$

$\Rightarrow x^2 - y^2 = a'$

$\frac{dy}{zx} = \frac{dz}{xy}$

$\Rightarrow y\,dy = z\,dz$

$\Rightarrow \int y\,dz = \int z\,dz$

$\therefore y^2 - z^2 = b'$

$\therefore \phi(x^2 - y^2, y^2 - z^2) = 0$

* Complete integral of $q^2 = z^2p^2(1-p^2)$

* Charpits method:

$$f(x, y, z, p, q) = 0 \quad \text{---} (1)$$

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}$$

working rule:

↳ Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial p}, \frac{\partial f}{\partial q}$

using (1)

↳ Find Charpits auxilary eqn for (1)

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

$$\text{---} (11)$$

Solve (11) with (1) and find value of p, q

↳ Find complete soln of (1) by

$$p\,dx + q\,dy = dz$$

The integrating we get complete soln.

Ex $p - 3x^2 = q^2 - y$

Sol^n: Let, $f(x, y, z, p, q) = p - 3x^2 - q^2 + y = 0$ ——— (1)

$\frac{\partial f}{\partial x} = -6x$, $\frac{\partial f}{\partial y} = 1$, $\frac{\partial f}{\partial z} = 0$, $\frac{\partial f}{\partial p} = 1$, $\frac{\partial f}{\partial q} = -2q$

Charpit's eq^n are →

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

$$\Rightarrow \frac{dp}{-6x + p.0} = \frac{dq}{1 + q.0} = \frac{dz}{-p - q(-2q)} = \frac{dx}{-1} = \frac{dy}{2q}$$ ——— (II)

eq^n (II) →

$$\frac{dq}{1} = \frac{dy}{2q}$$

$\Rightarrow 2q\,dq = dy$

$\Rightarrow q^2 = (y + a)$ [Integrating]

$\Rightarrow q = \pm (y + a)^{1/2}$

value of q in (1) →

$p = 3x^2 + (y + a) - y$

$= 3x^2 + a$

$dz = p\,dx + q\,dy$

$= (3x^2 + a)\,dx \pm (a + y)^{1/2}\,dy$

By integrating,

$$z = \int (3x^2 + a)\,dx + \int (a + y^{1/2})\,dy$$

$$= (x^3 + ax) + \frac{2}{3}(a+y)^{3/2} + b$$

required value where a, b are consts.

H.W $q = 3p^2$

* $Pxy + Pq + qy - yz = 0$

Soln: Let, $f(x, y, z, p, q) = Pxy + Pq + qy - yz = 0$

$$\frac{\partial f}{\partial x} = Py, \quad \frac{\partial f}{\partial y} = Px + qz, \quad \frac{\partial f}{\partial z} = -y, \quad \frac{\partial f}{\partial p} = xy + q,$$

$$\frac{\partial f}{\partial q} = P + q$$

Charpit's eqn,

$$\frac{dP}{Py - Py} = \frac{dq}{Px + qz - qy} = \frac{dz}{-P(xy+q) - q(P+q)} = \frac{dx}{-xy - q} = \frac{dy}{-P - y}$$

$$\frac{dP}{0} = \frac{dx}{-xy - q}$$

$$\Rightarrow (-xy - q)\,dP = 0$$

$$\Rightarrow dP = 0 \Rightarrow P = a \quad [\text{Integrating}]$$

value of p in ① $\rightarrow$

$$axy + aq + qy - yz = 0$$

$$\Rightarrow q(a+y) = y(z-ax)$$

$$\Rightarrow q = \frac{y(z-ax)}{(a+y)}$$

$$dz = p\,dx + q\,dy$$

$$\Rightarrow dz = a\,dx + \frac{y(z-ax)}{(a+y)}\,dy$$

$$\Rightarrow \frac{dz - a\,dx}{z-ax} = \frac{a-ay\,dy}{a+y}$$

$$= dy - \frac{a}{a+y}\,dy$$

Integrating,

$$\ln(z-ax) = y - a\ln(a+y) + \ln b$$

$$\Rightarrow \ln(z-ax) + \ln(a+y)^a = y + \ln b$$

$$\Rightarrow (z-ax)(a+y)^a = b e^y$$

H.W $\quad 2z + p^2 + qy + 2y^2 = 0$

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Charpits method:

$f(x, y, z, p, q) = 0$ ——— (1)

$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

working Rule:

* Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}, \frac{\partial f}{\partial p}, \frac{\partial f}{\partial q}$ using (1)

* Find charpit's auxiliary eqn for (1)

formula

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}}$$

$$= \frac{dy}{-\frac{\partial f}{\partial q}}$$

Solve (2) with (1) and find the value of p and q

* Find complete soln of (1) by $p\,dx + q\,dy = d$

After integrating we get complete soln.

$\left(\frac{\partial z}{\partial y}\right)^2$ এরকম হলে NON - Linear

Apply charpit's method to find a complete integral $p - 3x^2 = q^2 - y$

Soln ⇒ Let, $f(x, y, z, p, q) = p - 3x^2 - q^2 + y = 0$ — (1)

Here, $\frac{\partial f}{\partial x} = -6x$, $\frac{\partial f}{\partial y} = 1$, $\frac{\partial f}{\partial z} = 0$, $\frac{\partial f}{\partial p} = 1$

$\frac{\partial f}{\partial q} = -2q$

Charpit's auxiliary eq's are

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{\frac{-\partial f}{\partial p}} = \frac{dy}{\frac{-\partial f}{\partial q}}$$

p, q এর value বসে বসে

$$\Rightarrow \frac{dp}{-6x + p \cdot 0} = \frac{dq}{1 + q \cdot 0} = \frac{dz}{-p - q(-2q)} = \frac{dx}{-1} = \frac{dy}{2q} \quad \text{— (2)}$$

From (2)

$$\frac{dq}{1} = \frac{dy}{2q}$$

$$2q \cdot dq = dy$$

$$q^2 = (y + a)$$

$$q = \pm \sqrt{(y+a)}$$

teacher এটা করতেন

$\pm (y+a)^{1/2}$ কেবল করতে পারে

Putting the value of q in (1)

$$p = 3x^2 + (y + a) - y = 3x^2 + q$$

We have, $dz = p\,dx + q\,dy$

$= (3x^2 + a)\,dx \pm (a+y)^{1/2}\,dy$

By integrating,

$$z = \int (3x^2 + a)\,dx \pm \int (a+y)^{1/2}\,dy$$

$$= (x^3 + ax) \pm \frac{2}{3}(a+y)^{3/2} + b$$

which is complete integral, where a, b are constants.

PQ → Non-Linear

Apply charpits method to find a complete integral of $pxy + pq + qy - yz = 0$

⇒ Let, $f(x, y, z, p, q) = pxy + pq + qy - yz = 0$

Here,

$\frac{\partial f}{\partial x} = py$, $\frac{\partial f}{\partial y} = px + q - z$, $\frac{\partial f}{\partial p} = xy + p$, $\frac{\partial f}{\partial z} = -y$, $\frac{\partial f}{\partial q}$

charpit's auxiliary eqns

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}}$$

$$\frac{dp}{py - py} = \frac{dq}{px + q - z - qy} = \frac{dz}{-pxy - pq - qp - zy} = \frac{dx}{-xy - q} = \frac{dy}{-p}$$

$$\Rightarrow \frac{dp}{0} = \frac{dx}{-xy - q}$$

$$(-xy - q)\,dp = 0$$

$dp = 0 \Rightarrow p = a$

putting the value of p in (1)

$$axy + aq + ay - yz = 0$$

$$q(a + y) = y(z - ax) \Rightarrow q = \frac{y(z - ax)}{a + y}$$

we have, $dz = p\,dx + q\,dy$

$$dz = a\,dx + \frac{y(z-ax)}{a+y}\,dy$$

$$dz - a\,dx = (z-ax)\frac{y}{a+y}\,dy$$

$$\frac{dz - a\,dx}{z-ax} = \frac{a+y-a}{a+y}\,dy$$

[z এর ax বাদ দিয়ে ভাগ]

$$\frac{dz - a\,dx}{z-ax} = dy - \frac{a}{a+y}\,dy$$

Integrating, $\ln(z-ax) = y - a\ln(a+y) + \ln b$

$\Rightarrow \ln(z-ax) + \ln(a+y)^a = y + \ln b$

$\Rightarrow (z-ax)(a+y)^a = be^y$

which is the required complete integral

30/11/23

Linear Partial differential eq^{n}s of second order :

