Military Institute of Science and Technology (MIST)

Department of Electrical Electronic and Communication Engineering (EECE), MIST

Course Code & Title: PHY 101, Waves and Oscillations, Optics and Modern Physics

Problem Set #02 (W&O), Topic: Damped and Forced Oscillator

Solve the Numerical Problems

NP#1. A particle of mass 3 moves along the x-axis attracted toward origin by a force whose magnitude is numerically equal to 12x. The particle is also subjected to a damping force whose magnitude is numerically equal to 12 times the instantaneous speed. If it is initially at rest at x = 10, find the position and the velocity of the particle at any time.

NP#2. A particle of mass 1 g moves along the x-axis under the influence of two forces: (i) a force of attraction toward origin which is numerically equal to 4x dynes, and (ii) a damping force whose magnitude in dynes is numerically equal to twice the instantaneous speed. Assuming that the particle starts from rest at a distance 10 cm from the origin, (a) set up the differential equation of motion of the particle, (b) find the position of the particle at any time, and (c) determine the amplitude, period, and frequency of the damped oscillation.

NP#3. In a damped oscillatory motion an object oscillates with a frequency of 1 Hz and its amplitude of vibration is halved in 5 s. Find the differential equation for the oscillation.

NP#4. The energy of recoil of a rocket launcher of mass m = 4500 kg is absorbed in a recoil spring. At the end of the recoil, a damping dashpot is arranged in such a way that the launcher returns to the firing position without any oscillation (critical damping). The launcher recoils 3 m with an initial speed of 10 m/s. Find (a) the recoil's spring constant and (b) the dashpot's coefficient of critical damping.

NP#5. Solve the problem of simple pendulum if a damping force proportional to the instantaneous tangential velocity is taken into account (over, critically and under damped motion).

NP#6. (a) A steady force of 50 N is required to lift a mass of 2 kg vertically through water at a constant velocity of 2.5 m/s. Assuming that the effect of viscosity can be described by a force proportional to velocity, determine the constant of proportionality. (The effect of buoyancy is neglected.)

(b) The same mass is then suspended in water by a spring with spring constant $k = 120N/m$. Determine the equilibrium extension of the spring. The mass is further pulled through a small distance below its equilibrium position and released from rest at time $t = 0$. Show that it will vibrate about the equilibrium position according to an equation of the form,

$$\ddot{x} + 2b\dot{x} + \omega^2 x = 0$$

and determine b and ω for this system. Show that the motion is under damped and find its period of oscillation. Find the time in which the amplitude of oscillation falls by a factor of e.

NP#7. An automobile suspension system is critically damped and its period of oscillation with no damping is one second. If the system is initially displaced by an amount x0, and released with zero initial velocity, find the displacement at t = 2s.

NP#8. An LCR circuit has L = 10 mH, C = 1.0 μF, and R = 1 Ω. (a) After what time t will the amplitude of the charge oscillations drop to one-half of its initial value? (b) To how many periods of oscillation does this correspond?

NP#9. A periodic external force acts on a 3 kg mass suspended from the lower end of a vertical spring having spring constant 75 N/m. The damping force is proportional to the instantaneous speed of the mass and is 20 N when the speed is 1 m/s. Find the frequency at which amplitude resonance occurs.

NP#10. The mass on a vertical spring undergoes forced vibrations according to the Eqn.

$$\ddot{x} + \omega^2 x = f \sin \omega t$$

where there is no damping, and the impressed frequency is equal to the natural frequency of oscillation. (a) Obtain the solution of the above differential equation (b) Give a physical interpretation.

NP#11. A vertical spring has a spring constant of 50 N/m. At t = 0 a force given in newtons by $F(t) = 48 \cos 7t$, t ≥ 0 is applied to a 20 N weight which hangs in equilibrium at the end of the spring. Neglecting damping find the position of the weight at any later time t.

NP#12. A particle of mass m moves under the influence of external periodic force $F \sin pt\, \hat{\imath}$ in addition to the restoring force $-kx\hat{\imath}$ and damping force $-\beta\dot{x}\hat{\imath}$. In the steady state, show that the amplitude of the driven system is maximum when $p = (\omega^2 - 2b^2)^{1/2}$ and the value of the maximum amplitude is $\frac{f}{2b\sqrt{\omega^2 - b^2}}$.

Answer the Following Short Questions

SQ#1. What is the difference between underdamped and overdamped oscillation?

SQ#2. What causes damping in oscillatory motion? What are the types of damping in oscillatory motion?

SQ#3. Distinguish between free and forced oscillation.

SQ#4. What is forced oscillation? Explain resonance in forced oscillation.

SQ#5. Write down the differential equation of damped forced oscillator and explain all terms.

SQ#6. Explain how the phase of a driven oscillator depends on the relative values of natural and driven frequency.