Optics

(Lec-6)

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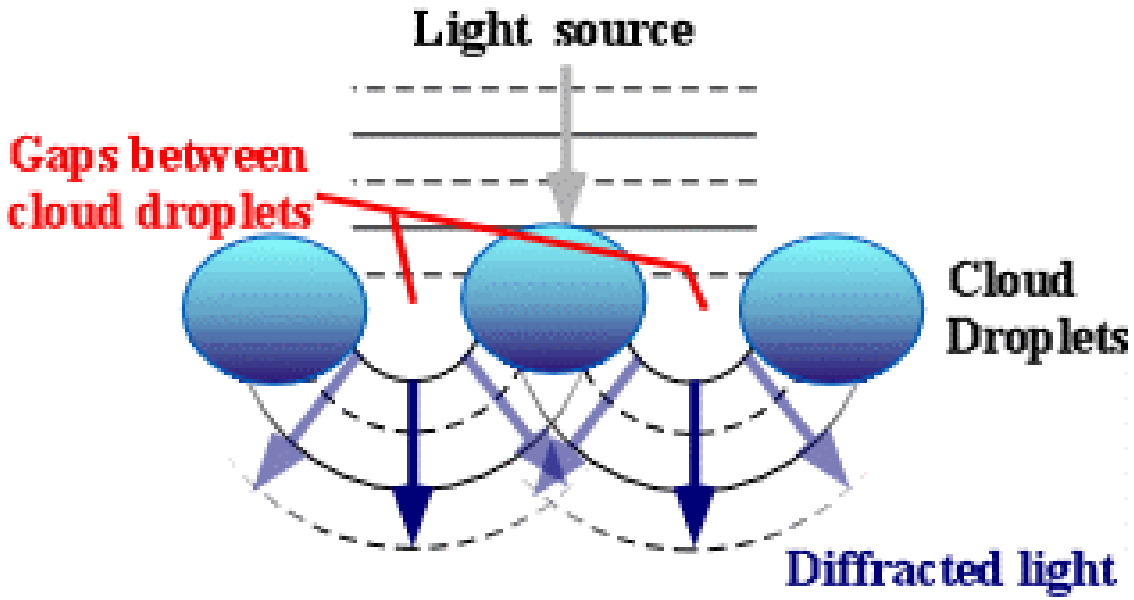
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OUTLINES

- Diffraction
- Fresnel & Fraunhofer diffraction
- Fraunhofer diffraction by a single slit

Diffraction

Diffraction is the slight bending of light as it passes around the edge of an object. The amount of bending depends on the relative size of the wavelength of light to the size of the opening. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. However, if the two are closer in size or equal, the amount of bending is considerable, and easily seen with the open eye.



Condition of diffraction

There are two conditions for the production of diffraction

- In case of straight edge: The edge should be very sharp and its width is to be equal to or is of the order of the wavelength, λ of light.
- In case of thin hole: the diameter of the hole should be extremely very small such that it is equal to or is of the order of the wavelength λ of light.

Diffraction is of two types

(1) Fresnel's class of diffraction

(2) Fraunhofer's class of diffraction

Fresnel's class of diffraction

When the source of light and the screen are at finite distance from the obstacle, then the diffraction observed due to the obstacle is called the Fresnel class of diffraction.

In this type of diffraction wave fronts are generally spherical or cylindrical. This type of diffraction occurs in a straight edge, fine wire and narrow slit.

Fraunhofer's class of diffraction

When the source of light and the screen are effectively at infinite distance from the obstacle or aperture causing diffraction, then that type of diffraction is called Fraunhofer's class of diffraction.

In this type of diffraction wave fronts incident on the obstacle or aperture is plane.