Consider the PDE of second order in two independent variable x and y as

$$A\frac{\partial^2 z}{\partial x^2} + B\frac{\partial^2 z}{\partial x \partial y} + C\frac{\partial^2 z}{\partial y^2} + f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0$$

or, $Ar + Bs + Ct + f(x, y, z, p, q) = 0$

where, A, B, C are constants or continuous func^c of and y and $A > 0$. Now, eq^n (1) is

① Elliptic if $B^2 - 4AC < 0$

② Parabolic if $B^2 - 4AC = 0$

③ Hyperbolic if $B^2 - 4AC > 0$

Classify the followings,

$$4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} = 0$$

Sol^n: $A = 4, \quad B = 4, \quad C = 1$

$\therefore B^2 - 4AC = 4^2 - 4 \cdot 4 \cdot 1 = 0$

Hence, the PDE is parabolic

Classify the PDE.

$$\frac{\partial^2 u}{\partial t^2} + t\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + 6u = 0$$

Sol^n: Here, $A = 1$, $B = t$, $C = x$

$$B^2 - 4AC = t^2 - 4x$$

① The eq^n is elliptic if

$$t^2 - 4x < 0$$

$$\Rightarrow t^2 < 4x$$

② The eq^n parabolic if $t^2 - 4x = 0$

$$\Rightarrow t^2 = 4x$$

③ The eq^n is Hyperbolic if $t^2 - 4x > 0$

$$\Rightarrow t^2 > 4x$$

Linear com➡️L.M

Classify the PDE.

$$xy\frac{\partial^2 z}{\partial x^2} - (x^2-y^2)\frac{\partial^2 z}{\partial x \partial y} - xy\frac{\partial^2 z}{\partial y^2} + y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 2(x^2-y^2)$$

Here $A = xy$, $B = -(x^2-y^2)$, $C = -xy$

$$B^2 - AC = \{(x^2-y^2)\}^2 - 4x^2y^2$$

$$= x^4 + y^4 - 2x^2y^2 + 4x^2y^2$$

$$= x^4 + y^4 + 2x^2y^2$$

$$= (x^2+y^2)^2 > 0 \rightarrow \text{Hyperbolic.}$$

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0 \quad [B=0, A=x, C=y]$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} = 0$$

$$\Rightarrow y^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2}$$

$$= \frac{y^2}{x}\frac{\partial u}{\partial x} + \frac{x^2}{y}\frac{\partial u}{\partial y}$$

More than two variables:

[Linear partial differential of second order]

$$\sum\sum a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} \frac{\partial^2 u}{\partial x_i \partial x_j}$$

$$\sum_{j=1}^{3} \sum_{i=1}^{3} a_{ij} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{3} b_i \frac{\partial u}{\partial u_1} + Cu = 0$$

Consider, a Matrix A = Coff. of

$$\begin{matrix} U_{xx}, & U_{xy}, & U_{xz} \\ U_{xy} & U_{yy} & U_{yz} \\ U_{zx} & U_{zy} & U_{zz} \end{matrix}$$

$$\left[U_x = \frac{\partial u}{\partial x} \qquad U_{xy} = \frac{\partial^2 u}{\partial x \partial y} \right]$$

$$\frac{\partial^2 u}{\partial x_1 \partial x_j} + \frac{\partial^2 u}{\partial x_2 \partial x_2} + \frac{\partial u^2}{\partial x_3^2} + \frac{\partial u^2}{\partial x_1 \partial x_2} + \frac{\partial^2 u}{\partial x_1 \partial x_2} + \frac{\partial^2 u}{\partial x_1 \partial x_3} + \frac{\partial^2 u}{\partial x_2 \partial x_3} + \cdots$$

$$\Rightarrow U_{x_1x_1} + U_{x_2x_2} + U_{x_3x_3} + U_{x_1x_2} + U_{x_1x_3}\delta + \delta U_{x_2x_3} + \cdots$$

