SB 26/09/23

Waves & Oscillations

14 weeks → 14 classes

9 Lecture notes

3 Problem set

Ref book:

Physics for Engineers

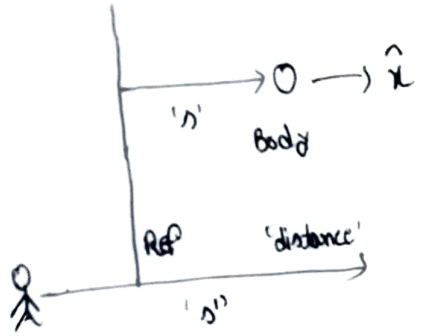
by Dr Gias Uddin

SB

01/10/23

Motion

Def Reference frame + set up coord. + time



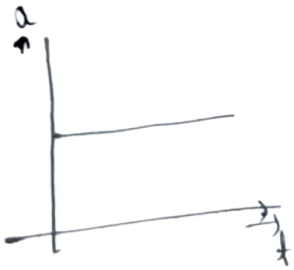
$v = u + at$

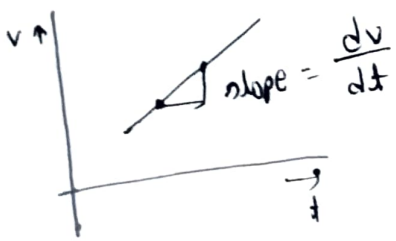
$x = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2ax$

} 'a' const

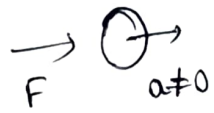
velocity, $\vec{v}_x = \frac{dx}{dt}\hat{x}$ & acceleration, $\vec{a} = \frac{d^2x}{dt^2}\hat{x} = \frac{dv}{dt}\hat{x}$





y : motc

Force



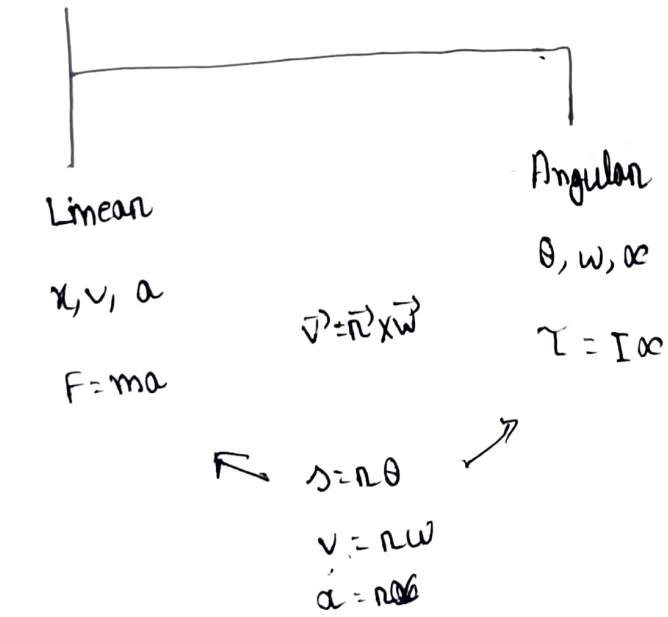
$F = ma$

law 1 → $\Sigma F = 0 \Rightarrow u = v$

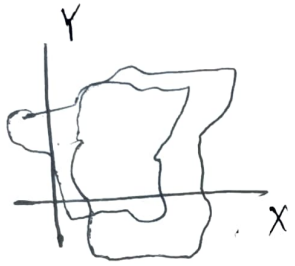
law 2 → $\frac{d\vec{p}}{dt} \propto \vec{F}$

law 3 → $F_1 = -F_2$

Motion

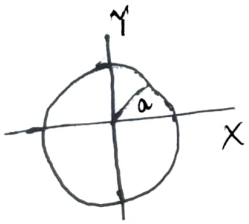


Constraints / Restrictions



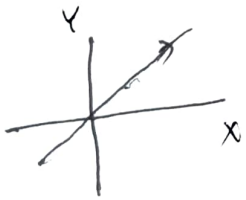
DOF = 2

$x^2+y^2=a^2$ (Restriction)



DOF = 1

$Y = mx + c$



DOF = 1

① Motion + Fixed time repeatation

"Periodic Motion" → 

point repeatation



② Periodic ~~motion~~ + fixed ~~time interval~~

"oscillatory motion" → 

③ + ④ $a \propto x$ & $a = -\hat{x}$

Simple ~~Harmonic~~ motion

$F_N = ma$

~~$F_H = -kx$~~

$F_D = -bv$

$F_H = -kx$

$m, k \rightarrow$ material

$b \rightarrow$ medium

Next class:

SHM

Linear

Angular



mass-spring system



Pendulum

SB 10/10/23

Simple Harmonic motion

→ Fixed point

→ Fixed time period

→ $\alpha \propto x$

→ $\hat{a} = -\hat{x}$

type 1 linear: Mass-spring system

type 2 angular: Pendulum system

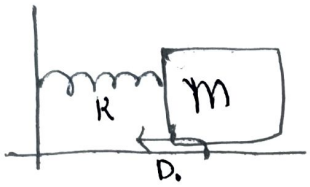
$F_n = ma$

$F_D = -bv$

$F_H = -kx$

Midterm full set ques from today's class

Mass - spring System



1 dim motion

$\Sigma F = 0 \Rightarrow F_N = F_D + F_H$

$\Rightarrow ma = -bv - kx$

$\Rightarrow m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

differential equation of simple harmonic motion

$$\frac{d^2x(t)}{dt^2} + \frac{b}{m}\frac{dx(t)}{dt} + \frac{k}{m}x(t) = 0$$

standard form

b → drag coeff. (medium const)

k → spring const (material const)

