LECTURE 09-10

NUMERICAL INTEGRATION

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MIST

RICHARDSON'S EXTRAPOLATION FORMULA FOR TRAPEZOIDAL RULE

$$TV \cong I_{2n} + \frac{I_{2n} - I_n}{3}$$

Example 4.8

The vertical distance covered by a rocket from t= 8 to t = 30 seconds is given by

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8\, t \right) dt$$

a. Use Richardson's extrapolation to find the distance covered. Use the 2- segment and 4-segment Trapezoidal rule.

b. Find the true error.

c. Find the absolute relative true error.

BASIS OF ROMBERG RULE

- Integration

The process of measuring the area under a curve.

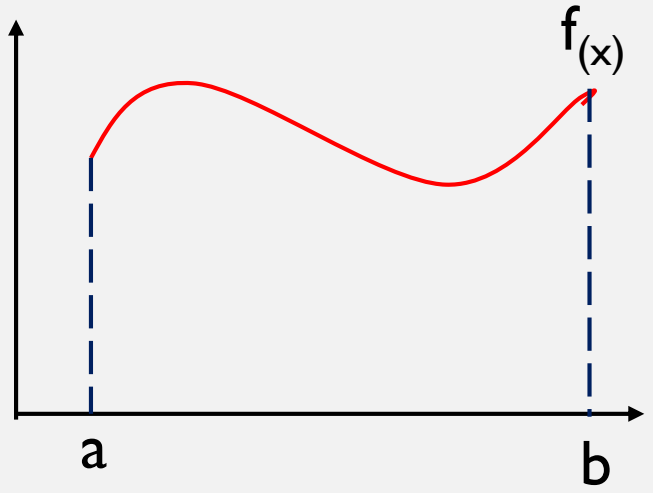
$I=\int_a^b f(x)dx$

Where,

f(x) is the integrand

a= lower limit of integration

b= upper limit of integration



WHAT IS THE ROMBERG RULE?

- Romberg integration is the extrapolation formula of the Trapezoidal Rule for integration. It provides a better approximation of the integral by reducing the True error.
- We use two estimates of an integral to compute a third integral that is more accurate than the previous integrals.
- Using the Romberg integration, it is possible to improve the accuracy of the trapezoidal rule when the function is known at an equidistant intervals.
- This method is highly suitable for computer application.

ROMBERG INTEGRATION

General Form:

$$I_{k,j} = I_{k-1,j+1} + \frac{I_{k-1,j+1} - I_{k-1,j}}{4^{k-1}-1} \; ; k \geq 2$$

Where, k= order of interpolation

(i.e. k=1, values for regular trapezoidal rule

k=2, values using error estimate $O(h^2)$)

j=more/less accurate estimate of integral

(i.e. value of integral w/ index j+1 is more accurate than the index with j)

IMPROVED ESTIMATES OF INTEGRAL VALUES

		1st Order	2nd Order	3rd Order
1 segment	$I_{1,1}$			
		$I_{2,1}$		
2 segment	$I_{1,2}$		$I_{3,1}$	
		$I_{2,2}$		$I_{4,1}$
4 segment	$I_{1,3}$		$I_{3,2}$	
		$I_{2,3}$		
8 segment	$I_{1,4}$			

EXAMPLE 4.9

Using the value given in the table find $\int_0^{0.8} e^{-x^2} dx$ by Romberg integration. Use the 1,2,4 and 8 segment Trapezoidal results.

x	f(x)
0	1.00
0.1	0.99005
0.2	0.960789
0.3	0.913931
0.4	0.852144
0.5	0.778801
0.6	0.697697
0.7	0.612626
0.8	0.527292