Consider a matrix A = coff of

$$\begin{bmatrix} U_{xx} & U_{xy} & U_{xz} \\ U_{xy} & U_{yy} & U_{yz} \\ U_{zx} & U_{zy} & U_{zz} \end{bmatrix}$$

then if

i) λ of A are non zero and all have same sign [elliptic]

ii) If any λ is zero then [parabolic]

iii) λ are non zero and have same sign except one then [Hyperbolic]

* $U_{xx} + U_{yy} + U_{zz} + U_{yz} + U_{zy} = 0$

- The matrix A of given eqn,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

if det = 0 then parabolic

$\Rightarrow 1(1-1) = 0$

* Classify $U_{xx} + U_{yy} = U_{zz}$

A classify $U_{xx} + U_{yy} + 2U_{zz} = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\text{Det} = 1(1+1) - 0$

$= 2$ (Hyperbolic)

Homogeneous linear PDE with const co-eff :

$$\frac{\partial^n z}{\partial x^n} + k_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} + k_2 \frac{\partial^n z}{\partial x^{n-2} \partial y^2} + \cdots + k_n \frac{\partial^n z}{\partial y^n} = F(x,y)$$

Complete sol^n = Complementary fun^c + particular integral

$$= C.F + P.I$$

→ To find complementary fun^c,

Symbolic form : $\frac{\partial}{\partial x} = D$, $\frac{\partial}{\partial y} = D'$; $f(D, D') = F(x,$

Auxilliary form : $D = m$, $D' = 1$; $f(m, 1) = 0$

Solve the auxilliary eq^n and find value of m

1. If roots are distinct $m_1, m_2, \cdots$

$$C.F = f_1(y + m_1 x) + f_2(y + m_2 x) + \cdots$$

2. If roots are $m_1, m_1, m_2 \cdots$ two are repeated,

$$C.F = f_1(y + m_1 x) + x f_2(y + m_1 x) + f_3(y + m_2 x)$$

3. Three are repeated,

$$C.F = f_1(y + m_1 x) + x f_2(y + m_1 x) + x^2 f_3(y + m_1 x$$

xyz

19/12/23

Rules for finding particular integral

$f(D, D')z = F(x, y)$

$D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$

$z = \frac{1}{f(D, D')} \boxed{F(x, y)}$

Types of $F(x, y)$	Working rule
$e^{(ax+by)}$	Put $D = a$ and $D' = b$
$\sin(ax+by)/\cos(ax+by)$	Put $D^2 = a^2$, $D'^2 = -b^2$, $DD' = -ab$
$x^m y^n$	$\frac{1}{f(D^2, DD', D'^2)}$ $\sin()/\cos$
	$f(D, D')^{-1} F(x, y)$

Binomial expansion

$e^{ax+by} V(x, y) : e^{ax+by} \frac{1}{f(D+a, D'+b)} V(x, y)$

put $D = D+a$, $D' = D'+b$

otherwise : $\frac{1}{(D - mD')} f(x, y)$

factor

Solve the PDE

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

$$(D^3 - 3D^2D' + 4D'^2)z = e^{x+2y} \qquad f(D, D')$$

$$f(D, D') = D^3 - 3D^2D' + 4D'^3 \qquad [D = m, D' = 1]$$

$$f(m, 1) = m^3 - 3m^2 + 4 = 0$$

$m = -1, 1, 2$

$$m^3 + m^2 - 4m^2 - 4m + 4m + 4 = 0$$

$$m^2(m+1) - 4m(m+1) + 4(m+1) = 0$$

$$(m+1)(m^2 - 2 \cdot 2m + 2^2) = 0$$

$$\boxed{m = -1, 2, 2}$$

Complementary function.