Simple case: $b = 0$

$$\frac{d^2x(t)}{dt^2} + \frac{k}{m}x(t) = 0 \; ; \quad \text{undamped SHM}$$

(i) Undamped: $b = 0 \longrightarrow$ 1 ques for final

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

(ii) Damped SHM: $b \neq 0 \longrightarrow$ 1 ques for final

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = 0$$

(iii) Forced SHM: $F_{ext}(t)$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = F_{ext}(t)$$

(iv) Coupled SHM: For simplicity let $b = 0$, N number of oscill- / mass

$$\ddot{x}_1 + \omega^2 x_2 = 0 \;\; \& \;\; \ddot{x}_2 + \omega^2 x_1 = 0 \; ; \text{ for } N = 2$$

① → $\frac{d^2x(t)}{dt^2} = -\omega^2 x(t)$

$\sin x \xrightarrow{d/dx} \cos x \xrightarrow{d/dx} -\sin x$

$\cos x \xrightarrow{d/dx} -\sin x \xrightarrow{d/dx} -\cos x$

} suitable for (i): double derivatives

$e^x \xrightarrow{d/dx} e^x \xrightarrow{d/dx} e^x$ } suitable for (ii): single derivative

---

(1) Soln type: sin/cos/exp

(2) 2nd order DE: 2 const;

(3) Sol: exp

$e^x \to e^x \to e^x$ ✗

$e^{-x} \to -e^{-x} \to e^{-x}$ ✗

$e^{ix} \to ie^{ix} \to -e^{ix}$ ✓

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$x(t) = A \sin(\omega t + \delta)$; ($\omega t + \delta$: dimless)

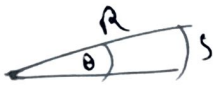
$x(t) = A \cos(\omega t + \delta)$

$x(t) = A e^{i\omega t} + B e^{i\omega t}$

$x(t) = A \sin(\omega t) + B \cos(\omega t)$

$\theta = x^\circ$

dim of degree = 0



$S = R\theta$ → zero dim

m m

SB 17/10/23

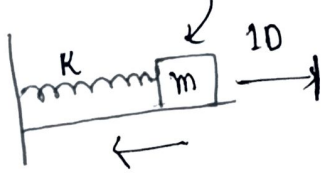
Undamped SHM

$$m\frac{d^2x}{dt^2} + kx(t) = 0$$

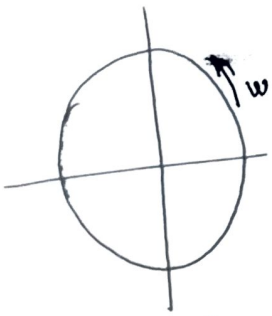
$$\frac{d^2x(t)}{dt^2} + \omega^2 x(t) = 0$$

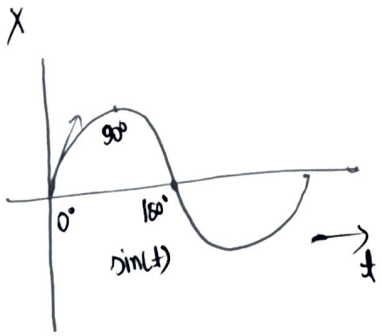
Solution : $x(t) = A\sin(\omega t + \phi)$

$$\omega^2 = \frac{k}{m}$$



linear motion
but has a hidden angular description
hence sin, cos

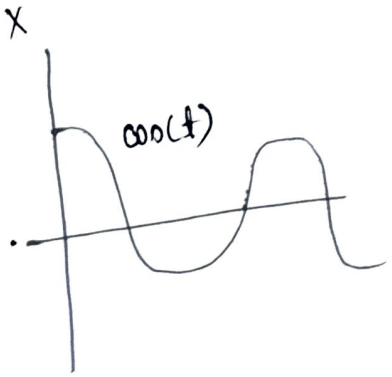




$$\omega = \frac{\theta}{t}$$

$t = T \Rightarrow \theta = 2\pi$

$$\omega = \frac{2\pi}{T}$$



$x(t) = A\sin(\omega t+\phi)$ ·

① $A\cos(\omega t+\phi)$ } explain why soln is like this → prev page

$A\sin(\omega t)+B\cos(\omega t)$

$Ae^{i\omega t} + Be^{-i\omega t}$ } explain why → implicitly involves sin / cos

$e^{i\theta} = \cos\theta + i\sin\theta$

derive to ①

Superposition principle

$x(t) = \sin(\omega t), \cos(\omega t), e^{i\omega t}, e^{-i\omega t}$; possible solutions

$\frac{d^2x}{dt^2} = -\omega^2 x$

$\frac{d^2}{dt^2}(e^{-i\omega t}) = (-i\omega)^2 e^{-i\omega t}$

$= -\omega^2 e^{-i\omega t}$

$X(t) = X_1(t) + X_2(t)$ → superpos principle

of power 1

General soln:

$X_1 = A\sin(\omega t)$

$X_2 = B\cos(\omega t)$

$X(t)_{gen} = X_1(t) + X_2(t)$

$= A\sin(\omega t) + B\cos(\omega t)$

In a nutshell;

double integration: two consts ; soln must have two const

possible soln addition

↳ whichever satisfies eqn

x_{sol}: $A\sin(\omega t)$, $B\cos(\omega t)$, $\alpha e^{i\omega t}$, $\beta e^{-i\omega t}$

