Optics

(Lec-4)

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OUTLINES

- Interference of light
- Young's double slit experiment

Young's Double Slit Experiment

In 1801 an English physicist named Thomas Young performed an experiment that strongly inferred the wave-like nature of light.

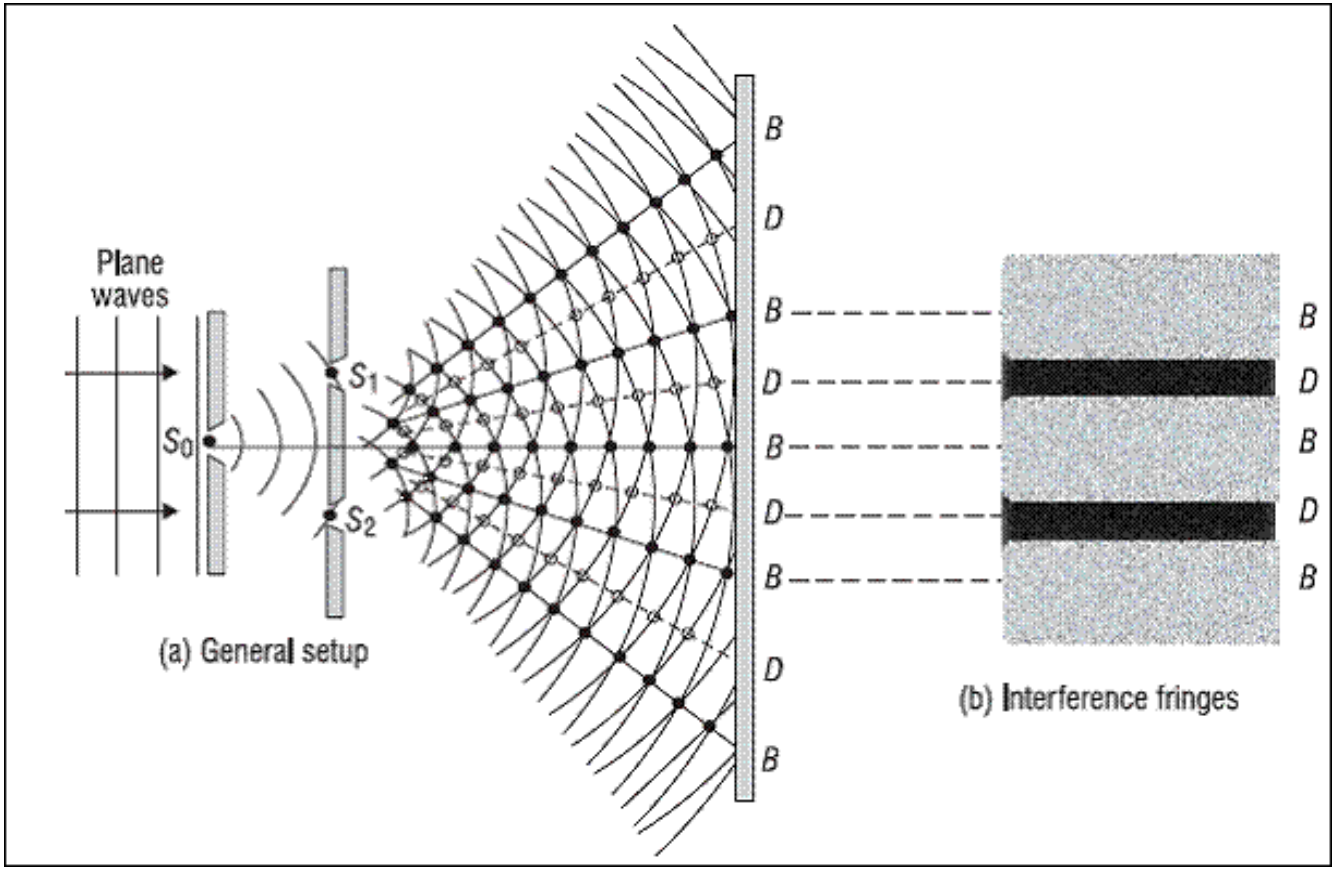


Fig: Young's double slit experiment

Thomas Young demonstrated the experiment on the interference of light. He allowed light to fall on a pinhole S_0 and then at some distance away on two pinholes S_1 and S_2. S_1 and S_2 are equidistant from S_0 and are close to each other. Spherical waves spread out from S_0. Spherical waves also spread out from S_1 and S_2. These waves are of the same amplitude and wavelength.

