Modern Physics

(Lec-2)

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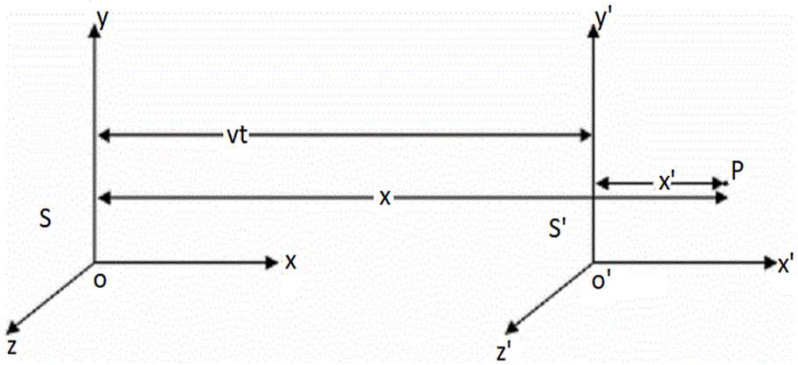
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OUTLINES

- Lorentz Transformations
- Length Contraction
- Time dilation

Derivation of Lorentz Transformation:

Let us consider inertial frames S and S' in figure. S' is moving with respect to S. Initially when t=0 the origin of S and S' coincide then the value of t will be zero i.e., t'=0. Suppose when t=t'=0 an event (may be explosion of flash bulb) occurred and the observers of both the frames measure the speed of light. According to second postulates both of will measure the same speed c.



That is in S framein S frame

$$x=ct\text{----------}(1)$$

And in S' frame

$$x'=ct'\text{---------}(2)$$

Now we have to find out which equation is valid for both (1) and (2).
The new equations should be like that an event in S frame will correspond to an event in frame S'.As for x coordinate the classical transformation equations are x'=x-vt and x=x'+vt, so we can consider the relationship between x and x' is

$$x'=k(x-vt)\text{ ----------}(3)$$

$$\text{And } x=k(x'+vt')\text{ ----------}(4)$$

Cont....

Here k is a propotional constant and does not depend on x and t but may be a function of v. As there is no relative motion along y and z direction, therefore

y'=y and z'=z. But t and t' are not equal. Putting value of x' from eqn.(3) to (4) we get

$$x=k\,[k(x-vt)+vt']$$

$$\text{or, } x=k^2x-k^2vt+kvt'$$

$$\text{or, } t'=kt+\left(\frac{1-k^2}{kv}\right)x \text{----------(5)}$$

now putting the value of x' and t' in eqn.(2) we get

$$k\,(x-vt)=c\,[kt+\left(\frac{1-k^2}{kv}\right)x]$$

$$\text{or, } kx-kvt=ckt+\left(\frac{1-k^2}{kv}\right)cx$$

$$\text{or, } x[k-\left(\frac{1-k^2}{kv}\right)c]=ckt+kvt$$

$$\text{or, } x=\frac{ct(k+\frac{kv}{c})}{[k-(\frac{1-k^2}{kv})\,c]}$$

Cont....

again we get from eqn.(1)

$$x=ct$$

Therefore, $$\frac{ct(k+\frac{kv}{c})}{[(k-\frac{1-k^2}{kv})\,c]}=ct$$

$$\text{or, } k+\frac{kv}{c}=k-(\frac{1-k^2}{kv})c$$

$$\text{or, } k^2v^2=-(1-k^2)c^2$$

$$\text{or, } k^2=\frac{c^2}{c^2-v^2}$$

$$\text{or, } k^2=\frac{1}{1-\frac{v^2}{c^2}}$$

$$\text{or, } k=\sqrt{1-\frac{v^2}{c^2}}$$

Now putting value of k in eqn. (3) and (5) we get the relations between time and coordinate of S frame with time and coordinates of S' frame,

Cont…

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Inverse Lorentz Transformation:

$$x = \frac{x' + vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

LENGTH CONTRACTION

The length of an object in motion with respect to an observer appears to be shorter than when it is rest with respect to him, this phenomenon is known as length contraction or Lorentz-Fitz Gerald contraction.

Let us consider a rod lying along the $x' - axis$ in the moving frame S'. An observer in this frame determines the coordinated of its ends to be $x_1' and\ x_2{}'$ and so the proper length of the rod is,

$$L_0 = x_2' - x_1'$$

In order to find $L = x_2 - x_1$, the length of the rod as measured in the stationary frame S at the time t, we make use of the Lorentz transformation to give,

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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Subtracting these two equations we get,

$$x_2' - x_1' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x_2 - vt - x_1 + vt}{\sqrt{1 - v^2/c^2}}$$

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

i.e.

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Since, $\sqrt{1 - v^2/c^2}$ is always less than unity, so,$L < L_0$. i.e. the length of the appears to be contracted according to the observer in reference frame S.

Problem: The proper length of a rod is 5m. What would be its length for an observer if it be moving relative to him in a direction parallel to its own length (i) with velocity 0.8c, (ii) moving with velocity c? What would be its length for an observer who is himself moving along with it?

TIME DILATION

The time interval between the occurrences of two events which are in motion with respect to an observer will be longer than the time interval of occurrence of the same two events which are at rest with respect to the observer; this phenomenon is known as time dilation.

Consider a gun placed at point x_1' in the frame S'. Suppose it fires two shots at times t_1' *and* t_2' as respect to the clock placed in S'. Let t_1 *and* t_2 be the corresponding times as respect to the clock carried by S.

Now, the time interval in frame S' is given by,

$$t_0 = t_2' - t_1'$$

And the time interval in frame S is given by,

$$t = t_2 - t_1$$

Now, according to the inverse Lorentz transformation,

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

CONTI…

And,

$$t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore,

$$t = t_2 - t_1$$

$$t = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{t_2' - t_1'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\therefore t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As, $1/\sqrt{1 - \frac{v^2}{c^2}}$ is greater than unity, we have, $t > t_0$. Thus the time interval t in the moving reference frame S' appears to be longer or dilated by a factor $1/\sqrt{1 - \frac{v^2}{c^2}}$ to the observer in the stationary frame S.

Problem: A clock in a spaceship emits signal at intervals of 1 s as observed by an astronaut in the spaceship. If the spaceship travels with a speed of $3 \times 10^7\ ms^{-1}$, what is the interval between successive signals as seen by n observer at the control center on the ground?