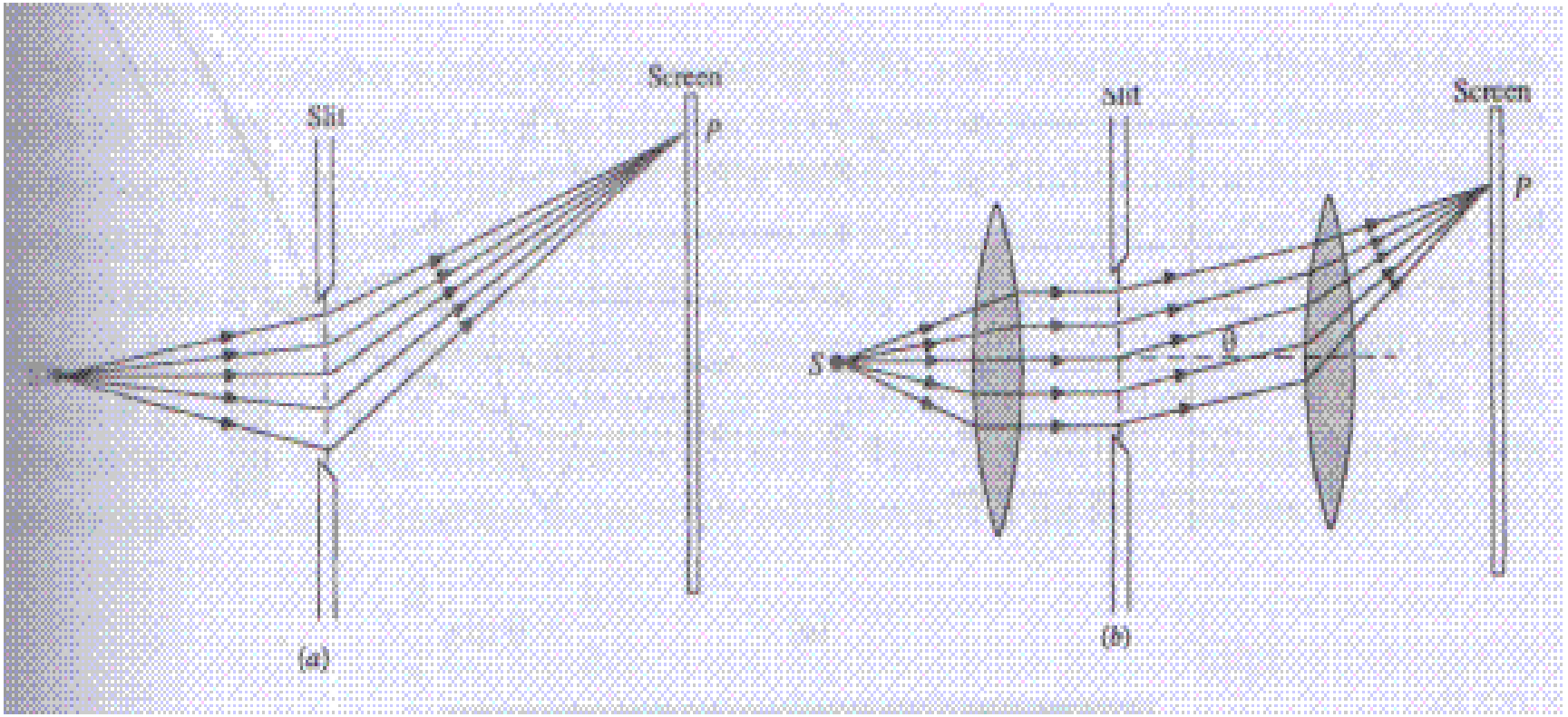


Fig: (a) Fresnel class diffraction, (b) Fraunhofer class diffraction.

Fraunhofer diffraction by a single slit

Let a beam of parallel monochromatic light be incident normally on an opaque plate in which there is a slit. Let **a** be the width of the slit and **ds** be an element of width of the wavefront in the plane of the slit, at a distance s from the center O, which shall be referred to as the origin. The part of each wavelet travelling normally to the slit, are brought to focus at P_o by the lens while the parts, travelling at an angle θ with the normal are brought to focus at P. AB, CD, EF etc. can be regarded as parallel rays reflected at an angle θ.

Let us first consider the wavelet emitted by the element **ds** situated at the origin then the equation of a spherical wave as,

$$y = \frac{a}{r} \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

where a is the amplitude. The wavelet emitted by the element ds produce an infinitesimal displacement dy at p_o represented by the equation,

$$dy = \frac{r.ds}{x} \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Where r.ds is the amplitude of the wave.