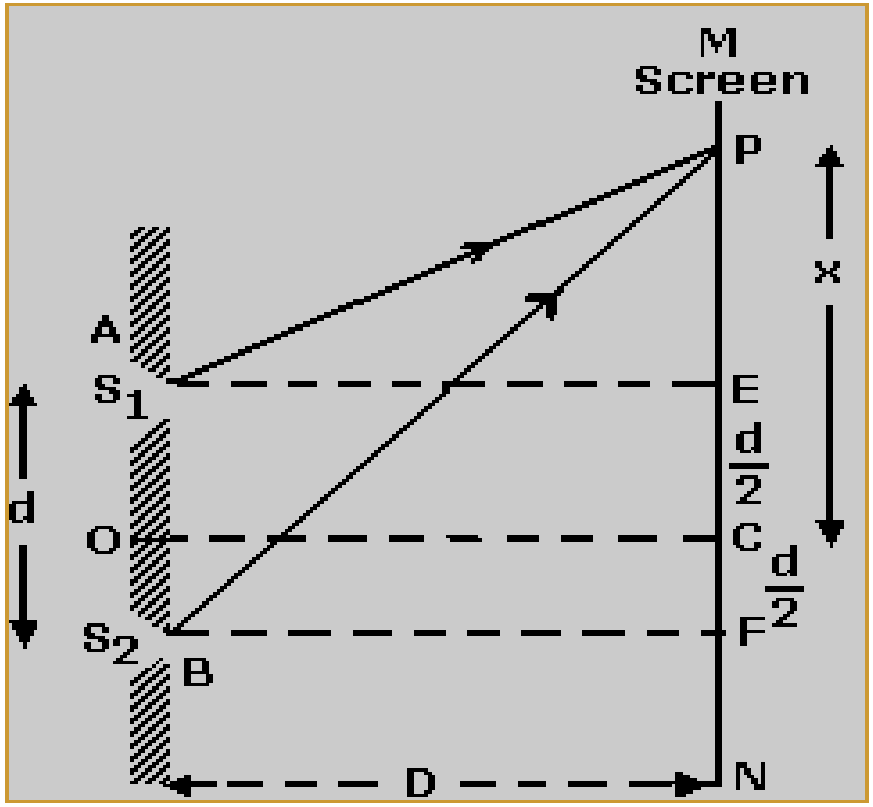
On the screen, interference bands are produced which are alternatively dark and bright. The points such as B are bright because the crest due to the one wave coincides with the rest due to the other and therefore, they reinforce with each other. The points such as D are dark because of the crest of one fall to on the trough of the other and they neutralize the effect of each other. Points, similar to B, where the trough of one fall on the trough of the other, are also bright because the two waves reinforce.

It is not possible to show interference due to two independent sources of light, because a large number of difficulties are evolved. The two sources may emit light waves of largely different amplitude and wave length and the phase difference between the two may change with time.

Theory of Interference Fringes

Let us consider a narrow monochromatic source S and two pinholes S_1 and S_2 equidistant from S. S_1 and S_2 acts as two coherent sources separated by a distance d.

Let a screen be placed at a distance D from the coherent source. The point C on the screen is equidistant from S_1 and S_2. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.



Let us consider a point P at a distance x from C. The waves reach at the point P from A and B.

