Modern Physics

(Lec-4)

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OUTLINES

- Mass and Energy equivalence equation
- Concept of Massless particle and its expression

Energy mass relationship

In classical physics, force is defined as the time rate of change of momentum. Or

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

since mass is constant. This definition of force is kept in relativistic mechanics, but now mass is also a variable. Hence

$$F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

Suppose the kinetic energy of a body is increased by an amount dT by exerting a force through a distance. Then the change in kinetic energy can be found, as in classical physics, by calculating the work done by the force. Or,

$$dT = F \cdot dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

where F is the force acting on the body and dx is the distance through which it works.

Substituting eqn. (ii) in eqn. (iii), we get

$$dT = m.\frac{dv}{dt}.dx + v.\frac{dm}{dt}.dx \quad \ldots\ldots\ldots \text{(iv)}$$

Since dx/dt = v, eqn. (iv) can be written as

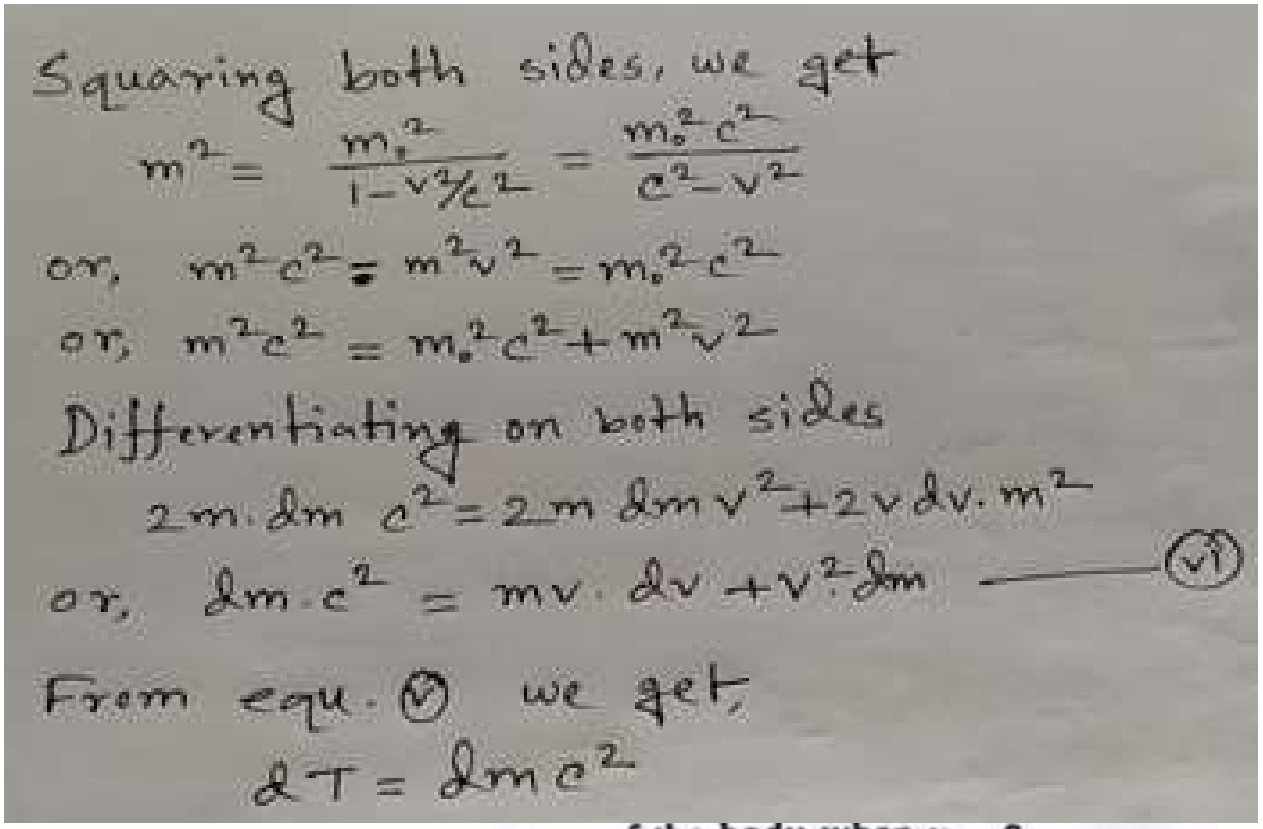
$$dT = mvdv + v^2dm \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

$$= v\,(mdv + vdm) = v.d(mv) \ldots\ldots \text{(v.a)}$$

Eqn. (v) can be simplified by using mass - transformation equation. Differentiating both sides of equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Cont....



