Modern Physics

(Lec-7)

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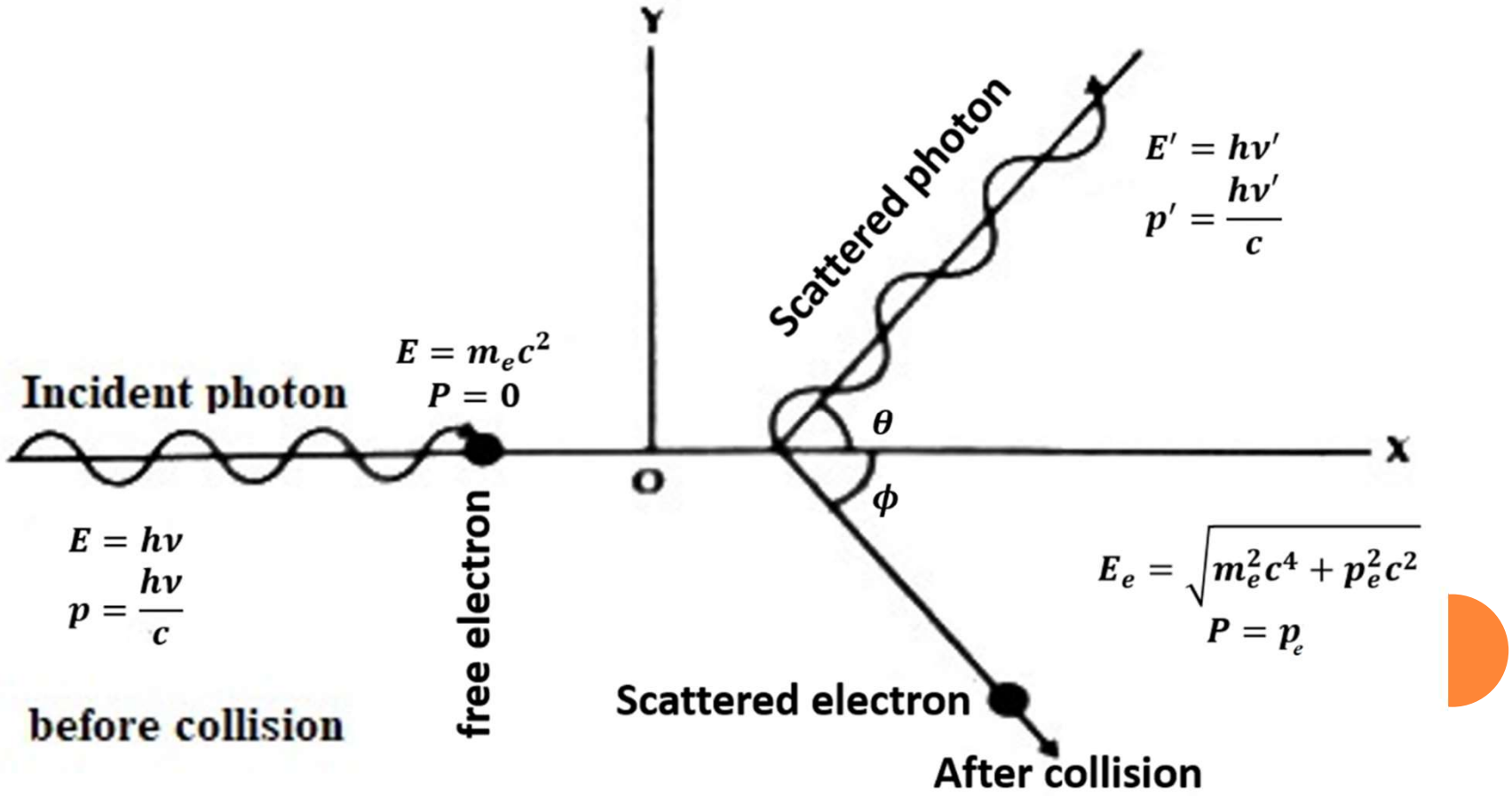
OUTLINES

- Compton Effect
- Compton wavelength shift
- The kinetic energy of the recoil electron

Compton Effect:

A phenomenon in which a collision between a photon and a free electron take place, as a result the kinetic energy of the electron increase and the wavelength of the photon also increase. As a result of the Compton Effect, the photons transfer some of their energy to the electrons.

Compton wavelength shift:



Cont....

