Method of False Position

- This is the oldest method for finding the real root of a nonlinear equation f(x)=0. and it closely resembles the Bisection method.

- In this method, also known as "**Regula-falsi method"**, we choose two points a and b such that f(a) and f(b) are of opposite sings. Hence a root must lie in between these two points.

Step-1: Find points x0 and x1 such that $x0<x1$ and $f(x0)\cdot f(x1)<0$.

Step-2: Take the interval [x0,x1] and
find next value using
Formula :

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

Step-3: If $f(x2)=0$ then x2 is an exact root,
else if $f(x0)\cdot f(x2)<0$ then $x1=x2$,
else if $f(x2)\cdot f(x1)<0$ then $x0=x2$.

Step-4: Repeat steps 2 & 3 until $f(xn)=0$ or $|f(xn)|\leq$Accuracy

Example: Find a root of an equation $f(x)=2x^3-2x-5$ using False Position method (regula falsi method). Desired Accuracy 0.001

Solution: $2x^3-2x-5=0$

Let $f(x)=2x^3-2x-5$

Here,

x	0	1	2
f(x)	-5	-5	7

So, root lies between _____

1*st* iteration :

Here $f(1)= -5 <0$ and $f(2)= 7 >0$

∴ Now, Root lies between $x0=1$ and $x1=2$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x2 = 1 - (-5) \cdot \frac{2 - 1}{7 - (-5)}$$

$$x2 = 1.41667$$

∴ $f(x2) = f(1.41667)$

$= -2.14699 <0$

2*nd* iteration : Here $f(1.41667)= -2.14699<0$ and $f(2)= 7>0$

$\therefore$ Now, Root lies between $x0=1.41667$ and $x1=2$

$$x3 = x0 - f(x0) \cdot (x1 - x0)/f(x1) - f(x0)$$

$$x3 = 1.41667 - (-2.14699) \cdot \frac{2 - 1.41667}{7 - (-2.14699)}$$

$$x3 = 1.55359$$

$\therefore f(x3)= f(1.55359)$

$=2 \cdot 1.55359^3 - 2 \cdot 1.55359 - 5$

= -0.60759 <0

3*rd* iteration :

Here $f(1.55359) = -0.60759 < 0$ and $f(2) = 7 > 0$

∴ Now, Root lies between $x0 = 1.55359$ and $x1 = 2$

$$x4 = x0 - f(x0) \cdot (x1 - x0)/f(x1) - f(x0)$$

$x4$ = 1.58924

∴ $f(x4)$=

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	Update
1	1	-5	2	7	1.41667	-2.14699	$x_0 = x_2$
2	1.41667	-2.14699	2	7	1.55359	-0.60759	$x_0 = x_2$
3	1.55359	-0.60759	2	7	1.58924	-0.15063	$x_0 = x_2$
4	1.58924	-0.15063	2	7	1.59789	-0.0361	$x_0 = x_2$
5	1.59789	-0.0361	2	7	1.59996	-0.00858	$x_0 = x_2$
6	1.59996	-0.00858	2	7	1.60045	-0.00203	$x_0 = x_2$
7	1.60045	-0.00203	2	7	1.60056	-0.00048	$x_0 = x_2$



Answer: Approximate root of the equation $2x3-2x-5=0$ using False Position method is 1.60056.

Example 2:

Find a root of an equation $f(x)=\sqrt[3]{48}$ using False Position method (regula falsi method). Desired accuracy 0.001.

Solution:

Here $48^{\frac{1}{3}} = 0$

Let $f(x) = 48^{\frac{1}{3}}$

$x = cube\sqrt{48}$

$\therefore x^3 = 48$

$\therefore x^3 - 48 = 0$

Here

x	0	1	2	3	4
$f(x)$	-48	-47	-40	-21	16

1^{st} iteration :

Here $f(3) = -21 < 0$ and $f(4) = 16 > 0$

$\therefore$ Now, Root lies between $x_0 = 3$ and $x_1 = 4$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = 3 - (-21) \cdot \frac{4 - 3}{16 - (-21)}$$

$$x_2 = 3.5676$$

$$f(x_2) = f(3.5676) = 3.5676^3 - 48 = -2.5936 < 0$$

2^{nd} iteration :

Here $f(3.5676) = -2.5936 < 0$ and $f(4) = 16 > 0$

$\therefore$ Now, Root lies between $x_0 = 3.5676$ and $x_1 = 4$

$$x_3 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

Approximate root of the equation $x^3 - 48 = 0$ using False Position mehtod is 3.6342 (After 5 iterations)

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$
1	3	-21	4	16	3.5676	-2.5936
2	3.5676	-2.5936	4	16	3.6279	-0.2513
3	3.6279	-0.2513	4	16	3.6336	-0.0237
4	3.6336	-0.0237	4	16	3.6342	-0.0022
5	3.6342	-0.0022	4	16	3.6342	-0.0002

Example 3:

Using Regula-falsi method, find a real root of the equation $x\log_{10} x - 1.2 = 0$ in 3 steps.

Find a root of an equation $f(x) = x\log x - 1.2$ using False Position method (regula falsi method)

Solution:

Here $x\log(x) - 1.2 = 0$

Let $f(x) = x\log(x) - 1.2$

Here

x	0	1	2	3
$f(x)$	◆	-1.2	-0.5979	0.2314

1st iteration :

Here $f(2) = -0.5979 < 0$ and $f(3) = 0.2314 > 0$

$\therefore$ Now, Root lies between $x_0 = 2$ and $x_1 = 3$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_2 = 2 - (- 0.5979) \cdot \frac{3 - 2}{0.2314 - (- 0.5979)}$$

$$x_2 = 2.721$$

$$f(x_2) = f(2.721) = 2.721\log(2.721) - 1.2 = -0.0171 < 0$$

2^{nd} iteration :

Here $f(2.721) = -0.0171 < 0$ and $f(3) = 0.2314 > 0$

$\therefore$ Now, Root lies between $x_0 = 2.721$ and $x_1 = 3$

$$x_3 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$x_3 = 2.721 - (-0.0171) \cdot \frac{3 - 2.721}{0.2314 - (-0.0171)}$$

$$x_3 = 2.7402$$

$$f(x_3) = f(2.7402) = 2.7402\log(2.7402) - 1.2 = -0.0004 < 0$$

Approximate root of the equation $x\log(x) - 1.2 = 0$ using False Position mehtod is 2.7402 (After 2 iterations)

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	**Update**
1	2	-0.5979	3	0.2314	2.721	-0.0171	$x_0 = x_2$
2	2.721	-0.0171	3	0.2314	2.7402	-0.0004	$x_0 = x_2$