Introduction to Simple Harmonic Motion

Waves and oscillations are a fascinating and important aspect of physics, and they are present in many natural and man-made systems. Simply put, a wave is a disturbance that travels through space and time, carrying energy with it. Oscillations, on the other hand, refer to repetitive motions around a fixed position or equilibrium.

Waves and oscillations are found in a wide range of physical phenomena, from the motion of sound waves through air to the oscillation of a pendulum. They play a crucial role in many fields of science and engineering, including acoustics, optics, and electronics.

Coordinate System

A coordinate system is a mathematical system that is used to define the position and location of points in space. It consists of a set of reference points, called the origin, and a set of axes that intersect at the origin and extend to form a grid. The axes are labeled with numerical values that represent distances or magnitudes along each axis.

There are several different types of coordinate systems, each with its own set of conventions and uses. The most commonly used coordinate system is the Cartesian coordinate system, also known as the rectangular coordinate system. In this system, the origin is located at the point (0,0), and the x- and y-axes extend horizontally and vertically, respectively. Points are identified by their distances from the origin along each axis, expressed as ordered pairs (x,y).

Other types of coordinate systems include polar coordinates, in which points are identified by their distances from the origin and the angles they make with a reference axis, and spherical coordinates, in which points are identified by their distances from the origin, their angles with respect to two reference axes, and their polar angles.

Coordinate systems are used in a wide variety of fields, including mathematics, physics, engineering, and geography, to describe the location and motion of objects and to analyze data. They provide a useful framework for representing complex data sets and visualizing relationships between variables.

Cartesian Coordinate System vs Polar Coordinate System

In the Cartesian coordinate system, points are identified by their distances from two perpendicular axes, typically labeled as the x-axis and y-axis. The axes intersect at the origin, which is often marked as (0,0). Points in this system are described using ordered pairs (x, y), where x is the horizontal distance from the origin along the x-axis, and y is the vertical distance from the origin along the y-axis.

In the polar coordinate system, points are identified by their distance from a fixed point called the pole, and the angle they make with a fixed reference direction, often the positive x-axis. The distance from the pole is called the radius, denoted by r, and the angle is denoted by θ. Points in this system are described using ordered pairs (r, θ).

Each system has its own advantages and disadvantages depending on the situation. The Cartesian coordinate system is more intuitive for measuring distances and angles in linear relationships, while the polar coordinate system is more intuitive for circular and periodic relationships.

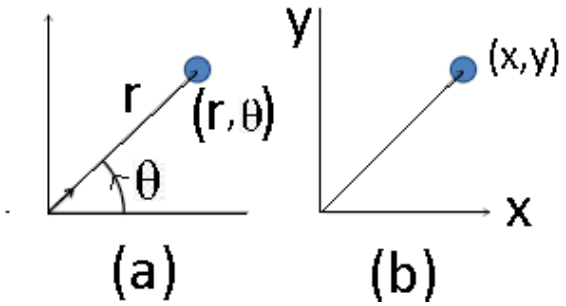
Mathematical Relation between Cartesian and Polar Coordinate System

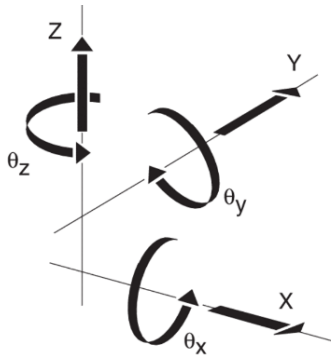
Given a point P with Cartesian coordinates (x, y) and polar coordinates (r, θ):

To convert from Cartesian to polar coordinates	To convert from polar to Cartesian coordinates
$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}(y/x)$	$x = r\cos\theta$ $y = r\sin\theta$

Note that in the polar coordinate system, the angle θ is usually measured in radians, whereas in the Cartesian coordinate system, it is measured in degrees or radians depending on the context.

Coordinate systems are used to specify the state or DOF of a system at any given time. For example, in Euclidean space, we use cartesian coordinate or polar coordinates etc. There are many possible coordinate systems of a given geometry or space. They are equally good as they are equivalent. Depending on the problem we are given, we can choose a coordinate system smartly that will make our calculation simpler.





