Optics

(Lec-7)

Abdul Barik
Lecturer
Department of Science & Humanities
Military Institute of Science and Technology (MIST)

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OUTLINES

- Fraunhofer diffraction by a double slit
- Diffraction Grating

Fraunhofer diffraction by a double slit

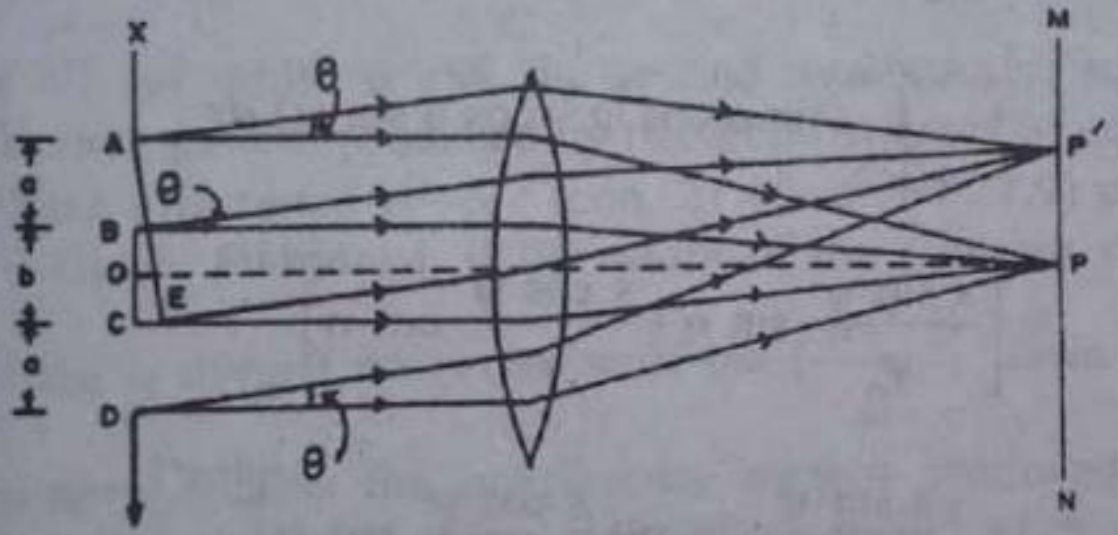


Fig. 28.5

In Fig. 28.5 *AB* and *CD* are two rectangular slits parallel to one another and perpendicular to the plane of the paper. Let *a* be the width of each slit and *b* be the width of the opaque portion separating them. *P* is a point on the screen *MN* such that *OP* is perpendicular to the screen. When a plane wavefront is incident on the surface of the slits, the transmitted rays are brought to focus at different points on the screen by the convex lens *L*. All secondary waves travelling in a direction parallel to *OP* will be brought to focus at *P*, which, therefore, corresponds to the position of the central bright maximum.

Thus, if the origin is chosen as the centre of one of the slits, then the integration is to extend from $s = -\frac{a}{2}$ to $+\frac{a}{2}$ and from $(d - \frac{a}{2})$ to $(d + \frac{a}{2})$, where d = a + b.

$$y = \int_{-a/2}^{+a/2} \sin(\omega - \psi)\, ds + \int_{d-a/2}^{d+a/2} \sin(\omega - \psi)\, ds$$

where $\omega = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ and $\psi = 2\pi s \frac{\sin\theta}{\lambda}$

$$y = \int_{-a/2}^{+a/2} (\sin\omega\cos\psi - \cos\omega\sin\psi)\, ds$$

$$+ \int_{d-a/2}^{d+a/2} (\sin\omega\cos\psi - \cos\omega\sin\psi)\, ds$$

$$= \left[\frac{s\sin\psi}{\psi}\sin\omega + \frac{s\cos\psi}{\psi}\cos\omega\right]_{-a/2}^{+a/2}$$

$$+ \left[\frac{s\sin\psi}{\psi}\sin\omega + \frac{s\cos\psi}{\psi}\cos\omega\right]_{d-a/2}^{d+a/2}$$

Substituting in the limits and combining the terms, we obtain

$$y = 2a\,\frac{\sin(\pi a\sin\theta)/\lambda}{(\pi a\sin\theta)/\lambda}\cos(\pi d\sin\theta)/\lambda$$

$$\times \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda} - \frac{d\sin\theta}{2\lambda}\right)$$

$$= 2a\,\frac{\sin\beta}{\beta}\cos\gamma\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda} - \frac{d\sin\theta}{2\lambda}\right)$$

where, as before,

$$\beta = \pi a \sin\theta/\lambda$$

and where $\gamma = \dfrac{\pi d \sin\theta}{\lambda}$

Replacing a by R_o as before,

$$y = 2R_o \frac{\sin\beta}{\beta} \cos\gamma \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} - \frac{d \sin\theta}{2\lambda}\right) \qquad (28.10)$$

The intensity is proportional to the square of the amplitude in the above equation. Hence,

$$I = 4R_o^2 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma \qquad (28.11)$$

Positions of maxima and minima

Eqn. (28.11) tells us that the resultant intensity will be zero when either of the two terms is zero. For the first term this will occur when

$$\beta = \pi, 2\pi, 3\pi, \ldots\ldots$$

and for the second term, when

$$\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2},$$

Since by definition $\gamma = (\pi/\lambda)\, d \sin\theta$, and

$$\beta = (\pi/\lambda)\, a \sin\theta,$$

the corresponding angles of diffraction are given by the following relations

$$a \sin\theta = \lambda, 2\lambda, 3\lambda, \ldots\ldots = m\lambda \qquad \text{minima}$$

where $m = 1, 2, 3 \ldots\ldots\ldots\ldots$

and $$d \sin\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2},$$

$$= (n + \frac{1}{2})\,\lambda \qquad \text{minima}$$

where $n = 0, 1, 2, 3$, etc. This equation simply expresses the condition that for minima, [redacted] positions of the maxima will then be determined solely by the $\cos^2\gamma$ factor. The interference maxima then occur

when $\gamma = 0, \pi, 2\pi$

or, when $d \sin\theta = 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots n\lambda$ maxima

The whole number n represents physically the number of wavelengths in the path difference from corresponding points in the two slits and represents the order of interference. *Points separated by a distance $(a + b)$ in the two slits are known as corresponding points.*

Diffraction Grating

Any arrangement which is equivalent in its action to a large number of parallel equidistant slits of the same width is known as a diffraction grating. The deflection gratings are of two types transmission type and the reflection type.

“The diffraction grating is a useful device for analyzing light sources. It consists of a large number of equally spaced parallel slits. “It’s working principle is based on the phenomenon of diffraction. The space between lines act as slits and these slits diffract the light waves thereby producing a large number of beams which interfere in such a way to produce spectra.