Squaring both sides, we get

$$m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2 c^2}{c^2 - v^2}$$

or, $m^2 c^2 - m^2 v^2 = m_0^2 c^2$

or, $m^2 c^2 = m_0^2 c^2 + m^2 v^2$

Differentiating on both sides

$$2m \cdot dm\, c^2 = 2m\, dm\, v^2 + 2v\, dv \cdot m^2$$

or, $dm \cdot c^2 = mv \cdot dv + v^2 dm$ ——— (v)

From equ. (v) we get,

$$dT = dm\, c^2$$

[Alternately: when m_0 is the mass of the body when $v = 0$ and m is the mass of the body when $v = v$, we have

$$T = \int_0^v dT = c^2 \int_{m_0}^{m} dm = c^2 (m - m_0) \quad \ldots\ldots\ldots\ldots \text{(x.a)}]$$

Thus, the kinetic energy of a body at high speeds is not equal to $\frac{1}{2}mv^2$; nor can we substitute $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ into the classical expression

and get a correct result *i.e.*, K.E. $\neq \frac{\frac{1}{2}m_0 v^2}{\sqrt{1 - \frac{v^2}{c^2}}}$

CONT…

When the body is at rest, the internal energy store in the body is $m_o c^2$. Einstein called $m_0 c^2$, the rest mass energy of the body. The total energy E possessed by the body is then the sum of the rest mass energy and kinetic energy of the body.

$$E = T + m_0 c^2 \qquad \text{(xi)}$$

$$= mc^2 - m_0 c^2 + m_0 c^2$$

$$= mc^2 \qquad \text{(11.17)}$$

$$= \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Eqn. (11.17) gives Einstein's famous formula $E = mc^2$ and is referred to as Einstein's mass-energy relation.

CONT…

This is the Einstein famous mass-energy relation.

(i) For low speeds, $v/c \ll 1$, Then we can write

$$K = mc^2 - m_0c^2 = \frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \qquad (8)$$

Using the binomial approximation $(1+x)^n \approx 1+nx$ with $|x| \ll 1$, we get

$$K = m_0c^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

$$= m_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)$$

$$= \frac{1}{2}m_0v^2 \qquad (9)$$

At low speed, the relativistic expression for the K.E. of a moving body reduces to the classical one.

(ii) Mass-less particles: We know, total energy

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \dots\dots\dots\dots (1)$$

and relativistic momentum, $p = mv = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \dots\dots\dots\dots (2)$

When $m_0 = 0$ and $v<c$, i.e. $E = p = 0$ i.e. **A mass less particle with a speed less than that of light can have neither energy nor momentum.**

(iii) When $m_0 = 0$ and $v = c$, $E = 0/0$ and $p = 0/0$ which are indeterminate: E and p can have any values. The equations [1] & [2] are consistent with the existence of mass-less particles that possess energy and momentum provided that they travel with the speed of light.

(iv) There is another restriction on mass-less particles.

we can write $E^2 = \frac{m_0^2 c^4}{1-\frac{v^2}{c^2}}$

Also we can write $p^2 = \frac{m_0^2 v^2}{1-\frac{v^2}{c^2}}$, or, $p^2c^2 = \frac{m_0^2 v^2 c^2}{1-\frac{v^2}{c^2}}$

Subtracting p^2c^2 from E^2 yields

$$E^2 - p^2c^2 = \frac{m_0^2 c^4}{1-v^2/c^2} - \frac{m_0^2 v^2 c^2}{1-v^2/c^2}$$

$$= m_0^2 c^2 \left(\frac{c^2}{c^2-v^2} - \frac{v^2}{c^2-v^2}\right) c^2$$

$$= m_0^2 c^4 \left(\frac{c^2-v^2}{c^2-v^2}\right)$$

$$= m_0^2 c^4$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

CONTI…

Therefore, for all particles, we have

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} = \sqrt{E_0^2 + p^2 c^2}$$

According to this formula, if a particle exists with $m_0 = 0$, the relationship between its energy and momentum must be given by

mass-less particles $E = pc$

In fact, mass-less particles of two different kinds - the photon and the neutrino have indeed been discovered and their behavior is as expected.