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Course Code & Title: PHY 101, Waves and Oscillations, Optics and Modern Physics

Problem Set #01 (W&O), Topic: SHM (undamped), Lissajous Figure

Multiple Choice Question

MCQ#01: At which position of a simple harmonic oscillator is the kinetic energy maximum?

(A) At the maximum displacement
(B) At the equilibrium position
(C) At the minimum displacement
(D) At all positions

MCQ#02: Which of the following equations represents the displacement of a simple harmonic oscillator?

(A) $x(t) = A\sin(\omega t)$
(B) $x(t) = A\exp(\omega t)$
(C) $x(t) = A\tan(\omega t)$
(D) $x(t) = A\log(\omega t)$

MCQ#03: The period of a simple pendulum depends on:

(A) The length of the pendulum
(B) The mass of the pendulum bob
(C) The amplitude of the oscillation
(D) Both A and B

MCQ#04: Which of the following is not a characteristic of simple harmonic motion?

(A) The motion is periodic
(B) The motion is oscillatory
(C) The motion is damped
(D) The motion is a result of a restoring force

MCQ#05: Which of the following factors does not affect the period of a simple harmonic oscillator?

(A) Mass of the oscillator
(B) Spring constant
(C) Amplitude of oscillation
(D) None of the above

MCQ#06: The restoring force in a simple harmonic oscillator is proportional to

(A) the velocity of the oscillator
(B) the acceleration of the oscillator
(C) the displacement from the equilibrium position
(D) the period of the oscillator

MCQ#07: Torsional pendulum shows SHM if the restoring torque is proportional to

(A) the angular displacement
(B) the angular acceleration
(C) the angular velocity
(D) square of the angular displacement

MCQ#08: Which of the following factors affect the period of a torsional pendulum?

(A) Length of the pendulum
(B) Mass of the bob
(C) Distance between COM and axis of rotation
(D) All of the above

MCQ#09: In a Lissajous figure, the number of lobes formed depends on:

(A) The ratio of the frequencies of the two SHM
(B) The amplitude of the oscillations
(C) The phase difference between the oscillations
(D) The period of the oscillations

MCQ#10: To form straight line in Lissajous Figure, the frequency ratio and the phase difference between two SHO must be,

(A) 1:2 and $\pi/2$
(B) 2:1 and $\pi/4$
(C) 1:1 and 0
(D) 1:2 and $\pi/4$

Answer the Following Short Questions

SQ#1. What is conservation of energy? Write down the energy conservation law for a SHM.

SQ#2. How does the presence of air resistance affect the motion of a pendulum? How can we ignore this?

SQ#3. What is meant by the center of oscillation in a compound pendulum?

SQ#4. What is damping? How does damping affect the motion of an oscillator?

SQ#5. Write down the differential equation of a forced, damped SHO. Explain all the terms.

SQ#6. What is the difference between Periodic motion and oscillatory motion?

SQ#7. What is a Lissajous figure? How is a Lissajous figure produced?

SQ#8. How can Lissajous figures be used to determine the frequency and phase difference between two signals?

SQ#9. What is the significance of the ratio of frequencies in a Lissajous figure?

SQ#10. How can pendulums be used to measure the acceleration due to gravity?

Solve the Numerical Problems

NP#1. A point is executing SHM with a period π sec. When it is passing through the center of its path, its velocity is 0.1 m/s. What is its velocity when it is at a distance of 0.03 m from the mean position?

NP#2. A mass of 1 g vibrates through 1 mm on each side of the middle point of its path and makes 500 complete vibrations per second. Assuming its motion to be simple harmonic, show that the maximum force acting on the particle is π^2 N.

NP#3. A particle performs harmonic oscillations along the x-axis according to the law $x = A\cos(\omega t)$. Assuming the probability P of the particle to fall within an interval from $-A$ to A to be equal to unity, find how the probability density dP/dx depends on x. Here dP denotes the probability of the particle within the interval from x to $x + dx$.