$x_{gen} = A\sin(\omega t) + B\cos(\omega t)$

$A = x_0\cos\phi$ & $B = x_0\sin\phi$

$x_{gen} = x_0[\sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi]$

$= x_0\sin(\omega t + \phi)$

$A = x_0\sin\phi$ & $B = -x_0\cos\phi$

$x(t) = A\sin(\omega t + \phi)$

$v(t) = \frac{dx(t)}{dt} = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$

$a(t) = \frac{dv(t)}{dt} = -A\omega^2\sin(\omega t + \phi) = -\omega^2 x(t)$

$K = \frac{1}{2}mv^2$

$P = \frac{1}{2}kx^2$

$T.E. = K + P$

$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

SB 31/10/23

H.W conservation of energy

show that energy is constant

Pendulum

Calc until page 3 first line / simple pendulum

compound "

until time period

SB 07/11/23

Damped Harmonic Oscillator

$\Sigma \vec{F} = 0$ Equilibrium

$F_N + F_D + F_H = 0$

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

$\frac{d^2x}{dt^2} + \boxed{\frac{b}{m}}\frac{dx}{dt} + \boxed{\frac{k}{m}}x = 0$ ——— (1)

$\boxed{\frac{b}{m}} = 2\beta$ $\quad \boxed{\frac{k}{m}} = \omega^2$

$b = 0 \Rightarrow \beta = 0$

$\omega^2 = k/m$

$2\pi f = \omega = \sqrt{k/m}$

ω, ω_0

$\omega = f(\omega_0, \beta)$

i.e, $\omega = \omega_0 + \cdots \beta$

Guess soln: $x(t) = Ae^{\alpha t}$

verify:

(i) $\rightarrow A\alpha^2 e^{\alpha t} + 2\beta A\alpha e^{\alpha t} + \omega_0^2 A e^{\alpha t} = 0$

$$Ae^{\alpha t}(\alpha^2 + 2\beta\alpha + \omega_0^2) = 0$$

Either $\underline{Ae^{\alpha t} = 0}$ or $\alpha^2 + 2\beta\alpha + \omega_0^2 = 0$
$\quad X$

$$\therefore \alpha_1 = \frac{-2\beta + \sqrt{4\beta^2 - 4\omega_0^2}}{2}$$

$$\alpha_1 = -\beta + \sqrt{\beta^2 - \omega_0^2}$$

$$\alpha_2 = \frac{-2\beta - \sqrt{4\beta^2 - 4\omega_0^2}}{2}$$

$$\alpha_2 = -\beta - \sqrt{\beta^2 - \omega_0^2}$$

Solutions:

$x_1(t) = Ae^{\alpha_1 t}$

$x_2(t) = Ae^{\alpha_2 t}$

General soln:

$$x(t) = x_1(t) + x_2(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

↑ Superposition principle

2 DOF bcz order 2 DE

A, B are two constants determined from the initial condition or boundary condition of the system.

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Case 1: $B^2 > 4AC \rightarrow$ 2 real soln. x_1 & x_2

Case 2: $B^2 = 4AC \rightarrow$ 1 real soln. x_1

Case 3: $B^2 < 4AC \rightarrow$ 2 complex soln. z_1 & z_2

1. Under Damped soln:

$\beta^2 < \omega_0^2 \rightarrow$ 2 complex soln (SHM)

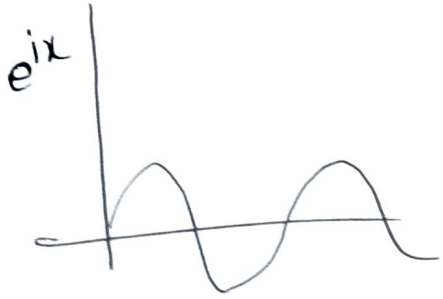
2. Critically Damped soln

$\beta^2 \approx \omega_0^2 \rightarrow$ 1 real soln (Not SHM)

3. Over Damped soln.

$\beta^2 > \omega_0^2 \rightarrow$ 2 real soln. (Not SHM)





Under Damped Solution

$$X_{Gen}(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$\alpha_1 = -\beta + \sqrt{\beta^2 - \omega_0^2} \quad \& \quad \alpha_2 = -\beta - \sqrt{\beta^2 - \omega_0^2}$$

$$\Rightarrow \alpha_1 = -\beta + i\sqrt{\omega_0^2 - \beta^2} \quad \& \quad \alpha_2 = -\beta - i\sqrt{\omega_0^2 - \beta^2}$$

$$X_{Gen}(t) = Ae^{(-\beta + i\sqrt{\omega_0^2 - \beta^2})t} + Be^{(-\beta - i\sqrt{\omega_0^2 - \beta^2})t}$$

$$= e^{-\beta t}\left[Ae^{i\sqrt{\omega^2 - \beta^2}t} + Be^{-i\sqrt{\omega_0^2 - \beta^2}t}\right]$$

$$\omega = \sqrt{\omega_0^2 - \beta^2}$$

$$X(t) = e^{-\beta t}\left[Ae^{i\omega t} + Be^{-i\omega t}\right]$$

H.W

Euler Formula!

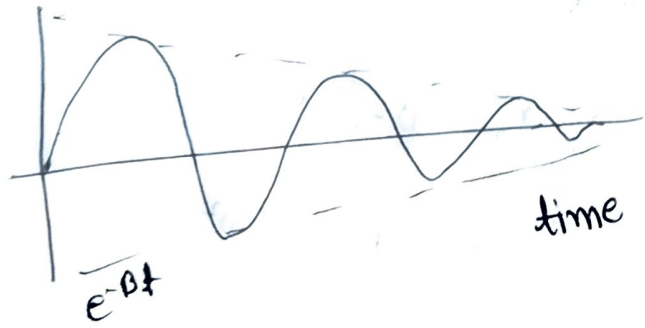
$$X(t) = e^{-\beta t}\left[\alpha \sin(\omega t) + \beta \cos(\omega t)\right]$$

