Optics

(Lec-6)

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Date: 26.06.2023

OUTLINES

- Brewster's Law
- Malus' Law
- Mathematical Problem

Brewster's Law

- In 1811 Brewster's proposed it. The law states that "*When unpolarized light is incident at polarizing angle, the tangent of polarizing angle is numerically equal to the Refractive index of the material*"

If θ_p is the angle and μ is the refractive index of the medium, then

$$\mu = \tan \theta_p$$

This is known as Brewster's law.

Mathematical derivation:

If natural light is incident on a smooth surface at the polarizing angle, it is reflected along PC and refracted along PB, as shown in figure. Brewster found that the maximum polarization of reflected ray occurs when it is at right angle to the refracted ray.

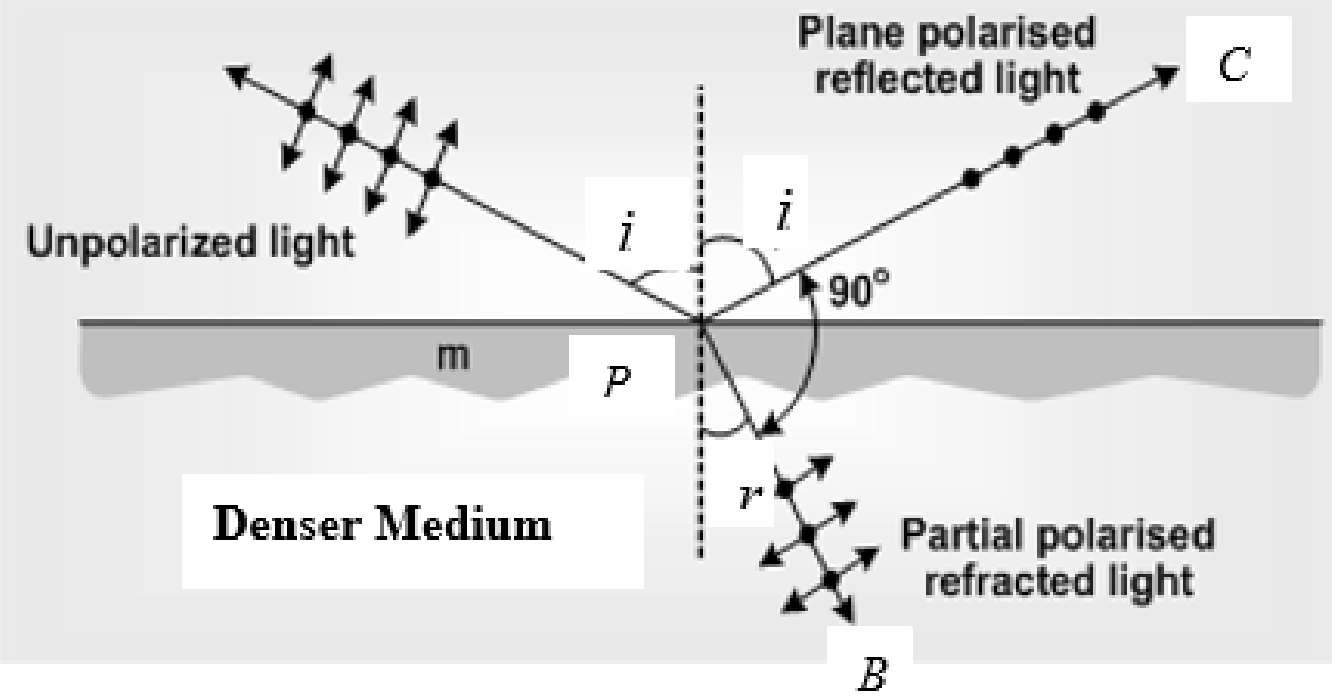


Fig. Production of plane polarized light by the method of reflection.

It means that,

$$i + r = 90°$$

$$\therefore r = 90° - i \ldots\ldots\ldots\ldots (1)$$

Where, i and r represents the incident angle and the refractive angle respectively.

According the Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \ldots\ldots\ldots\ldots (2)$$

Where μ_2 and μ_1 are the absolute refractive index of the reflecting surface and the surrounding medium.

It follows from equation (1) and (2) that

$$\frac{\sin i}{\sin(90° - i)} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\sin i}{\cos i} = \frac{\mu_2}{\mu_1}$$

$$\therefore \tan i = \frac{\mu_2}{\mu_1}$$

Equation (3) shows that the polarizing angle depends on the refractive index of the reflecting surface. The polarizing angle i is also known as Brewster angle and denote as θ_B. Light reflected from any angle other than Brewster angle is partially polarized.