NP#4. A 100 g mass vibrates horizontally without friction at the end of a horizontal spring for which the spring constant is 10 N/m. The mass is displaced 0.5 cm from its equilibrium and released. Find: (a) Its maximum speed, (b) Its speed when it is 0.3 cm from equilibrium. (c) What is its acceleration in each of these cases?

NP#5. A mass M attached to a spring oscillates with a period of 2 s. If the mass is increased by 2 kg, the period increases by one second. Find the initial mass M assuming that Hooke's law is obeyed.

NP#6. The potential energy of a particle of mass m is given by

$$V(x) = \frac{a}{x^2} - \frac{b}{x}$$

where a and b are positive constants. Find the minimum of $V(x)$ and expand $V(x)$ about the point of minimum. Find the period of small oscillations that the particle performs about the position of minimum of $V(x)$.

NP#7. A thin rod of length 10 cm and mass of 100 g is suspended at its midpoint from a long wire. Its period T_A of angular SHM is measured to be 2s. An irregular object, which we call object X, is then hung from the same wire, and its period T_X is found to be 3s. What is the rotational inertia of object X about its suspension axis?

NP#8. At $t = 0$, the displacement of a point in a linear oscillator is –8.6 cm, its velocity – 0.93 m/s and its acceleration is 48 m/s^2. (a) What are the angular frequency and the frequency? (b) What is the phase constant? (c) What is the amplitude of the motion? ~~a~~ A is not given

NP#9. The time period of a body executing SHM is 2s. After how much time interval from $t = 0$ will its displacement be half the amplitude?

NP#10. A block whose mass is 700g is fastened to a spring whose spring constant is 63 N/m. The block is pulled a distance 10 cm from its equilibrium position and released from rest. (a) Find the time period of oscillation of the block, (b) what is the mechanical energy of the oscillator? (c) What are the potential energy and kinetic energy of this oscillator when the particle is halfway to its end point? [Neglect gravitational P.E.]

NP#11. A particle performing S.H.M. has a period of 6 s and amplitude of 8 cm. The particle starts from the mean position and moves towards the positive extremity. Find its displacement, velocity, and acceleration 0.5 s after the start.

NP#12. A linear harmonic oscillator of force constant $2 \times 10^6\ N/m$ and amplitude 0.01 m has a total mechanical energy of 160 J. Show that its (a) maximum potential energy is 160 J (b) maximum kinetic energy is 100 J.

NP#13. The 100 g mass shown in Fig. 1.5 is pushed to the left against a light spring of spring constant k = 500 N/m and compresses the spring 10 cm from its relaxed position. The system is then released and the mass shoots to the right. If the friction is ignored how fast will the mass be moving as it shoots away?

NP#14. A mass-spring system has a spring constant of 100 N/m and a mass of 1 kg. The system is set in motion with an initial displacement of 0.2 meters and a velocity of 0.1 m/s. Determine the amplitude, frequency, and phase angle of the resulting motion.

NP#15. An LC circuit consists of a 50 mH inductor and a 1 μF capacitor. If the circuit is initially charged to a voltage of 10 V, calculate the frequency of the resulting oscillations.

Problem sheet-1

Short questions:

1. The conservation of energy is a fundamental principle in physics that states that the total energy of an isolated system remains constant over time.

For ~~SMH~~ SHM, which is a type of periodic motion often seen in system like a mass-spring system or a pendulum.

-total mechanical energy in SHM is,

$$E = KE + PE$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= \text{constant}$$

2. The presence of air resistance can significantly affect the motion of a pendulum. In a vacuum, a simple pendulum would continue to swing back and forth indefinitely. However, in the presence of air resistance, the motion of the

pendulum is gradually damped, meaning that the amplitude of the oscillations decreases over time.