Here,

$$PE = x - \frac{d}{2}$$

$$PF = x + \frac{d}{2}$$

Now from the triangle PAE we get,

$$(AP)^2 = D^2 + \left(x - \frac{d}{2}\right)^2 \text{ ----------(1)}$$

Similarly, from the triangle PBF we get

$$(BP)^2 = D^2 + \left(x + \frac{d}{2}\right)^2 \text{ ----------(2)}$$

Subtracting equation (1) from equation (2)

$$(BP)^2 - (AP)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

$$or, (BP)^2 - (AP)^2 = D^2 + x^2 + xd + \frac{d^2}{4} - D^2 - x^2 + xd - \frac{d^2}{4}$$

$$or, (BP)^2 - (AP)^2 = 2xd$$

$$or, (BP)^2 - (AP)^2 = 2xd$$

$$or, (BP - AP)(BP + AP) = 2xd$$

$or, (BP - AP) = 2xd/(BP + AP)$----------(3)

As, $d \ll D$. So, we can assume that $BP \approx AP \approx D$. So, from equation (3) we can write-

$$(BP - AP) = 2xd/(D + D)$$

$$or, (BP - AP) = 2xd/(D + D)$$

$$\therefore (BP - AP) = xd/D$$

But $(BP - AP)$ is the path difference of the light at point P.

4 The path difference $= xd/D$ ----------(4)

Again, we know, the phase difference $= \frac{2\pi}{\lambda} \times$ the path difference.

So, the phase difference of the light at point P $= \frac{2\pi}{\lambda} \times \frac{xd}{D}$ ----------(5)

Condition for bright fringes:

If the path difference is a whole number multiple of wavelength λ, then the points will be bright.

$$\text{i.e. } \frac{x\,d}{D} = n\,\lambda \qquad \text{Where n} = 0, 1, 2, 3\ldots\ldots\ldots\ldots$$

$$4x = \frac{n\,\lambda\,D}{d} \text{----------(6)}$$

This equation gives the distances of the bright fringes from the point C. At C, the path difference is zero and a bright fringe is formed.

$$\text{When } n = 0, \quad x_0 = 0$$

$$n = 1, \quad x_1 = \frac{\lambda\,D}{d}$$

$$n = 2, \quad x_2 = \frac{2\,\lambda\,D}{d}$$

$$n = 3, \quad x_3 = \frac{3\,\lambda\,D}{d}$$

.....

$$n = n, \quad x_n = \frac{n\,\lambda\,D}{d}$$

Therefore, the distance between any two consecutive bright fringes

$$x_2 - x_1 = \frac{2\,\lambda\,D}{d} - \frac{\lambda\,D}{d} = \frac{\lambda\,D}{d} \text{ ----------(7)}$$

Condition for dark fringes:

If the path difference is an odd number multiple of half wavelength, then the point will be dark.

$$\text{i.e., } \frac{x\,d}{D} = \frac{2n+1}{2}\lambda$$

$$4x = \frac{(2n+1)\,\lambda\,D}{2d}$$

This equation gives the distances of the dark fringes from the point C.

$$\text{When, } n = 0,\ x_0 = \frac{\lambda D}{2d}$$

$$n = 1,\ x_1 = \frac{3\,\lambda\,D}{2\,d}$$

$$n = 2,\ x_2 = \frac{5\lambda\,D}{2d}$$

$$n = 3,\ x_3 = \frac{7\,\lambda\,D}{2\,d}$$

.....

$$n = n,\ x_n = \frac{(2n+1)\,\lambda\,D}{2d}$$

The distance between any two consecutive dark fringes,

$$x_2 - x_1 = \frac{5\,\lambda\,D}{2d} - \frac{3\lambda\,D}{2d} = \frac{\lambda\,D}{d} \text{----------(8)}$$

The distance between any two-consecutive bright or dark fringes is known as fringe width.

Cont….

Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of C. Moreover, from equations (7) and (8), it is clear that the width of the bright fringe is equal to the width of the dark fringe. All the fringes are equal in width and are independent of the order of the fringe. The breadth of a bright or a dark fringe is, however, equal to half the fringe width and is equal to $\frac{\lambda D}{2d}$.

$$\text{The fringe width} = \frac{\lambda D}{d}$$

Therefore, (i) the width of the fringe is directly proportional to the wavelength of light, $\propto \lambda$. (ii) The width of the fringe is directly proportional to the distance between the two sources, $\propto \frac{1}{d}$. Thus, the width of the fringe increases (a) with increase in wavelength (b) with increase in the distance D and (c) by bringing the two sources A and B close to each other.



H. W. : Mathematical Problems