Modern Physics

(Lec-3)

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OUTLINES

- Relativistic Velocity Transformation
- Relativistic Mass Transformation

Relativistic Velocity Transformation:

The complete set of relativistic Lorentz transformation is

$$x' = k(x-ut), \quad y' = y, \quad z' = z \text{ and } \quad t' = k(t - ux/c^2) \qquad (1)$$

Let v_x, v_y, and v_z are the velocity components of a particle with respect to the S frame along x, y and z directions respectively, while v_x', v_y', and v_z' are the velocity components of that particle with respect to S' frame along x′, y′, and z′ directions respectively.

By differentiating the Lorentz transformation equations for x′, y′, z′ and t′, we get

$$dx' = k(dx - udt), \; dy' = dy, \qquad dz' = dz \text{ and } dt' = k(dt - u/c_2 dx)$$

Therefore

$$v_x' = \frac{dx'}{dt'} = \frac{k(dx - udt)}{k(dt - \frac{u}{c^2}dx)} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2}\frac{dx}{dt}} = \frac{v_x - u}{1 - \frac{u}{c^2}v_x} \qquad (5)$$

Cont....

We may write the complete set of relativistic velocity transformation as

$$v'_x = \frac{v_x - u}{1 - \frac{u}{c^2}v_x}, \qquad v'_y = \frac{v_y\sqrt{1-u^2/c^2}}{1 - \frac{u}{c^2}v_x} \text{ and}, \qquad v'_z = \frac{v_z\sqrt{1-u^2/c^2}}{1 - \frac{u}{c^2}v_x}$$

The inverse velocity transformation equations are

$$v_x = \frac{v'_x + u}{1 + \frac{u}{c^2}v'_x}, \qquad v_y = \frac{v'_y\sqrt{1-u^2/c^2}}{1 + \frac{u}{c^2}v'_x} \text{ and}, \qquad v_z = \frac{v'_z\sqrt{1-u^2/c^2}}{1 + \frac{u}{c^2}v'_x}$$

(i) If u and v_x are very small compared to c, the velocity transformations satisfy the classical results.

$$v'_x = v_x - u, \qquad v'_y = v_y \text{ and}, \qquad v'_z = v_z$$

(ii) If a ray of light is emitted in the moving frame S′ in its direction of motion relative to S, then $v'_x = c$, and no matter what the value of u of the observer, an observer in frame S will measure the speed

$$v_x = \frac{v'_x + u}{1 + \frac{u}{c^2}v'_x} = \frac{c + u}{1 + \frac{u}{c^2}c} = c$$

Thus, the observers in the car and on the road both find the same value for the speed of light, as they must.

Relativistic Mass Transformation

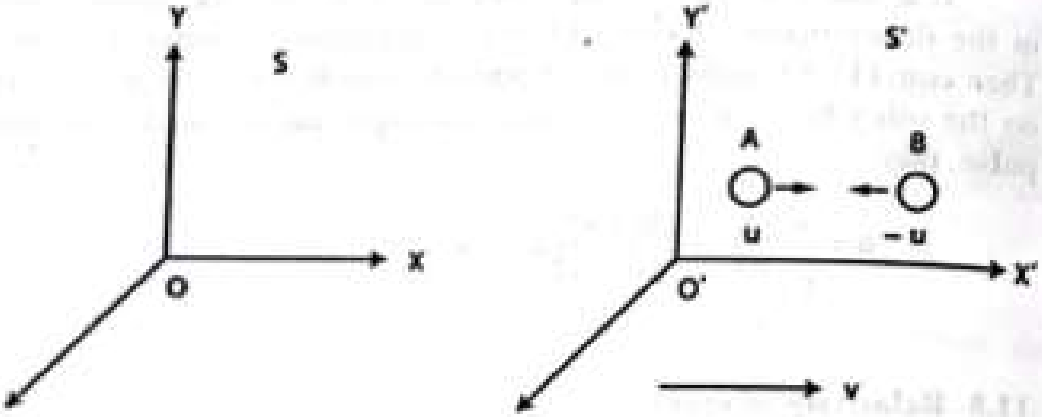


Fig. 11.4

Let the collision be considered with reference to the frame S. Let u_1 and u_2 be the velocities of the balls as observed from S. Then

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

and

$$u_2 = \frac{-u+v}{1-\frac{uv}{c^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

After collision, velocity of the coalesced mass is v relative to the frame S. As observed from the frame S, let the mass of the ball traveling with velocity u_1 be m_1 and that of B with velocity u_2 be m_2. Since the total momentum of the balls must be conserved, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \quad \ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

substituting the value of u_1 and u_2 as given by equations (i) and (ii) in equation (iii), we get

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$$m_1\left[\frac{u+v}{1+\frac{uv}{c^2}}\right]+m_2\left[\frac{-u+v}{1-\frac{uv}{c^2}}\right]=(m_1+m_2)v$$

$$\text{or, } m_1\left[\frac{u+v}{1+\frac{uv}{c^2}}-v\right]=m_2\left[v-\frac{-u+v}{1-\frac{uv}{c^2}}\right]$$

$$\text{or, } m_1\left[\frac{u+v-v-\frac{uv^2}{c^2}}{1+\frac{uv}{c^2}}\right]=m_2\left[\frac{v-\frac{uv^2}{c^2}+u-v}{1-\frac{uv}{c^2}}\right]$$

$$\text{or, } m_1\left[\frac{u\left(1-\frac{v^2}{c^2}\right)}{1+\frac{uv}{c^2}}\right]=m_2\left[\frac{u\left(1-\frac{v^2}{c^2}\right)}{1-\frac{uv}{c^2}}\right]$$

$$\text{or, } \frac{m_1}{m_2}=\frac{1+\frac{uv}{c^2}}{1-\frac{uv}{c^2}} \quad \text{..... (iv)}$$

$$\text{Also, } 1-\frac{u_1^2}{c^2}=1-\frac{\left(\frac{u+v}{c}\right)^2}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$=\frac{1+\frac{u^2v^2}{c^4}+\frac{2uv}{c^2}-\frac{u^2}{c^2}-\frac{v^2}{c^2}-\frac{2uv}{c^2}}{\left(1+\frac{uv}{c^2}\right)^2}$$

CONT…

$$= \frac{\left(1-\frac{u^2}{c^2}\right)-\frac{v^2}{c^2}\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\therefore \quad 1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots \text{(v)}$$

Similarly, $1-\frac{u_2^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{uv}{c^2}\right)^2}$ ..... (vi)

Dividing equation (vi) by (v)

$$\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}} = \frac{\left(1+\frac{uv}{c^2}\right)^2}{\left(1-\frac{uv}{c^2}\right)^2}$$

or, $$\frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}} = \frac{1+\frac{uv}{c^2}}{1-\frac{uv}{c^2}} \quad \ldots\ldots\ldots\ldots\ldots \text{(vii)}$$

From equation (vii) and (iv), we get

$$\frac{m_1}{m_2} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

CONTI….

or, $$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} \quad \text{..... (viii)}$$

Since the L.H.S. and R.H.S. of eqn. (viii) are independent of one another, the above result can be true only if each is a constant.

Therefore,

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_0 \text{ ..... (ix)}$$

The constant denoted by m_0, is called the rest mass of the body and corresponds to *zero velocity*.

Thus $$m_1 = \frac{m_0}{\sqrt{1-\frac{u_1^2}{c^2}}}; \quad \text{or} \quad m_2 = \frac{m_0}{\sqrt{1-\frac{u_2^2}{c^2}}}$$

In general, if m denotes the mass of a body when it is moving with a velocity v, then,

$$m_1 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (11.14)$$