Figure shows the collision between a single photon and an electron which is assumed to be at rest. Initially, photon has energy E given by,

$$E = h\nu = \frac{hc}{\lambda}$$

And linear momentum p,

$$p = \frac{E}{c}$$

The electron, at rest, has rest energy $m_e c^2$. After scattering photon moves at an angle θ with energy E' and momentum p'. And the electron moves at an angle ϕ with energy E_e and momentum p_e.

From conservation of energy,

$$E + m_e c^2 = E' + E_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From conservation of momentum, in x direction

Initial momentum=final momentum

$$p = p_e cos\phi + p' cos\theta$$

$$\Rightarrow p_e cos\phi = p - p' cos\theta \ldots\ldots\ldots\ldots (2)$$

Cont…

And, in y direction

$$0 = p'sin\theta - p_e sin\phi$$

$$\Rightarrow p_e sin\phi = p'sin\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Squaring and adding equations (2) and (3),

$$p_e^2 \cos^2\phi + p_e^2 \sin^2\phi = (p - p'cos\theta)^2 + p'^2 \sin^2\theta$$

$$p_e^2(\cos^2\phi + \sin^2\phi) = p^2 - 2pp'cos\theta + p'^2 \cos^2\theta + p'^2 \sin^2\theta$$

$$\therefore p_e^2 = p^2 - 2pp'cos\theta + p'^2 \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Now, from Einstein's relation,

$$E_e^2 = m_e^2c^4 + p_e^2c^2$$

$$\Rightarrow (E + m_ec^2 - E')^2 = m_e^2c^4 + (p^2 - 2pp'cos\theta + p'^2)c^2$$

$$\Rightarrow E^2 + m_e^2c^4 + E'^2 + 2Em_ec^2 - 2E'm_ec^2 - 2EE'$$

$$= m_e^2c^4 + p^2c^2 - 2pp'c^2cos\theta + p'^2c^2$$

$$\Rightarrow E^2 + m_e^2c^4 + E'^2 + 2Em_ec^2 - 2E'm_ec^2 - 2EE'$$

$$= m_e^2c^4 + E^2 - 2EE'cos\theta + E'^2 \qquad [E = pc]$$

$$2Em_ec^2 - 2E'm_ec^2 = 2EE' - 2EE'cos\theta$$

$$m_ec^2(E - E') = EE'(1 - cos\theta)$$

$$\frac{E - E'}{EE'} = \frac{1}{m_ec^2}(1 - cos\theta)$$

CONT…

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - cos\theta)$$

$$\frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1}{m_e c^2}(1 - cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta)$$

Here, the quantity $h/m_e c$ is called Compton wavelength. It is also denoted by λ_C, where λ_C =2.42pm.

The kinetic energy of the recoil electron:

The kinetic energy of the recoil electron is,

$$E_r = h\nu - h\nu'$$

Where, $h\nu$ is the energy of the incident photon and $h\nu'$ is that of the scattered photon.

From equation (5), the change in wavelength in a Compton scattering is given by,

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta)$$

$$\Rightarrow \frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_e c}(1 - cos\theta)$$

$$\Rightarrow \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_e c^2}(1 - cos\theta)$$

$$\Rightarrow \frac{1}{h\nu'} - \frac{1}{h\nu} = \frac{1}{m_e c^2}(1 - cos\theta)$$

$$\Rightarrow \frac{1}{h\nu'} = \frac{1}{h\nu} + \frac{1}{m_e c^2}(1 - cos\theta)$$

$$\Rightarrow \frac{1}{h\nu'} = \frac{m_e c^2 + h\nu(1-cos\theta)}{h\nu m_e c^2}$$

$$\Rightarrow h\nu' = \frac{h\nu m_e c^2}{m_e c^2 + h\nu(1-cos\theta)}$$

$$\Rightarrow h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1-cos\theta)}$$

$$\Rightarrow h\nu' = \frac{h\nu}{1 + \alpha(1-cos\theta)}$$

$$\text{Where, } \alpha = \frac{h\nu}{m_e c^2}$$

Hence, the kinetic energy of the recoil electron is,

$$E_r = h\nu - h\nu'$$

$$= h\nu - \frac{h\nu}{1 + \alpha(1-cos\theta)}$$

$$= h\nu \left[1 - \frac{1}{1 + \alpha(1-cos\theta)}\right]$$

$$= h\nu \left[\frac{1 + \alpha(1-cos\theta) - 1}{1 + \alpha(1-cos\theta)}\right]$$

$$= h\nu \left[\frac{\alpha(1-cos\theta)}{1 + \alpha(1-cos\theta)}\right]$$

$$= h\nu \left[\frac{\alpha 2 sin^2 \theta/2}{1 + \alpha 2 sin^2 \theta/2} \right]$$

$$Er = h\nu \left[\frac{2\alpha sin^2 \theta/2}{1 + 2\alpha sin^2 \theta/2} \right]$$