Problems:

1) The refractive index for plastic is 1.25. Calculate the angle of refraction for a ray of light incident at polarizing angle.

2) The critical angle for certain wavelength of light in the case of a piece of glass is 40^o. Find the polarizing angle for glass.

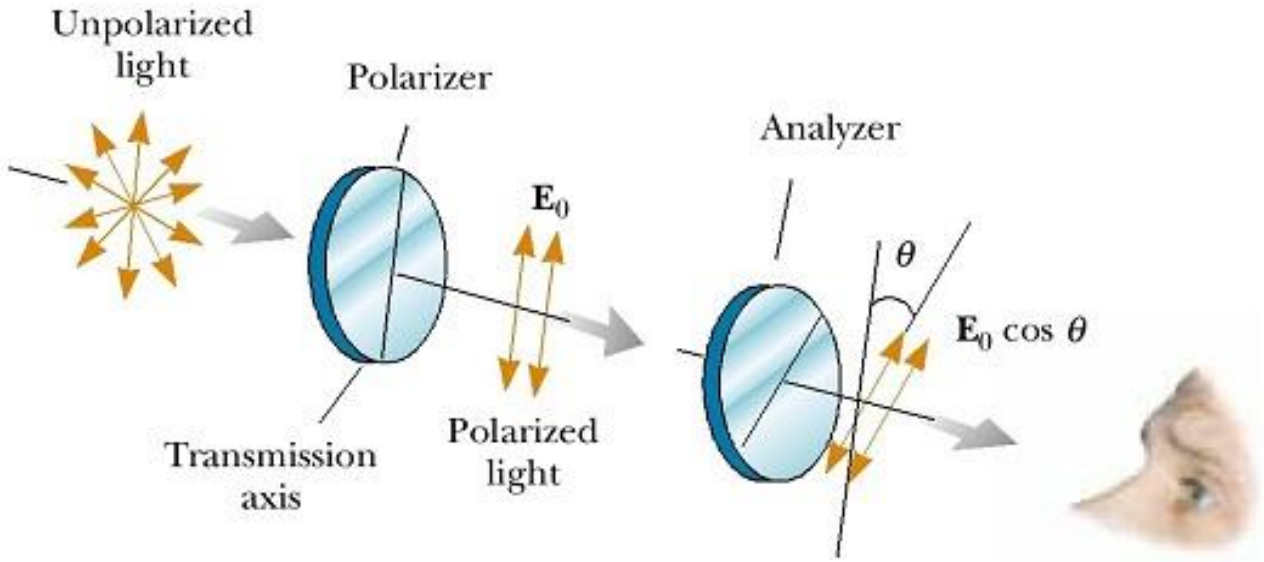
Malus's Law:

According to Malus, "*when completely plane polarized light is incident on the analyzer, the intensity I of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer.*

$$i.e. I \propto \cos^2 \theta$$

Suppose the angle between the transmission axes of the analyzer and the polarizer is θ. If E_0 is the amplitude of the electric vector transmitted by the polarizer, then intensity I_0 of the light incident on the analyzer is

$$I_0 \propto E_0^2$$



The electric field vector E_0 can be resolved into two components, $E_0 cos\theta$ and $E_0 sin\theta$. The analyzer will transmit only the component $(i.e\, E_0 cos\theta)$ which is parallel to its transmission axis. However, the component $E_0 sin\theta$ will be absorbed by the analyzer. Therefore, the intensity I of light transmitted by the analyzer is,

$$I \propto (E_0 cos\theta)^2$$

$$\Rightarrow I \propto E_0^2 \cos^2\theta$$

$$\Rightarrow I \propto I_0 \cos^2\theta$$

$$\therefore I \propto \cos^2\theta$$

This proves Malus' law.

When $\theta = 0°\ (or\ 180°), I = I_0 \cos^2 0° = I_0$ That is the intensity of light transmitted by the analyzer is maximum when the transmission axes of the analyzer and the polarizer are parallel.

When $\theta = 90°, I = I_0 \cos^2 90° = 0$ That is the intensity of light transmitted by the analyzer is minimum when the transmission axes of the analyzer and polarizer are perpendicular to each other.

Problem: If unpolarized light falls on two polarizing sheets placed one on top of the other, what must be the angle between the characteristic's directions of the sheet if the intensity of the transmitted light is 1/3 of the maximum intensity of the incident beam?