H.W

$$\beta = X_0 \sin\phi \quad \& \quad \alpha = X_0 \cos\phi$$

$$X(t) = X_0 e^{-\beta t} \sin(\omega t + \phi)$$

$$x_{Gen}(t) = X_0 e^{-\beta t} \sin(\omega t + \phi)$$



SB

21/11/23

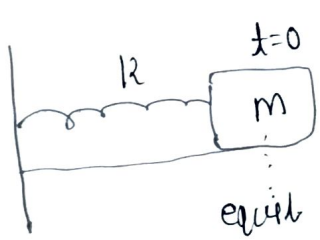
for test/ mid

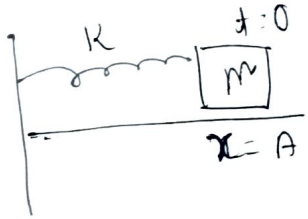
defⁿ's

$x(t) = A\sin(\omega t + \phi)$; show that total energy is const.

mass spring system at equilibrium to start oscil-- derive soln

$x(t) = A\sin(\omega t + \phi)$





$x_{gen}(t) = A\sin(\omega t + \phi)$

gen

$\phi = 0$

$A = A\sin(\phi)$

$\theta = 90°$

$x(t) = A\sin(\omega t)$

$\omega^2 = k/m$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$e^x = 1 + x$; x = very small

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^x \longrightarrow$ when real

$e^x = \cosh\theta + i\sinh\theta$

$\beta^2 < \omega_0^2$

$$x(t)_{Gen} = Ae^{\left(-\beta + \sqrt{\beta^2 - \omega_0^2}\right)t} + Be^{\left(-\beta - \sqrt{\beta^2 - \omega_0^2}\right)t}$$

$$= e^{\beta t}\left[A\left\{1 + \sqrt{\beta^2 - \omega_0^2}\, t\right\} + B\left\{1 - \sqrt{\beta^2 - \omega_0^2}\, t\right\}\right]$$

$e^x = 1 + x$ @ x = very small

$\beta^2 > \omega_0^2$

$$x(t) = e^{-\beta t}\left[Ae^{\sqrt{\beta^2 - \omega_0^2}\, t} + Be^{-\sqrt{\beta^2 - \omega_0^2}\, t}\right]$$

$$= e^{-\beta t}\left[(A+B)\cosh\theta + (A-B)i\sinh\theta\right]$$ → replace θ

$$= \ldots$$

$$= \gamma_0 e^{-\beta t}\sinh(\omega t + \phi)$$

H.W

Rough / midterm

Forces affecting SHM:

$F_n = ma$

$F_H = -Kx$

$F_D = -bv$

$F_n = F_D + F_H \Rightarrow ma + Kx + bv = 0$

$\Rightarrow a + {}^{K}/_{m}\, x + {}^{b}/_{m}\, v = 0$

$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

Undamped SHM: when $b = 0$; no drag force

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

To find the eqn of such SHM, there must be a func. such that its second derivative is the func. itself times $(-\frac{k}{m})$.

To begin, funcs that repeat itself on its second derivatives ~~are~~ w/ negative signs are

i) $\sin x \xrightarrow{\frac{d}{dx}} \cos x \xrightarrow{d/dx} -\sin x$

ii) $\cos x \xrightarrow{d/dx} -\sin x \xrightarrow{d/dx} -\cos x$

iii) $e^{ix} \xrightarrow{d/dx} ie^{ix} \xrightarrow{d/dx} -e^{ix}$

iv) $e^{-ix} \xrightarrow{d/dx} -ie^{-ix} \xrightarrow{d/dx} e^{-ix}$

---

i) Let ω be a const

$x(t) = \sin(\omega t) = y$

$\Rightarrow y_1 = \omega \cos(\omega t)$

$\Rightarrow y_2 = -\omega^2 \sin(\omega t)$

Hence, $\omega^2 = K/m \Rightarrow \omega = \sqrt{\frac{K}{m}}$

~~Again, if t is the time period (~~

ii) $x(t) = \cos(\omega t)$

iii) $x(t) = e^{i\omega t} = y \Rightarrow y_1 = i\omega e^{i\omega t} \Rightarrow y_2 = -\omega^2 e^{i\omega t}$

iv) $x(t) = e^{-i\omega t}$

Hence, the general soln,

$x_1(t) = \sin(\omega t)$

$x_2(t) = \cos(\omega t)$

$x_{gen}(t) = A\sin(\omega t) + B\cos(\omega t)$; double integration → two const

→ (1)

Let, $A = x_0 \cos\phi$ & $B = x_0 \sin\phi$

(1) $\Rightarrow x(t) = x_0 \sin(\omega t + \phi)$

Let, $A = x_0 \sin\phi$ & $B = x_0 \cos\phi$

(1) $\Rightarrow x(t) = x_0 \cos(\omega t - \phi)$

Similarly for exponential func.

$$x_{gen}(t) = \alpha e^{i\omega t} + \beta e^{-i\omega t}$$

Let us consider the eq^n

$$x(t) = A \sin(\omega t + \varphi)$$

At initial condition,

$$x(t) = A \sin(\omega t)$$

Let T is the time period (time for one complete oscillation)

$$x(t+T) = x(t)$$

$$\Rightarrow A \sin[\omega(t+T)] = A \sin(\omega t)$$

$$\Rightarrow \omega T = 2\pi$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Velocity