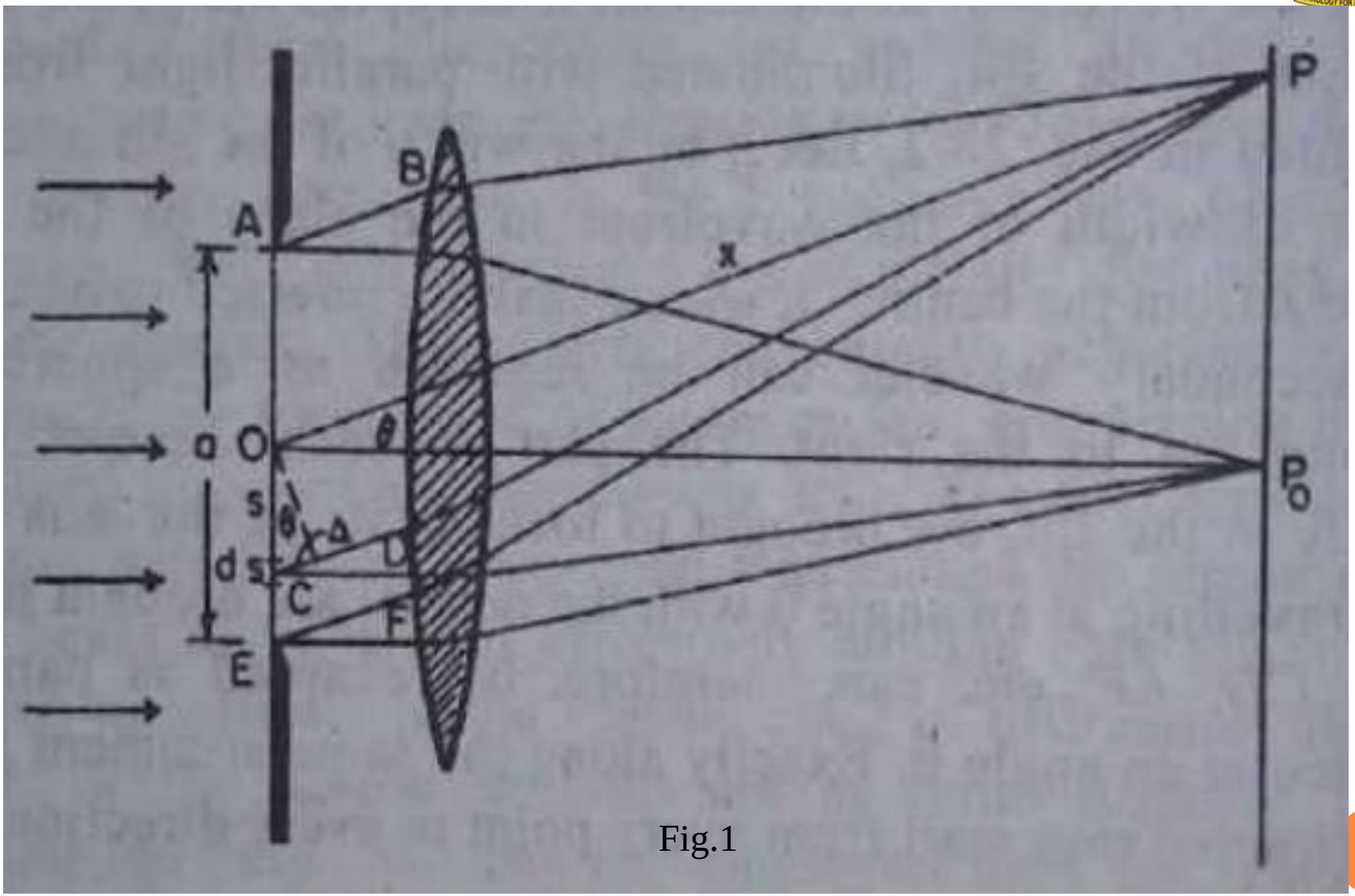


Fig.1

The part of this wavelet reaching P traverses an optical path X_n and produces a vibration at P given by

$$dy_o = \frac{rds}{x} \sin 2\pi \left(\frac{t}{T} - \frac{X_n}{\lambda}\right)$$

When ds is at a distance s below the origin then

$$dy_s = \frac{rds}{x} \sin 2\pi \left(\frac{t}{T} - \frac{X_n + \Delta x}{\lambda}\right)$$

$$= \frac{rds}{x} \sin 2\pi \left(\frac{t}{T} - \frac{X_n}{\lambda} - \frac{s \sin\theta}{\lambda}\right)$$

Where, $\Delta x = s \sin\theta$ is the additional distance.

Now, integrating between the limits $s = -\frac{a}{2}$ and $s = +\frac{a}{2}$ and amplitude factor r/x may dropped because it is practically same for all wavelets, then we get,

$$y = \int_{-\frac{a}{2}}^{+\frac{a}{2}} dy_s$$

$$= \int_{-a/2}^{+a/2} \sin 2\pi \left(\frac{t}{T} - \frac{x_n}{\lambda} - \frac{s \sin \theta}{\lambda} \right) ds \qquad (28.4)$$