Mathematical Problem

X-rays of wavelength 10.0 pm are scattered from a target. (*a*) Find the wavelength of the x-rays scattered through 45°. (*b*) Find the maximum wavelength present in the scattered x-rays. (*c*) Find the maximum kinetic energy of the recoil electrons.

Solution

(*a*) From Eq. (2.23), $\lambda' - \lambda = \lambda_C(1 - \cos\phi)$, and so

$$\begin{aligned}\lambda' &= \lambda + \lambda_C(1 - \cos 45°)\\ &= 10.0\text{ pm} + 0.293\lambda_C\\ &= 10.7\text{ pm}\end{aligned}$$

(*b*) $\lambda' - \lambda$ is a maximum when $(1 - \cos\phi) = 2$, in which case

$$\lambda' = \lambda + 2\lambda_C = 10.0\text{ pm} + 4.9\text{ pm} = 14.9\text{ pm}$$

(*c*) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$\text{KE}_{\max} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where λ' is given in (*b*). Hence

$$\begin{aligned}\text{KE}_{\max} &= \frac{(6.626 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^{8}\text{ m/s})}{10^{-12}\text{ m/pm}}\left(\frac{1}{10.0\text{ pm}} - \frac{1}{14.9\text{ pm}}\right)\\ &= 6.54 \times 10^{-15}\text{ J}\end{aligned}$$

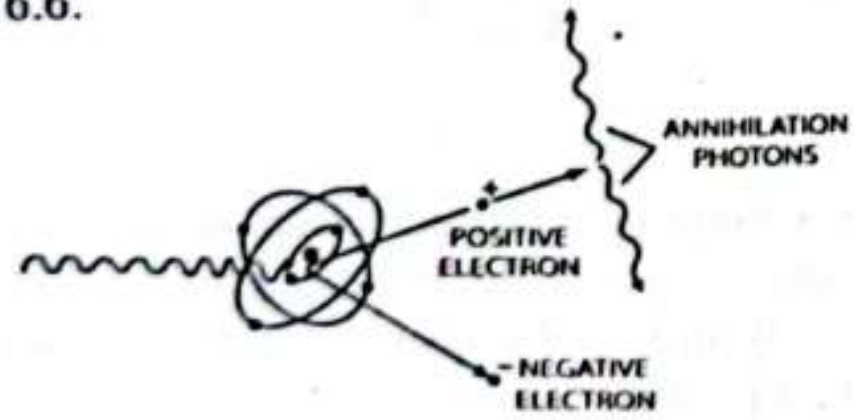
which is equal to 40.8 keV.

PAIR PRODUCTION

Pair production: In this process of absorption, a γ-ray photon disappears in the Coulomb field of an atomic nucleus and an electron-positron pair appears in its place. Since the energy equivalent to the mass of an electron is 0.51 MeV, the creation of two electrons requires 1.02 MeV. Consequently, photons with energy less than 1.02 MeV do not interact by pair production. During pair production, energy in excess of 1.02 MeV is released as kinetic energy of the two electrons.

$$h\nu\,(\text{MeV}) = 1.02 + (E_k)_{e^-} + (E_k)_{e^+}$$

Although the nucleus recoils slightly during pair production, the small amount of energy transferred to the recoiling nucleus is usually neglected. Pair production is depicted in Fig. 16.6.



PAIR PRODUCTION

Occasionally, pair production occurs near the coulomb field of an electron rather than that of the nucleus. An interaction near an electron is termed *triplet production*, because the electron receives energy from the photon and is ejected from the atom. Hence, three electrons, two negative and one positive, are released during triplet production. To conserve momentum, the threshold energy for triplet production must be 2.04 MeV. The ratio of triplet to pair production increases with the energy of the incident photons and decreases as the atomic number of the medium is increased.