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt} A\sin(\omega t+\delta)$$

$$= A\omega\cos(\omega t+\delta)$$

$$= \omega\sqrt{A^2\cos^2(\omega t+\delta)}$$

$$= \omega\sqrt{A^2(1-\sin^2(\omega t+\delta))}$$

$$= \omega\sqrt{A^2-\{x(t)\}^2}$$

Accelenation

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt} A\omega\cos(\omega t+\delta)$$

$$= -A\omega^2\sin(\omega t+\delta)$$

$$= -\omega^2 x(t)$$

Conservation of energy

Kinetic energy, $E_K = \frac{1}{2}mv^2$

Potential energy, $E_p = \int F dx$

$= \int Kx dx$

$= \frac{1}{2}Kx^2$

Total energy, $E = E_K + E_p$

$= \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

$= \frac{1}{2}\left[m\{\omega^2(A^2 - x^2)\} + Kx^2\right]$

$= \frac{1}{2}\left[m\frac{K}{m}(A^2 - x^2) + Kx^2\right]$

$= \frac{1}{2}[KA^2]$

$= \frac{1}{2}KA^2 = \text{const}$ $\left[\frac{dE}{dt} = 0\right]$

Conservation of energy (from $x(t) = A\sin(\omega t)$)

$$E_k = \frac{1}{2}mv^2, \quad E_p = \int F\,dx = \int kx\,dx$$

$$= \frac{1}{2}kx^2$$

$$E = E_k + E_p$$

$$= \frac{1}{2}m\left\{\frac{d}{dt}A\sin(\omega t+\delta)\right\}^2 + \frac{1}{2}kA^2\sin^2(\omega t+\delta)$$

$$= \frac{1}{2}\left[m A^2\omega^2\cos^2(\omega t+\delta) + kA^2\sin^2(\omega t+\delta)\right]$$

$$= \frac{1}{2}\left[kA^2\{1-\sin^2(\omega t+\delta)\} + kA^2\sin^2(\omega t+\delta)\right]$$

$$= \frac{1}{2}kA^2$$

$$\frac{dE}{dt} = \frac{d}{dt}\frac{1}{2}kA^2 = 0$$

Solve/set 01

NP#1. $v(t) = \omega\sqrt{A^2 - x^2}$

$= 0.08\ ms^{-1}$

$\omega = \frac{2\pi}{T}$

$= 2\ rads^{-1}$

$\omega A = 0.1$

$\therefore A = 0.05\ m$

2.

$F = ma_{max} = m(\omega^2 A)$

$= \pi^2\ N$

$f = 500\ Hz$

$\therefore T = \frac{1}{500}\ s$ $\Big\}$ $\omega = \frac{2\pi}{T} = 1000\pi$

$m = 1g = 0.001\ kg$

$A = 1\ mm = 0.001\ m$

3.

14/ At $t = 0$,

$0.20 = A \sin(\phi)$

$\therefore \phi = 90°$

$K = 100\ N/m$

$m = 1\ kg$

$\} \Rightarrow \omega = 10\ rad\ s^{-1}$

$0.1 = 10\sqrt{A^2 - 0.2^2}$

$\therefore A = 0.2\ m$

$f \Rightarrow \omega = 2\pi f$

$\therefore f = 1.6\ Hz$

SB/ Damped SHO

$$F_n = F_D + F_H$$

$$\Rightarrow ma + bv + Kx = 0$$

$$\Rightarrow a + \frac{b}{m}v + \frac{K}{m}x = 0$$

$$\Rightarrow \frac{d^2x(t)}{dt^2} + 2\beta \frac{dx(t)}{dt} + \omega^2 x(t) = 0 \quad - (1)$$

Solve for gen eqn.

$$x(t) = Ae^{\alpha t}$$

$$(1) \Rightarrow A\alpha^2 e^{\alpha t} + 2\beta A\alpha e^{\alpha t} + \omega_0^2 A e^{\alpha t} = 0$$

$$\Rightarrow Ae^{\alpha t}(\alpha^2 + 2\beta\alpha + \omega_0^2) = 0$$

Now,

$$\alpha^2 + 2\beta\alpha + \omega_0^2 = 0$$

$$\therefore \alpha_1 = \frac{-2\beta + \sqrt{4\beta^2 - 4\omega_0^2}}{2} \quad \& \quad \alpha_2 = \frac{-2\beta - \sqrt{4\beta^2 - 4\omega_0^2}}{2}$$

$$= -\beta + \sqrt{\beta^2 - \omega_0^2} \qquad \alpha = -\beta - \sqrt{\beta^2 - \omega_0^2}$$

Gen soln: $x(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$

Case 1: $\beta < \omega_0$; under damped

$$x(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$\alpha_1 = -\beta + \sqrt{\beta^2 - \omega_0^2} \quad \& \quad \alpha_2 = -\beta - i\sqrt{\omega_0^2 - \beta^2}$$

$$= -\beta + i\sqrt{\omega_0^2 - \beta^2}$$

$$\therefore x(t) = Ae^{[-\beta + i\sqrt{\omega_0^2 - \beta^2}]t} + Be^{[-\beta - i\sqrt{\omega_0^2 - \beta^2}]t}$$

$$= e^{-\beta t}\left[Ae^{i\sqrt{\omega_0^2 - \beta^2}\,t} + Be^{-i\sqrt{\omega_0^2 - \beta^2}\,t}\right]$$

$$= e^{-\beta t}\left[Ae^{i\omega t} + Be^{-i\omega t}\right]$$

$$= e \cdots$$

$$= e^{-\beta t}\left[(A+B)\cos\omega t + i(A-B)\sin\omega t\right]$$

$$= \cdots$$

$$= x_0 e^{-\beta t}\sin(\omega t + \delta)$$

case 2; $\beta \approx \omega_0$

$$x(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$= Ae^{\left[-\beta + \sqrt{\beta^2 - \omega_0^2}\right] t} + Be^{\left[-\beta - \sqrt{\beta^2 - \omega_0^2}\right] t}$$

$$= e^{-\beta t}\left[Ae^{\sqrt{\beta^2 - \omega_0^2}\, t} + Be^{-\sqrt{\beta^2 - \omega_0^2}\, t}\right]$$

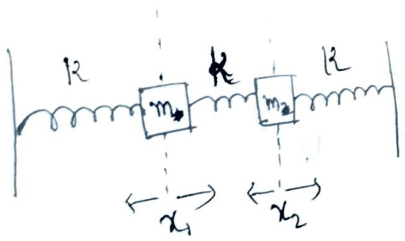
$$= e^{-\beta t}\left[A\left(1 + \sqrt{\beta^2 - \omega_0^2}\, t\right) + B\left(1 - \sqrt{\beta^2 - \omega_0^2}\, t\right)\right]$$

SB 28/11/23

Coupled oscillator

Refers to a system where two or more oscillators interact with each other.

