Lecture 03

System of Linear Algebraic Equations

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Jacobi Method

* The Jacobi method extends the idea of the fixed point iteration method to a system of equations.
* It is a direct substitution method where the values of the unknowns are improved by substituting the previous values.

Jacobi Method

Example 1.8:

Solve the equations by the Jacobi method:

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

Solution:

$$x_1 = 0.3 + 0.2x_2 + 0.1x_3 + 0.1x_4$$

$$x_2 = 1.5 + 0.2x_1 + 0.1x_3 + 0.1x_4$$

$$x_3 = 2.7 + 0.1x_1 + 0.1x_2 + 0.2x_4$$

$$x_4 = -0.9 + 0.1x_1 + 0.1x_2 + 0.2x_3$$

Initially take $x_1=x_2=x_3=x_4=0$. The results are given in the table below:

$$x_1 = 0.3 + 0.2x_2 + 0.1\,x_3 + 0.1\,x_4$$
$$x_2 = 1.5 + 0.2x_1 + 0.1x_3 + 0.1\,x_4$$
$$x_3 = 2.7 + 0.1x_1 + 0.1\,x_2 + 0.2x_4$$
$$x_4 = -0.9 + 0.1x_1 + 0.1\,x_2 + 0.2x_3$$

n	x_1	x_2	x_3	x_4
1	0.3	1.5	2.7	-0.9
2	0.78	1.74	2.7	-0.18
3	0.9	1.908	2.916	-0.108

n	x_1	x_2	x_3	x_4
1	0.3	1.5	2.7	-0.9
2	0.78	1.74	2.7	-0.18
3	0.9	1.908	2.916	-0.108
4	0.9624	1.9608	2.9592	-0.036
5	0.9845	1.9848	2.9851	-0.0158
6	0.9939	1.9938	2.9938	-0.006
7	0.9975	1.9975	2.9976	-0.0025
8	0.9990	1.9990	2.9990	-0.0010
9	0.9996	1.9996	2.9996	-0.0004
10	0.9998	1.9998	2.9998	-0.0002
11	0.9999	1.9999	2.9999	-0.0001
12	1.0	2.0	3.0	0.0

Gauss-Seidel Method

* Gauss-Seidel method is an improved version of the Jacobi method.

Example 1.9

Solve the equations by the Gauss-Seidel method:

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$

$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$

$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$

$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

Solution:

$$x_1 = 0.3+0.2x_2+0.1\ x_3+0.1\ x_4$$

$$x_2 = 1.5+0.2x_1+0.1x_3+0.1\ x_4$$

$$x_3 = 2.7+0.1x_1+0.1\ x_2\ +0.2x_4$$

$$x_4 = -0.9+0.1x_1+0.1\ x_2\ +0.2x_3$$

Initially take $x_1=x_2=x_3=x_4=0$. The results are given in the table below:

n	x_1	x_2	x_3	x_4
1	0.3	1.56	2.886	-0.1368
2	0.8869	1.9523	2.9566	-0.0248
3	0.9836	1.9899	2.9924	-0.0042

n	x_1	x_2	x_3	x_4
1	0.3	1.56	2.886	-0.1368
2	0.8869	1.9523	2.9566	-0.0248
3	0.9836	1.9899	2.9924	-0.0042
4	0.9968	1.9982	2.9987	-0.0008
5	0.9994	1.9997	2.9998	-0.0001
6	0.9999	1.9999	3.0	0.0
7	1.0	2.0	3.0	0.0

Practice Problem

1.10 Solve the following system of equations by

(i) the Gauss elimination method and

(ii)the Gauss-Jordan method:

$$2x+y+z=10$$
$$3x+2y+3z=18$$
$$x+4y+9z=16$$

1.11 Solve the following system of equations

$$2x_1+x_2+x_3= 5$$
$$3x_1+5x_2+2x_3=15$$
$$2x_1+x_2+4x_3= 8$$

by the Jacobi method and the Gauss-Seidel method.

THANK YOU