Air resistance acts as a ~~damped~~ damping force that opposes the motion of the pendulum bob through the air. As the bob moves through the air, it experiences a resistive force that depends on its velocity. This resistive force converts some of the KE of the pendulum ~~of~~ into thermal energy, leading to a gradual loss of energy and a decrease in the amplitude of the oscillations.

To minimize the impact of air resistance experiments with pendulums are often conducted in a controlled environment, such as a vacuum chamber or in a location with minimal air movement.

3. The centre of oscillation in a compound pendulum is the point where the centre mass of the pendulum could be concentrated to produce the same period of oscillation as the distributed mass system. In simpler terms, it is the theoretical point where the entire mass can be considered to be concentrated for the purpose of calculating the pendulum's oscillatory behaviour, as if it were a simple pendulum.

4. Damping refers to the gradual reduction of amplitude or energy in an oscillatory system over time. It is a dissipative force that opposes ~~that~~ the motion of the oscillator.

Damping affects the motion of an oscillator by influencing the amplitude and duration of oscillations. It introduces a resistive force that causes the oscillator to lose energy, leading to a decrease in amplitude

over time. This decreasing rate depends on the amount of damping present in the system

5. The differential eqn of a forced, damped SHO is,

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0 \cos(\omega_{force} t)$$

where,

m = mass of the oscillator

x = displacement of the oscillator from its equilibrium position

c = damping coefficient

k = spring constant

F_0 = amplitude of the ~~dire~~ driving force

ω_{force} = angular frequency of the driving force

t = time

6. Periodic motion	Oscillatory motion
① refers to any motion that repeats itself in equal intervals of time.	① refers to repetitive, back and forth motion around a central point or equilibrium position.
② It could involve a regular pattern of motion without necessarily having a back and forth or repetive oscillation.	② Oscillatory motion is the regular, repetive pattern having a centre point or position of rest.
③ Examples are Planetery motion, where a planet revolves around the sun.	③ Examples are a swinging pendulum, a vibrating guiter string

7. A Lissajous figure is a complex harmonic waveform pattern produced by the intersection of two perpendicular sinusoidal wave motions.

$$x(t) = A \sin(at + \delta)$$

$$y(t) = B \sin(bt)$$

here A and B are amplitudes and

a, b are the regular angular frequen harmo

8. General steps to determine the frequenc, and phase diff using Lissajous figures -

i. Selecting the signals
ii. Adjusting the Oscilloscope
iii. Vary the frequency
iv. Determining frequency ratio
v. Analyzing phase difference
vi. Determining phase difference

9. The significance of the ration of frequencies in a Lissajous figure lies in providing information about the relationship between the two harmonic signals. When the frequencies of the signals from a simple ratio, distinct and recognizable pattern emerge in the Lissajous figure. The specific shape of the figure corresponds to the frequency relationship between the signals, aiding in the analysis of their

harmonic interaction and helping determine the frequency ration between them.

10. The period of a simple pendulum is,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow g = \frac{4\pi^2 L}{T^2}$$

Numerical problems:

1. $T = \frac{2\pi}{\omega}$

$$\Rightarrow \frac{2\pi}{\pi} = 2 \text{ rad s}^{-1} \quad [T = \pi]$$

let, $v_{max} = 0.1 \text{ ms}^{-1}$

$A = 0.1 \text{m}$

$x = 0.03$

now, $v = \omega\sqrt{A^2 - x^2}$

$= 2\sqrt{0.1^2 - 0.03^2}$

$= 0.1911 \text{ ms}^{-1}$

2. $\omega = 2\pi f$

$= 2\pi \times 500 \text{ rad s}^{-1}$

$T = \frac{1}{500} s$

$m = 0.001 \text{ kg}$

$K = m\omega^2$

$= 0.001 \times (2\pi \times 500)^2$

$= 1 \times 10^3 \pi^2$

$f_{max} = K.A$

$= 1 \times 10^3 \times \frac{1}{1000}$

$= 1 N$

$f_{max} = K.A$

$= \{0.001 \times (2\pi \times 500)^2\} \times \frac{1}{1000}$

$= \frac{\pi^2}{2} N$

3. $x = A \cos(\omega t)$

$\Rightarrow v = -A \sin \omega t$

if the wavefunction $\psi(x,t) = A \cos(\omega t)$

$\psi(x,t) = A \cos(\omega t)$

$|\psi(x,t)|^2 = |A \cos(\omega t)|^2$

$\Rightarrow |\psi(x,t)|^2 = A^2 \cos^2(\omega t)$ ----- (1)