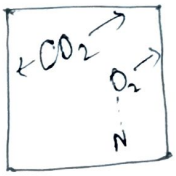
Simple case: Two body coupled oscillator



N=2

molecular level

m may be mass of Hydrogen atom.



expansion → −

compression → +

$F_{on\,2\,from\,1} = -kx_1 - kx_2$

$F_{11} = -kx_1 - kx_2$

$F_{12} = +kx_2$

$F_1 = F_{11} + F_{12} = -(k+k)x_1 + kx_2 = m\ddot{x}_1$

$F_2 = F_{21} + F_{22} = -(k+k)x_2 + kx_1 = m\ddot{x}_2$

$$m\ddot{x}_1 = -(k+k)x_1 + kx_2$$

$$m\ddot{x}_2 = -(k+k)x_2 + kx_2$$

Coupled diff eqn

Decoupling the diff eqn

(A)+(B) $\Rightarrow$ $m(\ddot{x}_1 + \ddot{x}_2) = -k(x_1 + x_2)$

$x_1 + x_2 \longrightarrow z$

$$\boxed{m\ddot{z} = -kz} \qquad \omega^2 = \frac{k}{m} = \frac{-z}{z}$$

(A) $-$ (B) $\Rightarrow$ $m(\ddot{x}_1 - \ddot{x}_2) = -(2k+k)(x_1 - x_2)$

$x_1 - x_2 \longrightarrow y$

$$m\ddot{y} = -(2k+k)y \; ; \quad \omega^2 = \frac{(2k+k)}{m}$$

$$\ddot{y} + \frac{(2k+k)}{m}y = 0$$

$$y = A_F \sin(\omega_F t + \varphi_F)$$

$$z = A_S \sin(\omega_S t + \varphi_S)$$

$$x_1, x_2 \longrightarrow y, z \longrightarrow x_1, x_2$$

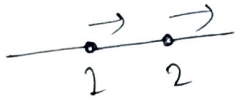
$$x_1 = \frac{1}{2}(z + y) = \frac{1}{2}\left[(x_1 + x_2) + (x_1 - x_2)\right]$$

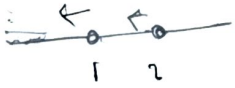
$$x_1 = \frac{1}{2}\left[A_F \sin(\omega_F t + \varphi_F) + A_S \sin(\omega_S t + \Phi)\right]$$

$$x_2 = \frac{1}{2}\left[A_S \sin(\omega_S t + \Phi) - A_F \sin(\omega_F t + \varphi_S)\right]$$

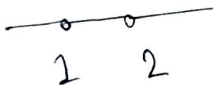
symmetric mode	Anti sym. mode
$x_1(t=0) = x_2(t=0)$	$x_1(t=0) = -x_2(t=0)$
$A_F = 0$	$A_S = 0$

modes











$A_f = 0$









$A_b = 0$

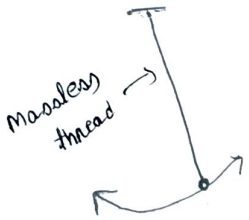


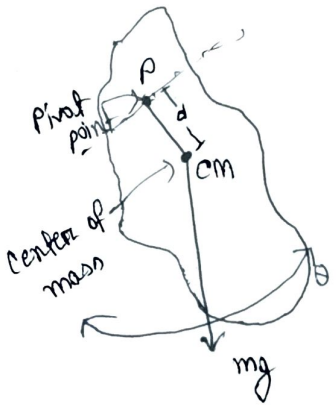
SB

05/12/23

Compound Pendulum

Defn: Any swinging rigid body free to oscillate about a fixed axis.





Motion angular

Coordinate system = Polar

(R, θ)

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau_1 = dmg \sin\theta$ [Hook term]

Newton's 2nd law of motion

$$\tau_2 = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{d\omega}{dt} \text{ & } \omega = \frac{d\theta}{dt}$$

$$I\frac{d^2\theta}{dt^2} + dmg\sin\theta = 0$$

$$\left[m\frac{d^2x}{dt^2} + Kx = 0\right]$$

$$\frac{d^2\theta}{dt^2} + \frac{mgd}{I}\sin\theta = 0 \; ; \text{ not SHM}$$

Small angle approximation

$$\sin\theta \approx \theta$$

$$\frac{d^2\theta}{dt^2} + \frac{mgd}{I}\theta = 0 \; ; \text{ SHM}$$

$$\omega^2 = \frac{mgd}{I} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$I_{cm} \longrightarrow I_p$

Apply parallel axis theorem

$$I_p = I_{cm} + md^2$$



I_{cm}

K = radius of gyration

$$I_{cm} = mK^2$$

$$T = 2\pi\sqrt{\frac{k^2+d^2}{gd}}$$

$$T_{eq} = 2\pi\sqrt{\frac{L}{g}}$$ ← effective length

$$T = T_{eq}$$

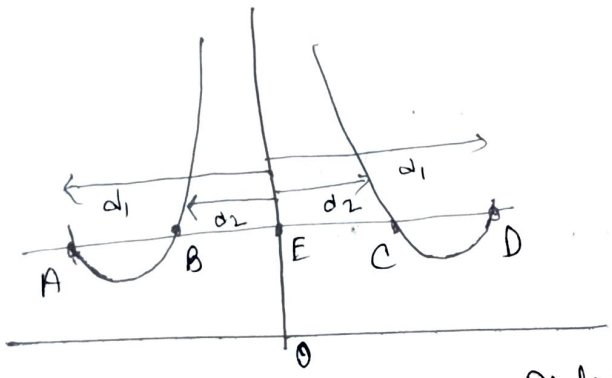
$$L = \frac{k^2+d^2}{d}$$

$$\Rightarrow \boxed{d^2 - Ld + k^2 = 0}$$

Quadratic equation in "d"

Let's consider the soln's are d_1 & d_2

$d_1 d_2 = k^2$ & $d_1 + d_2 = L$



Distance from COM.