$$f_1(y + m_1 x) + x f_2(y + m_1 x) + f_3(y - m_2 x)$$

$$= f_1(y + 2x) + x f_2(y + 2x) + f_3(y - x)$$

particular integral

$$z = \frac{1}{f(D,D')} F(x,y)$$

$$z = \frac{1}{D^3 - 3D^2D' + 4D'^3} e^{x+2y} \qquad [a=1, b=2]$$

$$z = \frac{1}{1^3 - 3\cdot 1^2\cdot 2 + 4\cdot 2^3} \left(e^{x+2y}\right)$$

$$= \frac{1}{27} e^{x+2y}$$

complete soln:

$$f_1(y+2x) + xf_2(y+2x) + f_3(y-x) + \frac{1}{27} e^{x+2y}$$

Solve the PDE :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \cos 2y$$

$$(D^2 - DD')z = \sin x \cdot \cos 2y$$

$$= \frac{1}{2}\{\sin(x+2y) + \sin(x-2y)\}$$

Formula: $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{1}{2}\{\sin(x+2y) + \sin(x-2y)\}$$

$$f(D, D') = D^2 - DD' \quad [D = m, D' = 1]$$

$$m^2 - m = 0$$

$$m(m-1) = 0$$

$$m = 0, 1$$

$$C.F = f_1(y+x) + f_2(y)$$

Partical integral,

$$z = \frac{1}{D^2 - DD'}\left\{\frac{\sin(x+2y)}{2}\right\} + \frac{\sin(x-2y)}{2}\} \frac{1}{2}$$

$$= \frac{1}{-1^2 - (-1 \cdot 2)}\left\{\sin(x+2y\right.$$

$[a = 1, b = 2]$

$$\begin{bmatrix} a = 1 \\ b = 2 \end{bmatrix}$$

$$z = \frac{1}{D^2 - DD'} \left\{ \frac{\sin(x+2y)}{2} \right\} + \frac{1}{D^2 - DD'} \left\{ \frac{\sin(x-2y)}{2} \right\}$$

$$= \frac{1}{-1^2 - (-1 \times 2)} \left(\frac{\sin(x+2y)}{2} \right) + \frac{1}{-1^2 - \{-1 \times 2\}} \frac{\sin(x-2y)}{2}$$

$$= \frac{\sin(x+2y)}{2} - \frac{1}{6} \sin(x-2y)$$

Homework

(1) $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$

(2) $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 12xy$

(3) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y-1)e^x$

21/12/2023

Solve the PDE

$$(D^2 + 2DD' + D'^2)z = x^2 + xy + y^2$$

$$m^2 + 2m + 1 = 0$$

$$m = -1, -1$$

$$CF = f_1(y - x) + x f_2(y - x)$$

$$PI = \frac{1}{D^2 + 2DD' + D'^2}(x^2 + xy + y^2)$$

$$(1+x)^n = 1 + nx + n(n-1)\frac{x^2}{2!} + n(n-1)(n-2)\frac{x^3}{3!} + \cdots$$

$$= \frac{1}{(D + D')^2}(x^2 + xy + y^2)$$

$$= \frac{1}{D^2\left(1 + \frac{D'}{D}\right)^2}(x^2 + xy + y^2)$$

$$= \frac{1}{D^2}\left(1 + \frac{D'}{D}\right)^{-2}$$

$$= \frac{1}{D^2}\left(1+\frac{D'}{D}\right)$$

$$= \frac{1}{D^2}\left\{1+(-2)\frac{D'}{D}+(-2)(-3)\frac{\frac{D^2}{D}}{2!}\right\}$$

$$(x^2+xy+y^2)$$

$$= \left(\frac{1}{D^2} - 2\frac{D'}{D^3} + \frac{6}{2}\frac{D'^2}{D^4}\right)$$

$$(x^2+xy+y^2)$$

$$= \left(\frac{1}{D^2} - 2\frac{D'}{D^3} + 3\frac{D'^2}{D^4}\right)(x^2+xy+y^2)$$