Assuming $\phi = 2\pi \left(\frac{t}{T} - \frac{x_n}{\lambda} \right)$ and $\psi = 2\pi s \frac{\sin\theta}{\lambda}$, we have

$$y = \int_{-a/2}^{+a/2} \sin (\phi - \psi)\, ds$$

$$= \int_{-a/2}^{+a/2} (\sin \phi \cos \psi - \cos \phi \sin \psi)\, ds$$

$$= \int_{-a/2}^{+a/2} \left[\sin \phi \cos \frac{2\pi \sin \theta}{\lambda} . s - \cos \phi \sin \frac{2\pi \sin \theta}{\lambda} s\right] ds$$

now $\int \sin mx\, dx = -\cos \frac{mx}{m}$ and $\int \cos mx\, dx = \frac{\sin mx}{m}$

Hence

$$y = \left[\sin \omega \frac{\frac{2\pi \sin\theta}{\lambda} s}{\frac{2\pi \sin\theta}{\lambda}} + \cos \omega \frac{\frac{\cos 2\pi \sin\theta}{\lambda} s}{\frac{2\pi \sin\theta}{\lambda}} \right]_{-a/2}^{+a/2}$$

$$= \left[\frac{s \sin\psi \sin\omega}{\psi} + \frac{s \cos\psi \cos\omega}{\psi} \right]_{-a/2}^{+a/2}$$

when $s = +a/2$ $\psi = \dfrac{\pi a \sin\theta}{\lambda}$

$s = -a/2$ $\psi = -\dfrac{\pi a \sin\theta}{\lambda}$

$$\therefore \quad y = \left[\frac{a}{2} \frac{\sin(\pi a \sin\theta)/\lambda}{(\pi a \sin\theta)/\lambda} \sin\omega + \frac{a}{2} \frac{\sin(\pi a \sin\theta)/\lambda}{(\pi a \sin\theta)/\lambda} \sin\omega \right.$$

$$\left. + \frac{a}{2} \frac{\cos(\pi a \sin\theta)/\lambda}{(\pi a \sin\theta)/\lambda} \cos\omega - \frac{a}{2} \frac{\cos(\pi a \sin\theta)/\lambda}{(\pi a \sin\theta)/\lambda} \cos\omega \right]$$

$$= a \frac{\sin(\pi a \sin\theta)/\lambda}{(\pi a \sin\theta)/\lambda} \sin\omega$$

$$= a \frac{\sin(\pi a \sin\theta)/\lambda}{(\pi a \sin\theta)/\lambda} \sin 2\pi \left(\frac{t}{T} \frac{x_n}{\lambda} \right)$$

This equation represent a new simple periodic motion of aplitude

$$R = (a \sin \beta) / \beta \quad \text{where } \beta = (\pi / \lambda)\, a \sin\theta$$

Due to the neglect of the factor r/x durig the summing up process, this expression for R differs by some arbitary constant from the true amplitude. It will be more correct to write

$$R = R_o \frac{\sin \beta}{\beta}$$

Position for principle maxima:

On writing the expansion of sin β in ascending powers of β, the expression for the resultant amplitude R becomes

$$R = \frac{R_o}{\beta}\left(\beta - \frac{\beta^3}{3!} + \frac{\beta^5}{5!} - \frac{\beta^7}{7!} + \ldots\ldots\ldots\ldots\right)$$

$$= R_o\left(1 - \frac{\beta^2}{3!} + \frac{\beta^4}{5!} - \frac{\beta^6}{7!} + \ldots\ldots\ldots\ldots\right)$$

It is obvious that R will be maximum, if the negative terms vanish. This is only possible when

$$\beta = \frac{\pi a \sin\theta}{\lambda} = 0; \quad \text{or,} \quad \theta = 0.$$

For β = 0, the quotient sin β/β becomes indeterminate but it should be remembered that sin θ approaches θ for small angles and is equal to it when θ vanishes. Hence for β = 0 (*i.e.*, θ = 0), R = R Thus the maximum value of R is R_o; it represents the amplitude when all the wavelets arrive in phase.

Position of minimum intensity

The intensity will be zero in the diffraction pattern at the positions where $\sin \beta = 0$. The values of β which satisfies the above condition are $\beta = m\pi$ where $m = \pm 1, \pm 2, \pm 3, \ldots\ldots$ which gives on substituting the value of β

$$\frac{a\pi \sin\theta}{\lambda} = \pm m\pi$$

or, $a \sin\theta = \pm m\lambda$.

where $m = 1, 2, 3,$

The value $m = 0$ is not admissible, because the value $\theta = 0$ corresponds to the principal maximum.

The first minimum occurs at

$\theta = \pm \sin^{-1}\left(\frac{\lambda}{a}\right)$, the second minimum at

$\theta = \pm \sin^{-1}\left(\frac{2\lambda}{a}\right)$, etc. In words, the rays diffracted at an angle θ interfere destructively if the path difference between the extreme diffracted rays is an integral multiple of λ.