$$\cos(\omega t) = \frac{x}{A}$$

Substituting $\cos(\omega t)$ value in eqn (1) $\rightarrow$

$$|\psi(x,t)|^2 = A^2 \left(\frac{x}{A}\right)^2$$
$$= x^2$$

4.a) We know,

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$
$$\Rightarrow v = \sqrt{\frac{k}{m}x^2}$$
$$= \sqrt{\frac{10}{0.1}(0.005)^2}$$
$$= 0.05 \text{ ms}^{-1}$$

b) If $x' = 0.3 \text{ cm}$
$= 0.003 \text{ m}$

$$\therefore v' = \sqrt{\frac{k}{m}x'^2}$$
$$= \sqrt{\frac{10}{0.1}(0.003)^2}$$
$$= 0.03 \text{ ms}^{-1}$$

c) $a = \frac{F}{m}$

$$= \frac{-kx}{m}$$
$$= \frac{-10 \times 0.005}{0.1}$$
$$= -0.5 \text{ ms}^{-2}$$

$a' = \frac{-kx'}{m}$

$$= \frac{-10 \times 0.003}{0.1}$$
$$= -0.3 \text{ ms}^{-2}$$

5.

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$\Rightarrow 2 = 2\pi\sqrt{\frac{M}{K}} \quad [m = M] \quad \text{---- (I)}$$

for 2kg,

$$m = M+2$$

$$T = T+1$$

$$T+1 = 2\pi\sqrt{\frac{M+2}{K}} \quad \text{---- (II)}$$

$$\Rightarrow (T+1)^2 = (2\pi)^2 \frac{M+2}{K}$$

$$\Rightarrow T^2+2T+1 = 4\pi^2 \frac{M+2}{K}$$

$$\Rightarrow T^2+2T+1 = 4\frac{4}{T^2}(M+2) \quad [\text{eq}^n \text{ (I)}]$$

$$\Rightarrow T^2+2T+1 = \frac{16}{T^2}(M+2)$$

$$\Rightarrow T^2+2T+1 = \frac{16M}{T^2} + \frac{32}{T^2}$$

$$= \frac{16M+32}{T^2}$$

$$\Rightarrow T^4+2T^3+T^2 = 16M+32$$

$$\Rightarrow T^4+2T^3+T^2-32 = 16M$$

$$\Rightarrow M = \frac{T^4+2T^3+T^2-32}{16}$$

$$\Rightarrow M = \frac{2^4+2\times2^3+2^2-32}{16}$$

$$\therefore M = 0.25 \text{ kg}$$

7. $T = 2\pi\sqrt{\frac{I}{mgd}}$

for rod,

$I_{rod} = \frac{1}{12} M L_{rod}^2$

for object X,

$I_X = \frac{1}{12} M L_X^2$

So, $\frac{T_{rod}}{T_X} = \frac{2\pi\sqrt{\frac{I_{rod}}{mgd}}}{2\pi\sqrt{\frac{I_X}{mgd}}}$

$\therefore \frac{I_X}{I_{rod}} = \left(\frac{T_X}{T_{rod}}\right)^2$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

8. a) $x(t) = A\cos(\omega t + \varphi)$

$v(t) = -A\omega\sin(\omega t + \varphi)$

$a(t) = -A\omega^2\cos(\omega t + \varphi)$

$t = 0$;