Degrees of Freedom

In physics, the degrees of freedom refer to the number of independent variables that are needed to describe the state of a system. These variables may include positions, velocities, and other physical quantities that define the system's behavior.

For example, a particle moving in three-dimensional space has three degrees of freedom, corresponding to its position in the x, y, and z directions. A pendulum has two degrees of freedom, corresponding to the angle and angular velocity of the pendulum's motion.

In some cases, the degrees of freedom may be restricted by physical constraints or symmetry considerations. For instance, a system of two particles connected by a rigid rod in three dimensions has only three degrees of freedom, since the distance between the particles and the angle of the rod can uniquely determine the configuration of the system.

Conservation of Energy

The principle of conservation of energy states that energy cannot be created or destroyed but can only be converted from one form to another. The principle of conservation of energy implies that the total energy of a system is constant, but it does not tell us anything about how that energy is distributed among the degrees of freedom of the system. However, the laws of physics do impose constraints on how energy can be distributed among the degrees of freedom of a system. For example, in a closed system, the total energy must be evenly distributed among all of the degrees of freedom available to the system, according to the equipartition theorem. This means that each degree of freedom must, on average, have the same amount of energy.

Linear Motion vs Angular Motion

Linear motion, also known as translational motion, refers to motion in which an object moves along a straight line. This motion is described by the object's position, velocity, and acceleration in terms of the Cartesian coordinate system. Linear motion can be either uniform, where the object moves at a constant speed in a straight line, or non-uniform, where the object's speed and/or direction change over time.

Angular motion, also known as rotational motion, refers to motion in which an object rotates about an axis. This motion is described by the object's angular displacement, angular velocity, and angular acceleration in terms of the polar coordinate system. Angular motion can be either uniform, where the object rotates at a constant rate, or non-uniform, where the object's rate of rotation changes over time.

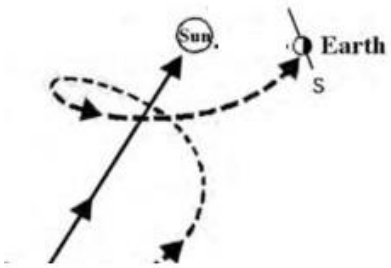
In the case of linear motion, there are three degrees of freedom, corresponding to the position of the object along the x, y, and z directions. In contrast, in the case of angular motion, there are three degrees of freedom associated with rotation, corresponding to the rotation around the x, y, and z axes.

Periodic Motion

Periodic motion refers to the repetitive and regular motion of an object or system over time, in which the motion returns to its initial state after a fixed time interval called the period. This means that the motion of the object or system repeats itself identically at regular intervals, exhibiting a characteristic pattern or cycle.

Example: Spiral motion of a planet in a galaxy.

Periodic Motion = Motion + ”Repeat in Equal Time Intervals”





Oscillatory Motion

Oscillatory motion is a type of periodic motion in which an object or system repeatedly moves back and forth around a central or equilibrium position. In other words, motion is characterized by a periodic variation in displacement, velocity, or acceleration with respect to time.

Oscillatory motion is a motion with an extra constraint that the system must pass through a fixed point along with repetition in equal time intervals. Example: A fan rotating with constant angular velocity.

Oscillatory Motion = Motion + ”Repeat in Equal Time Intervals” + ”Pass Through Same Point”

Oscillatory Motion = Periodic Motion + ”Pass Through Same Point”

Simple Harmonic Motion

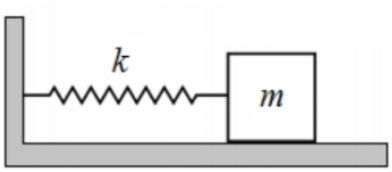
Simple harmonic motion (SHM) is a type of oscillatory motion in which an object or system moves back and forth around an equilibrium position with a constant amplitude and a sinusoidal (sine or cosine) displacement function.