$$= \frac{1}{D^2}(x^2+xy+y^2)$$

$$- 2\frac{D'}{D^3}(x^2+xy+y^2) + 3\frac{D'^2}{D^4}(x^2+xy+y^2)$$

$$= \frac{1}{D}\left(\frac{x^3}{3}+\frac{x^2}{2}y+y^2x\right)$$

$$- 2\frac{1}{D^3}(2x+2y+0)(x+2y)$$

$$+ 3\frac{D'}{D^4}(x+2y)$$

$$= \frac{x^4}{12}+\frac{x^3}{6}y+\frac{y^2x^2}{2}$$

$$- 2\frac{1}{D^2}\left(\frac{x^2}{2}+2xy\right)+\frac{3}{D^4}(2)$$

$$\Rightarrow \frac{x^4}{12} + \frac{x^3y}{6} + \frac{x^2y^2}{2} - \frac{2}{D}\left(\frac{x^3}{6} + yx^2\right) + 3\frac{1}{D^3}(2x)$$

$$\Rightarrow \frac{x^4}{12} + \frac{x^3y}{6} + \frac{x^2y^2}{6} - 2\left(\frac{x^4}{24} + \frac{yx^3}{3}\right) + 3\frac{1}{D^2}(x^2)$$

$$\Rightarrow \frac{x^4}{12} + \frac{x^3y}{6} + \frac{x^2y^2}{2} - \frac{x^4}{12} - \frac{2x^3y}{3} + 3\frac{1}{D}\left(\frac{x^3}{3}\right)$$

$$\Rightarrow \frac{x^3y}{6} - \frac{2x^3y}{3} + \frac{x^2y^2}{2} + 3\frac{x^4}{12}$$

$$\Rightarrow 3\frac{x^3y}{6} + \frac{x^2y^2}{2} + \frac{x^4}{4}$$

$$\Rightarrow \frac{x^4}{4} + x^2.$$

$$e^{ax+by} \frac{1}{(D+a)^2+(D'+b)^2} V(x,y)$$

$D \to D+a$
$D' \to D'+b$

p.1

$$z = \frac{1}{D^2-D'^2} e^{x-y} \sin(x+2y)$$

$$= e^{x-y} \frac{1}{(D+1)^2-(D'-1)^2} \sin(x+2y)$$

$$= e^{x-y} \frac{1}{D^2+2D-D'+2D'} \sin(x+2y)$$

$D^2 = -a^2$
$D' = -b^2$
$DD' = -ab$
$[a=1, b=2]$

$$= e^{x-y} \frac{1}{2D+3+2D'} \sin(x+2y)$$

$$= e^{x-y} \frac{1 \times D}{D \times (2D+3+2D')} \sin(x+2y)$$ [D দ্বারা লব ও হর গুণ]

$$= e^{x-y} \frac{D}{2D^2+3D+2DD'}$$

$$= e^{x-y} \frac{D}{2(-1^2)+3D+2(-1)(2)}$$

$$= e^{x-y} \frac{D}{3D-6} \sin(x+2y)$$

$$= \frac{e^{x-y}}{3} \frac{D}{D-2} \sin(x+2y)$$

$$= \frac{e^{x-y}}{3} \frac{D^2+2D}{D^2-4} \sin(x+2y)$$

$$= \frac{e^{x-y}}{3} \frac{D^2+2D}{-5} \sin(x+2y)$$

$$= -\frac{e^{x-y}}{15} D^2\{\sin(x+2y)\} + 2D\{\sin(x+2y)\}$$

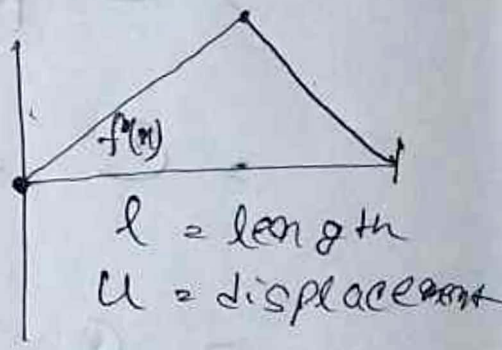
$$= +\frac{e^{x-y}}{15}\sin(x+2y) - \frac{2e^{x-y}}{15}\cos(x+2y)$$



Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{— (1)}$$





Boundary condition:

$$u(0,t) = u(l,t) = 0$$

Initial condition:

$$u(x,0) = f(x)$$

$$\frac{\partial u}{\partial t} = 0, \text{ at } t = 0$$

problem

Separation of variable:

$$u(x,t) = X(x)\,T(t)$$

Soln: Let, the complete soln of wave eqn is $u(x,t) = T(t)\,X(x)$ —— ②

$$\frac{\partial^2 u}{\partial x^2} = T(t)\,X''(x)$$

$$\frac{\partial^2 u}{\partial t^2} = T''(t)\,X(x)$$

$$T''X = c^2 X''T$$

$$\frac{T''}{T} = c^2\frac{X''}{X}$$ অথবা $$\frac{T''}{c^2T} = \frac{X''}{X}$$ ভাঙ্গা যায়

$$\frac{T''}{c^2T} = \frac{X''}{X} = K \rightarrow (\text{constant})$$

$$\boxed{T'' - Kc^2T = 0} \rightarrow ③$$ and $$\boxed{X'' - KX = 0} \rightarrow ④$$

Case-1: when $k=0$

integration

$T'' = 0$

$T' = c_1$

$T = c_1 t + c_2$

$X'' = 0$

$X' = c_3$

$X = c_3 x + c_4$

Complete sol^n: $u(x,t) = (c_1 t + c_2)(c_3 x + c_4)$

Case-2: when k is positive • $\boxed{k = p^2}$ here

p is any number

From ③ and ④

$\boxed{T'' - p^2 c^2 T = 0}$

and $\boxed{X'' - p^2 X = 0}$

$(D^2 - p^2 c^2) T = 0$

$D = \frac{d}{dt}$

$m^2 - p^2 c^2 = 0$

$m = \pm pc$

$T(t) = c_1 e^{pct} + c_2 e^{-pct}$

$(D^2 - p^2) X = 0$

$D' = \frac{d}{dx}$

$m'^2 - p^2 = 0$

$m = \pm p$

$X(x) = c_3 e^{px} + c_4 e^{-px}$

complete soln : $(c_1 e^{pct} + c_2 e^{-pct})$ ✗ ↗ सही

$c_3 e^{px} + c_4 e^{-px}$)

Case-3 : When K is negative, $K = -p^2$

From ③ and ④

$T'' + p^2 c^2 T = 0$

$m^2 + p^2 c^2 = 0$

$m = \pm ipc$

$m = a + ib$

$X'' + p^2 X = 0$

$m^2 + p^2 = 0$

$m = \pm ip$

$T(t) = e^{ax} (c_1 \cos pct + c_2 \sin pct)$

$X(x) = c_3 \cos px + c_4 \sin px$

complete sol^n :

$u(x,t) = (c_1 t + c_2)(c_3 x + c_4)$

$\therefore u(x,t) = (c_1 \cos pct + c_2 \sin pct) \cdot$

$(c_3 \cos px + c_4 \sin px)$

04.01.24

compulsory → PDE → 2 set
- ↳ cons/fanc elimination
- ↳ lagrange + theory
- ↳ charpit
- ↳ homo linearc pde
- ↳ 2nd orderc / weave

ortho,

Matrix
- ↳ defn (sym, harc, skeral
- ↳ system of eqn
- ~~↳ linearc eo~~ ↳ consistant
  - ↳ ∞, 1, → soln
- ↳ linearc com, depen, Inde + defn
- ↳ eigen vector + value
- ↳ caley (verification)
- ↳ rank nullity defn (short Q)

Section-A ← compulsory

1. a) Constant / function elemination
   b) Lagrenge
   c) Charpit

2) a)
   b) 2nd order / wave
   c) Lagrenge theorem

3) a) Def: Symetric, non sym ...
   ~~b) Linear combination~~
   b) System of eqn.
   c) unit, ~~in~~ infinite, non-solution for x.

4) Eigen value, vec, caley-hamilton theory (verification

verification