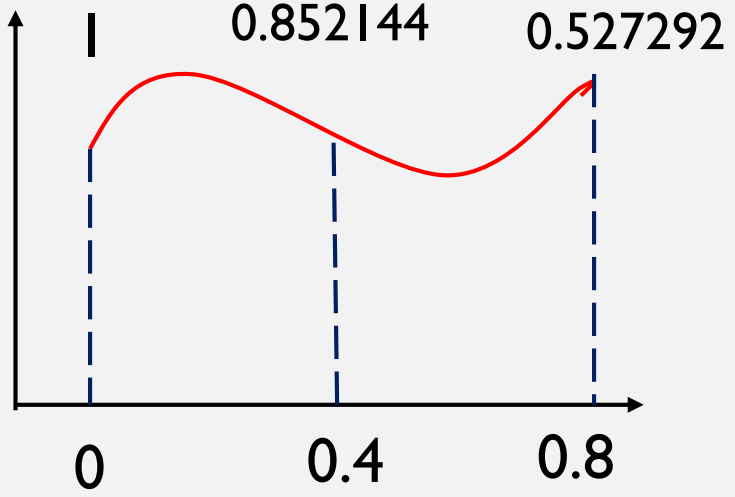
EXAMPLE 4.9

Considering number of strip=1, h=0.8,

Using trapezoidal rule, $I_{1,1} = \frac{h}{2}(f_0+f_1)$

$= \frac{0.8}{2}(1+0.527292)$

$= 0.6109168$



Considering number of strip=2, h=0.4,

Using trapezoidal rule, $I_{1,2} = \frac{h}{2}(f_0+2f_1+f_2)$

$= \frac{0.4}{2}[1+(2\times 0.852144)+0.527292]$

$= 0.646316$

EXAMPLE 4.9

Considering number of strip=4, h=0.2,

Using trapezoidal rule,

$$I_{1,3} = \frac{0.2}{2}\,(1+2(0.960789 + 0.852144 + 0.697676)+ 0.527292)$$

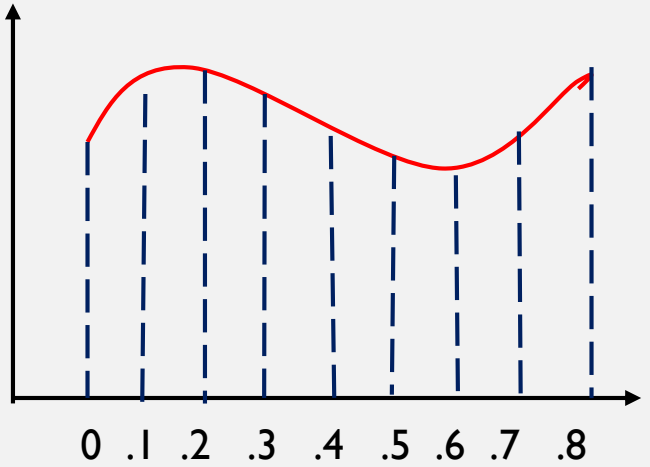
$$= 0.6548510$$

Considering number of strip=8, h=0.1,

Using trapezoidal rule,

$$I_{1,4} = \frac{0.1}{2}\,[1+2(0.99005 + 0.960789+.913931 + 0.852144 + 0.778801+.967697+.612626)+0.527292]$$

$$= 0.6569663$$



EXAMPLE 4.9

h	I_{1j} (j=1,2,3,4)
0.8	0.6109168
0.4	0.646316
0.2	0.654851
0.1	0.6569663

EXAMPLE 4.9

Romberg's Integration Formula:

$$I_{k,j} = I_{k-1,j+1} + \frac{I_{k-1,j+1} - I_{k-1,j}}{4^{k-1} - 1} \; ; k \geq 2$$

To get the first order extrapolation values,

$$I_{2,1} = I_{1,2} + \frac{I_{1,2} - I_{1,1}}{4^{2-1} - 1}$$

$$= 0.646316 + \frac{0.646316 - 0.6109168}{3}$$

$$= 0.658115$$

EXAMPLE 4.9

Similarly, $I_{2,2} = I_{1,3} + \frac{I_{1,3} - I_{1,2}}{4^{2-1} - 1}$

$$= 0.654851 + \frac{0.654851 - 0.646316}{3}$$

$$= 0.657696$$

$$I_{2,3} = I_{1,4} + \frac{I_{1,4} - I_{1,3}}{4^{2-1} - 1}$$

$$= 0.6569663 + \frac{0.6569663 - 0.654851}{3}$$

$$= 0.6576714$$

EXAMPLE 4.9

For the second order extrapolation values,

$$I_{3,1} = I_{2,2} + \frac{I_{2,2} - I_{2,1}}{4^{3-1} - 1}$$

$$= 0.657696 + \frac{0.657696 - 0.658115}{15}$$

$$= 0.657668$$

$$I_{3,2} = I_{2,3} + \frac{I_{2,3} - I_{2,2}}{4^{3-1} - 1}$$

$$= 0.6576714 + \frac{0.6576714 - 0.657696}{15}$$

$$= 0.6576698$$

EXAMPLE 4.9

For the second order extrapolation values,

$$I_{4,1} = I_{3,2} + \frac{I_{3,2} - I_{3,1}}{4^{4-1} - 1}$$

$$= 0.6576698 + \frac{0.6576698 - 0.657668}{63}$$

$$= 0.6576698$$

EXAMPLE 4.9

		1st order	2nd order	3rd order
1 segment	0.6109168			
		0.658115		
2 segment	0.646316		0.657668	
		0.657696		0.6576698
4 segment	0.654851		0.6576698	
		0.6576714		
8 segment	0.659663			

PRACTICE PROBLEM

- Use Romberg integration to evaluate

4.10 $\int_0^{3\pi/2} e^x \sin x \, dx$

4.11 $\int_0^{2\pi} (5 + 2\sin x) \, dx$

THANK YOU