$x(0) = -8.6\,cm = -0.086\,m = A\cos(\varphi)$

$v(0) = -.93\,ms^{-1} = -A\omega\sin(\varphi)$

$a(0) = 48\,ms^{-2} = -A\omega^2\cos(\varphi)$

b) $A\cos(\varphi) = -0.086$

$\Rightarrow \cos(\varphi) = \frac{-0.086}{A}$

$\Rightarrow \varphi = \cos^{-1}\left(\frac{-0.086}{A}\right)$

$a = -\omega^2 x$

$\Rightarrow 48 = -\omega^2 \times (-0.086)$

$\Rightarrow \omega^2 = 558.139$

$\Rightarrow \omega = 23.624$

$f = \frac{\omega}{2\pi}$

$= \frac{23.624}{2\pi}$

$= 3.76 \text{ Hz}$

b) $\omega = \sqrt{\frac{a(0)}{x(0)}}$

~~$v = x(0)$~~

$\frac{v(0)}{x(0)} = -\omega \tan\phi$

$\Rightarrow \tan\phi = -\frac{v(0)}{\omega x(0)}$

$= -0.458$

$\Rightarrow \phi = 155.4°$

b) $x(0) = -8.6 \text{ cm}$

$v(0) = -0.93 \text{ ms}^{-1}$

$x(t) = A\cos(\omega t + \phi)$

$A\cos(\phi) = -8.6$

$\Rightarrow \cos\phi = -\frac{8.6}{A}$

$\therefore \phi = \cos^{-1}\left(-\frac{8.6}{A}\right)$

b) $\tan\phi = -\frac{v(0)}{\omega x(0)}$

$\Rightarrow \phi = \tan^{-1}\left(-\frac{v(0)}{\omega x(0)}\right)$

$= \tan^{-1}\left(-\frac{0.93}{3.76 \times 0.086}\right)$

$= \tan^{-1}\left(-\frac{0.93}{23.62 \times 0.086}\right)$

$= 24.5997$

c) $A = \frac{x(0)}{\cos\phi}$

$= 0.09446 \text{ m}$

9.

$$x(t) = A\cos(\omega t + \varphi)$$

$$\Rightarrow x(T/2) = A\cos\left(\frac{\omega T}{2} + \varphi\right)$$

$$= -A/2$$

when $x = \frac{A}{2}$;

$$\frac{A}{2} = A\cos(\omega t + \varphi)$$

$$\Rightarrow \frac{1}{2} = \cos(\omega t + \varphi)$$

$$\Rightarrow \cos\frac{\pi}{3} = \cos(\omega t + \varphi)$$

$$\Rightarrow \omega t + \varphi = \frac{\pi}{3}$$

$$\Rightarrow \frac{2\pi}{T} \times t + \varphi = \frac{\pi}{3}$$

$$\Rightarrow t = \frac{\frac{\pi}{3} - \varphi}{\frac{2\pi}{T}}$$

10. a) $T = 2\pi\sqrt{\frac{m}{K}}$

$= 2\pi\sqrt{\frac{0.7}{63}}$

$= 0.6625$

b) $E = PE + KE$

$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$= \frac{1}{2} 63 \times (0.1)^2 + 0$ [V=0 initially]

$= 0.315\ J$

c) At max displacement,

$PE = KE$

$\Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv^2 \Rightarrow 0.07875 = \frac{1}{2}mv^2$

$\Rightarrow v^2 = \frac{0.07875 \times 2}{0.7}$

~~$KE = \frac{1}{2}mv^2 = 0$~~

$\Rightarrow v^2 = 0.225$

$PE = \frac{1}{2}k\left(\frac{A}{2}\right)^2$

$\Rightarrow v = 0.4743\ ms^{-1}$

$= \frac{1}{2} \times 63 \times \left(\frac{0.1}{2}\right)^2$

$= 0.07875\ J$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.7 \times 0.225$

$= 0.07875\ J$

11.

