Modern Physics

(Lec-9)

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OUTLINES

- Expression for Bohr radii
- Expression for orbital energy for hydrogen atom

Radius of and total energy of atom on the basis of Bohr atomic model:

Let us consider the motion of an electron of mass 'm' and charge 'e' moving uniformly with speed 'v', in a circular orbit of radius 'r' around a nucleus of charge Ze. Then it experiences an acceleration v^2/r towards the center of the circle.

The electrostatic force of attraction acts between the nucleus and the revolving electron and tend to attract the electron towards the nucleus. According to Coulomb's law this force is given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{Ze.e}{r^2}$$

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \qquad (1)$$

The centripetal force $F_c = \frac{mv^2}{r}$ is holding the electron in the orbit from the nucleus is provided by the electrostatic force F_e between them.

Bohr's postulates states that, the electrostatic force of attraction provides necessary centripetal force for revolution of electron in the orbit, *i.e.*

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{Ze^2}{4\pi\varepsilon_0 mr}$$

$$\therefore \quad v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 mr}} \qquad (2)$$

Again, according to the Bohr postulate, the orbital angular momentum of the revolving electron must be equal to the integer multiple of $\frac{h}{2\pi}$ *i.e.*,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad v = \frac{nh}{2\pi mr} \qquad (3)$$

where n (= 1, 2, 3, 4,.....)

Equating equations (2) and (3) we get

Cont....

$$\frac{nh}{2\pi mr} = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 mr}}$$

Squaring $$\frac{n^2h^2}{4\pi^2m^2r^2} = \frac{Ze^2}{4\pi\varepsilon_0 mr}$$

$$r = \frac{n^2h^2\varepsilon_0}{\pi mZe^2}$$

$$\therefore \quad r_n = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} \qquad (4)$$

Where r_n is the radius of the n^{th} orbital electron.

For Hydrogen atom $Z = 1$. Thus, the radius of the n^{th} orbit of Hydrogen atom is

$$\therefore \quad r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2} \qquad (5)$$

Now substituting the value of r from equation (4) in equation (2) we get

$$v_n = \frac{nh}{2\pi m \times \frac{n^2h^2\varepsilon_0}{\pi Zme^2}}$$

$$\therefore \quad v_n = \frac{Ze^2}{2\pi\varepsilon_0 nh} \qquad (6)$$

Where v_n is the velocity of the n^{th} orbital electron.

The kinetic energy of the revolving electron is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m \times \frac{Ze^2}{4\pi\varepsilon_0 mr}$$

$$\therefore \quad KE = \frac{Ze^2}{8\pi\varepsilon_0 r} \qquad (7)$$

Cont…

and the potential energy of the electron is

$$PE = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} \qquad (8)$$

The total energy E of the revolving electron in a stable orbit is given by

$$E = KE + PE$$

$$E = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$\therefore \quad E = -\frac{Ze^2}{8\pi\varepsilon_0 r} \qquad (9)$$

Now substituting the value of r from equation (4) in equation (9) we get

$$E_n = -\frac{Ze^2}{8\pi\varepsilon_0 \times \frac{n^2h^2\varepsilon_0}{\pi mZe^2}}$$

$$\therefore \quad E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2h^2} \qquad (10)$$

Where E_n is the energy of the n^{th} orbital electron.

For Hydrogen atom $Z = 1$. Thus, the energy of the n^{th} orbit of Hydrogen atom is

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2h^2} \qquad (11)$$

For $n = 1$, we can write equation (11) as

$$E_1 = -\frac{me^4}{8\varepsilon_0^2 h^2} \qquad (12)$$

From equation (11) and (12) we can write

$$E_n = \frac{E_1}{n^2}$$

When an electron jumps from upper energy level n_2 to lower energy level n_1, we can write

$$\Delta E = E_{n_2} - E_{n_1} \tag{13}$$

From equation (10) we can write

$$E_{n_2} = -\frac{mZ^2e^4}{8\varepsilon_0^2 n_2^2 h^2} \tag{14}$$

and

$$E_{n_1} = -\frac{mZ^2e^4}{8\varepsilon_0^2 n_1^2 h^2} \tag{15}$$

From equation (13), (14) and (15) we can write

$$\Delta E = -\frac{mZ^2e^4}{8\varepsilon_0^2 n_2^2 h^2} + \frac{mZ^2e^4}{8\varepsilon_0^2 n_1^2 h^2}$$

$$\Delta E = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{16}$$

Where, ΔE is the energy of the emitted photon.

For Hydrogen atom $Z = 1$. Thus, the energy of the Hydrogen atom in this case is

$$\Delta E = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{17}$$



Problem-1: Calculate the radius of the first Bohr orbit for H and He atoms.

Problem-2: Calculate the radius of the third orbit of hydrogen atom. Also calculate the energy of electrons in that orbit. (Given $h = 6.62 \times 10^{-34}\, JS,\ m = 9.1 \times 10^{-31} kg,$

$e = 1.6 \times 10^{-19}$ c)

Problem-3: How much energy is required to remove an electron from n=3 or n=2 state in Hydrogen atom.