The key defining feature of SHM is that the restoring force acting on the object or system is directly proportional to the displacement from the equilibrium position and is directed towards it. This means that the system experiences a force that pulls it back towards the equilibrium position whenever it is displaced from it, and the motion is repetitive and predictable.

SHM = Oscillatory Motion + ”$\boldsymbol{a} \propto \boldsymbol{x}$” + ”$\hat{\boldsymbol{a}} = -\hat{\boldsymbol{x}}$”

Examples of systems that exhibit SHM include a simple pendulum, a mass-spring system, and an LC circuit. The motion of these systems is governed by the mass, spring constant, damping factor, and other parameters, which determine the frequency, period, amplitude, and other characteristics of the motion.

Linear and Angular Simple Harmonic Motion

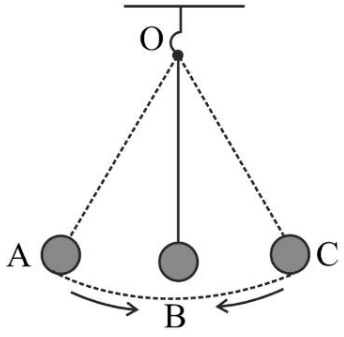


Mass-Spring System

Linear Simple Harmonic Motion: This type of SHM occurs when an object oscillates in a straight line, such as a mass-spring system. In this case, the displacement of the object from the equilibrium position varies sinusoidally with time, and the acceleration is directly proportional to the displacement and is always directed towards the equilibrium position.

Angular Simple Harmonic Motion: This type of SHM occurs when an object oscillates about an equilibrium point with an angular displacement, such as a simple pendulum. In this case, the displacement of the object from the equilibrium position varies sinusoidally with time, and the acceleration is directly proportional to the displacement and is always directed towards the equilibrium position.

Constraints of SHM



There are four constraints given on the DOF of the SHM system. They are given below.

1. Motion repetition in equal intervals of time
2. System pass through same point
3. Acceleration is proportional to displacement from equilibrium
4. Direction of acceleration is opposite to the direction of displacement.

Hooke Force

Hooke force is the force that acts on an object attached to the end of a spring or a rope that is being stretched or compressed. According to Hooke's law, the force exerted by a spring is proportional to the displacement of the spring from its equilibrium position, as long as the spring is not stretched or compressed beyond its elastic limit. The proportionality constant is called the spring constant (k).

$$\vec{F}_{Hook} = -k\vec{x}$$

where x is the displacement of the object from its equilibrium position, and the negative sign indicates that the force is in the opposite direction to the displacement.

Damping Force

Damping forces are a special type of force that is used to slow down or stop a motion. The damping effect caused by damping forces is due to the dissipation of energy. The damping force opposes the motion of the body. The magnitude of the damping force is directly proportional and opposite to the velocity of the body.

$$F_D = -bv$$

Here, v is the velocity of the body and b is the damping coefficient. This relation is true for small velocity.

Newtonian force

The fundamental principle in classical mechanics states that the net force acting on an object is equal to the object's mass times its acceleration. Mathematically, the principle can be written as:

$$F = ma$$

This principle is based on Newton's second law of motion, which states that the rate of change of an object's momentum is directly proportional to the net force acting on it. In other words, the greater the force applied to an object, the greater its acceleration, and the harder it is to change its motion.

Classification Based on Damping and External Force

Based on damping and external force applied to the system, we can classify simple harmonic motion as follows:

1. Undamped simple harmonic motion: This type of SHM occurs when there is no friction or damping force acting on the system. The motion continues indefinitely with a constant amplitude and frequency, as long as there is no external force acting on it. Examples of undamped SHM include the motion of a mass on a spring with no air resistance.

2. Damped simple harmonic motion: This type of SHM occurs when there is friction or a damping force acting on the system. The amplitude of the motion gradually decreases over time, and the frequency may also change. Examples of damped SHM include the motion of a mass on a spring in a viscous fluid.

3. Forced simple harmonic motion: This type of SHM occurs when an external periodic force is applied to the system, which can cause the amplitude and frequency of the motion to change. Examples of forced SHM include the motion of a mass on a spring that is being driven by an external force.