$AC = d_1 + d_2$

$BD = d_1 + d_2$

$\Big\}$ $L = \frac{AC + BD}{2}$

H.W

Torsional Pendulum



$I = \phi\theta$

$T = 2\pi \sqrt{\frac{I}{\phi}}$

Rough / CT-3

Damped H.O.

$$F_n = F_H + F_D$$

$$\Rightarrow ma + Kx + bv = 0$$

$$\Rightarrow a + \frac{b}{m}v + \frac{K}{m}x = 0$$

$$\Rightarrow \frac{d^2x(t)}{dt^2} + 2\beta\frac{dx(t)}{dt} + \omega_0^2 x(t) = 0 \quad ; \quad \omega_0 = K/m, \ \beta = \frac{b}{2m}$$

Solve for gen soln:

$$x(t) = Ae^{\alpha t} \qquad [\text{Guess}]$$

Hence, $\alpha^2 Ae^{\alpha t} + 2\alpha\beta Ae^{\alpha t} + \omega_0^2 Ae^{\alpha t} = 0$

$$\Rightarrow Ae^{\alpha t}\left[\alpha^2 + 2\alpha\beta + \omega_0^2\right] = 0$$

Now, $\alpha^2 + 2\alpha\beta + \omega_0^2 = 0$ $\qquad [Ae^{\alpha t} \text{ can't be zero}]$

$$\therefore \alpha_1 = \frac{-2\beta + \sqrt{4\beta^2 - 4\omega_0^2}}{2} \qquad \alpha_2 = \frac{-2\beta - \sqrt{4\beta^2 - 4\omega_0^2}}{2}$$

$$= -\beta + \sqrt{\beta^2 - \omega_0^2} \qquad = -\beta - \sqrt{\beta^2 - \omega_0^2}$$

For $\beta < \omega$,

$$\alpha_1 = -\beta + i\sqrt{\omega_0^2 - \beta^2} \qquad \alpha_2 = -\beta - i\sqrt{\omega_0^2 - \beta^2}$$

Gen sol^n: $x(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$

$$= Ae^{\left[-\beta + i\sqrt{\omega_0^2 - \beta^2}\right]t} + Be^{\left[-\beta - i\sqrt{\omega_0^2 - \beta^2}\right]t}$$

$$= e^{-\beta t}\left[Ae^{i\omega t} + Be^{-i\omega t}\right] \qquad \left[\omega^2 = \omega_0^2 - \beta^2\right]$$

Using $e^{i\theta} = \cos\theta + i\sin\theta$,

$$e^{-\beta t}\left[A(\cos\omega t + i\sin\omega t) + B(\cos(-\omega t) + i\sin(-\omega t))\right]$$

$$= e^{-\beta t}\left[A\{\cos\omega t + i\sin\omega t\} + B\{\cos\omega t - i\sin\omega t\}\right]$$