a) $\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{6}$

$= 1.047$

b) $x(t) = A\cos(\omega t)$

$= 0.08 \cos(1.047 \times 0.5)$

$= 0.0799\ m$

c) $v(t) = -A\omega \sin(\omega t)$

$= -0.08 \times 1.047 \sin(1.047 \times 0.5)$

$= -7.65 \times 10^{-4}\ ms^{-1}$

$a(t) = -A\omega^2 \cos(\omega t)$

$= -0.08 \times (1.047)^2 \cos(1.047 \times 0.5)$

$= -8.77 \times 10^{-2}\ ms^{-2}$

12. a) $PE_{max} = \frac{1}{2}kA^2$

$= \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2$

$= 160\ J$

b) $E = PE + KE$

$\Rightarrow 160 = \frac{1}{2}kA^2 + \frac{1}{2}mv^2$

$= \frac{1}{2} 2 \times 10^6 \times (0.01)^2 + \frac{1}{2}mv^2$

b) $KE_{max} = \frac{1}{2} KA^2$

$= \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2$

$= 100\ J$

13. $PE_{max} = KE_{max}$

$\Rightarrow \frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$\Rightarrow kx^2 = mv^2$

$\Rightarrow v = \sqrt{\frac{kx^2}{m}}$

$= \sqrt{\frac{500 \times (0.1)^2}{0.1}}$

$= 7.071\ ms^{-1}$

14. $\omega = \sqrt{\frac{k}{m}}$

$= \sqrt{\frac{100}{1}}$

$= 10\ rad/s$

at max displacement, $A = x_0$

$\Rightarrow A = 0.2m$

$v(t) = -A\omega \sin(\omega t + \varphi)$

$\Rightarrow 0.1 = -0.2 \times 10 \sin\varphi$

$\Rightarrow \frac{-0.5}{10} = \sin\varphi$

$\Rightarrow \varphi = \sin^{-1}\left(-\frac{0.5}{10}\right)$

$= -2.865$

$$f = \frac{\omega}{2\pi}$$

$$= \frac{10}{2\pi}$$

$$= 15.7 \text{ Hz}$$

15. $L = 50 \times 10^{-3} \text{ H}$

$C = 1 \times 10^{-6} \text{ F}$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{(50\times10^{-3})(1\times10^{-6})}}$$

$$= 711.7625 \text{ Hz}$$

3. $\frac{dx}{dt} = V = -A\omega \sin \omega t$

distance from $x+dx$ is

$$\frac{dx}{|x|} = \frac{dx}{A\omega\sqrt{1 - x^2/A^2}} = \frac{dx}{\omega\sqrt{A^2 - x^2}}$$

distance from $-A$ to A is $T/2$. So,

$$dP = \frac{dx}{T/2 \times \omega\sqrt{A^2 - x^2}}$$

$$\Rightarrow dP = \frac{dx}{\pi\sqrt{A^2 - x^2}}$$

$$\Rightarrow \frac{dP}{dx} = \frac{1}{\pi\sqrt{A^2 - x^2}}$$

6) $\left.\begin{aligned} &\frac{dV}{dx} > 0 \\ \Rightarrow\ &\frac{d^2V}{dx^2} > 0 \end{aligned}\right\}$ at $x = 2a/b$

$$V\left(\frac{2a}{b}\right) = -\frac{b^2}{4a}$$

$$V'\left(\frac{2a}{b}\right) = 0$$

$$V''\left(\frac{2a}{b}\right) = \frac{b^4}{8a^3}$$

$$V(x) = -\frac{b^2}{4a} + \frac{\left(x - \frac{2a}{b}\right)^2}{2!}\frac{b^4}{8a^3} + \cdots$$

putting, $y = x - \frac{2a}{b}$,

$$V(x)_{min} = my = -\frac{b^4}{8a^3}y$$

$$\omega = \sqrt{\frac{b^4}{8a^3}}$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{4\pi a}{b^2}\sqrt{2ma}$$