$$= e^{-\beta t}\left[\cos\omega t(A+B) + i\sin\omega t(A-B)\right]$$

Let, $A + B = x_0 \sin\varphi$

$$i(A-B) = x_0 \cos\varphi$$

Now, $x_0 e^{-\beta t} \sin(\omega t + \varphi)$

$x_0 e^{-\beta t}$ — amplitude, declining w/ time

NP1

$T = \pi$ sec

$\omega = \frac{2\pi}{\pi} = 2$ rad s^{-1}

$v_{max} = \omega A \Rightarrow A = 0.05$ m

$v_{0.03} = \omega\sqrt{A^2 - (0.03)^2}$

$= 0.8$ ms^{-1}

NP2

$A = 1$ mm $= 10^{-3}$ m

$f = 500$ Hz

$T = \frac{1}{500}$ s

$\omega = 2\pi f = 1000\pi$ rad s^{-1}

$a_{max} = \omega^2 A$

$= (1000\pi)^2 \times 10^{-3}$ ms^{-2}

$F_{max} = m a_{max}$

$= 10^{-3} \times (1000\pi)^2 \times 10^{-3}$ N

$= \pi^2$ N

NP14

$k = 100$ N m^{-1}

$m = 1$ kg

$\Big\} \omega = 10$ rad s^{-1}

~~$\phi = 90°$~~

At $t = 0$,

$0.2 = 0.2 \sin(\phi)$

$\therefore \phi = 90°$

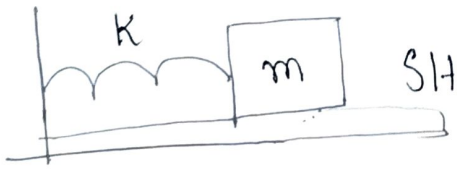
$v_{0.2} = 0.1$ ms^{-1}

$= \omega\sqrt{A^2 - (0.2)^2}$

$\therefore A = 0.2$ m

SA 26/12/23

Waves



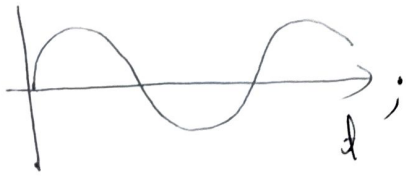
SHO

$$x(t) = A\sin(\omega t + \phi)$$

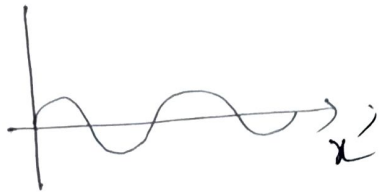
$\lambda \longrightarrow 2\pi$

$1 \longrightarrow \frac{2\pi}{\lambda}$

$x \longrightarrow \frac{2\pi}{\lambda} x = \phi$



; time based oscillation



; space based oscillation

$$y(t) = A\sin\left(\omega t \pm \frac{2\pi}{\lambda} x\right)$$

$$y(t) = A\sin\left(\frac{2\pi}{T} t \pm \frac{2\pi}{\lambda} x\right)$$

$$y(t) = A\sin(\omega t \pm kx)$$; Progressive wave

$\omega = \frac{2\pi}{T}$ & $k = \frac{2\pi}{\lambda}$

$\Phi = \omega t \pm kx$: phase of the wave

$$\frac{d\Phi}{dt} = \omega \pm k\frac{dx}{dt} = 0$$

↖ definition of wave

$$\frac{dx}{dt} = \mp \frac{\omega}{k}$$

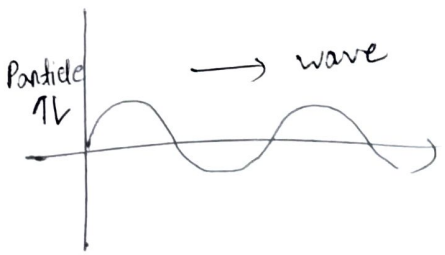
$y(t) = A\sin(\omega t + kx) \Rightarrow \frac{dx}{dt} = -\frac{\omega}{k}$; Backward moving

$y(t) = A\sin(\omega t - kx) \Rightarrow \frac{dx}{dt} = +\frac{\omega}{k}$; Forward moving

---

$y(t)$ denotes to

$\frac{dy}{dt}$ denotes to



~~$y(t) = A\sin(\omega t \pm kx)$~~

$y(x,t) = A\sin(\omega t - kx)$ & $\omega = \frac{2\pi}{T}$ & $k = \frac{2\pi}{\lambda}$ & $v = \frac{\omega}{k}$

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx)$$

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx)$$

$$= -A\omega^2 \sin(\omega t - kx)\frac{k^2}{\omega^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}\frac{1}{v^2}$$

when derived w/ $+kx$ same result is arrived

H.W 4.11 Distribution of energy

Prob set 1, 2, ~~3~~

SHM — characteristics

Mass spring system — diff. eq^n

show that avg KE, PE is ½ of TE

Compulsory — prob set 1

Damped Forced H.M — definition

difference

underdamped full derive

prob set 2 math

---

wave — from prev class theory

numerically from chap 4 (7/8 count) Giancoli

from chapter 8 exercise

SB 02/01/24

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$y_1(x,t) = A\sin(kx-\omega t)$ ⟶ forward moving wave $v = +\omega/k$

$y_2(x,t) = A\sin(kx+\omega t)$ ⟶ backward " " $v = -\omega/k$

$y_{total} = y_1 + y_2$

$= 2A\sin(kx)\cos(\omega t)$

~~$y'_{tot} = 2A[\omega \sin kx \sin\omega t$~~

~~y'_{tot}~~ ⟹

$$\frac{\partial y_{tot}}{\partial x} = 2A(\cos\omega t) \times k\cos kx \qquad \frac{\partial y_{tot}}{\partial t} = -2A(\sin kx)\,\omega \sin\omega t$$

$$\frac{\partial^2 y_{tot}}{\partial x^2} = -2Ak^2 \sin kx \cos\omega t \qquad \frac{\partial^2 y_{tot}}{\partial t^2} = -2A\omega^2 \sin kx \cos\omega t$$

$y_{tot} = \underbrace{2A\sin(kx)}_{\text{amplitude}}\cos(\omega t)$ —— wave D.E soln.

$$\frac{d}{dt}(\text{phase}) = 0$$

$$\frac{d}{dt}(kx - \omega t) = 0$$

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k}$$

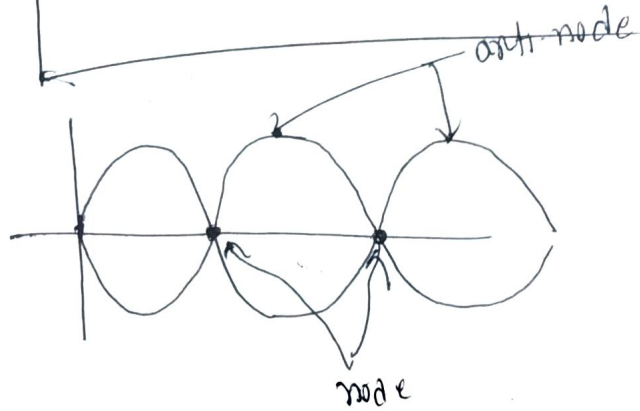
$$v_{tot}: \frac{d}{dt}(kx) = 0$$

$$\frac{dx}{dt} = 0$$

Check wave: wave DE

check standing wave: wave DE + V = 0

Check progressive wave: wave DE + V ≠ 0



Node points:

$$\sin kx = 0$$

$$\frac{2\pi}{\lambda}x = kx = n\pi$$

$$f\frac{2\pi x}{v} = n\pi \quad \left[\lambda = \frac{v}{f}\right]$$

$$f = \frac{n\pi v}{2\pi x} = \frac{nv}{2x}$$

Antinode

$$\sin kx = \pm 1$$