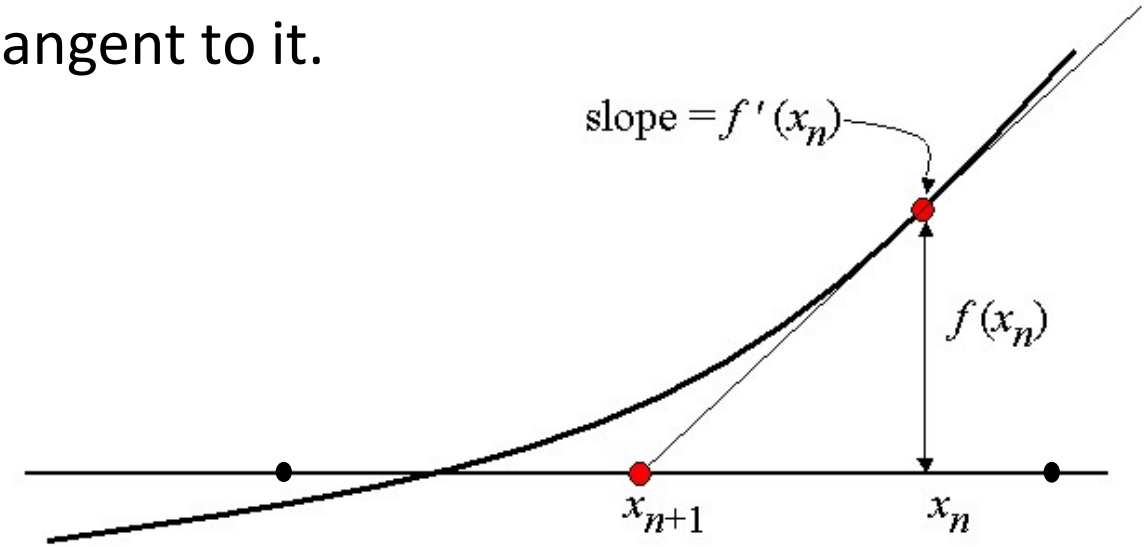
Newton-Raphson Method

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- The **Newton-Raphson method** (also known as **Newton's method**) is a way to quickly find a good approximation for the root of a real-valued function f(x)=0.
- It uses the idea that a continuous and differentiable function can be approximated by a straight line tangent to it.
- The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$$



Steps

Step-1:	Find points a and b such that a<b and f(a)·f(b)<0.
Step-2:	Take the interval [a,b] and find next value $x0 = (a + b)/2$
Step-3:	Find f(x0) and f ′(x0) $x1 = x0 - f(x0)/f'(x0)$
Step-4:	If f(x1)=0 then x1 is an exact root, else x0=x1
Step-5:	Repeat steps 2 to 4 until f(xn)=0 or |f(xn)|≤Accuracy

Example-1: Find a root of an equation f(x)=x3-2x-5 using Newton-Raphson method. Correct upto 3 decimal places.

- **Solution:**

Here $x^3 - 2x - 5 = 0$

Let $f(x) = x^3 - 2x - 5$

$d/dx(x^3 - 2x - 5) = 3x^2 - 2$

$\therefore f'(x) = 3x^2 - 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Here

x	0	1	2	3
f(x)	-5	-6	-1	16

- Here $f(2)= -1 <0$ and $f(3)= 16 >0$

  $\therefore$ Root lies between 2 and 3

$$x0 = 2 + 3/2 = 2.5$$

$$x0 = 2.5$$

1*st* iteration :

$f(x0) = f(2.5) = 2.5^3 - 2 \cdot 2.5 - 5 = 5.625$

now, $f'(x0) = f'(2.5) = 3 \cdot 2.5^2 - 2 = 16.75$

$$x1 = x0 - \frac{f(x0)}{f'(x0)}$$

$$x1 = 2.5 - (5.625/16.75)$$

$\therefore x1 = 2.1642$

2*nd* iteration :

$f(x1) = f(2.1642) = 2.1642^3 - 2 \cdot 2.1642 - 5 = 0.8079$

now $f'(x1) = f'(2.1642) = 3 \cdot 2.1642^2 - 2 = 12.051$

$$x2 = x1 - \frac{f(x1)}{f'(x1)}$$

$$x2 = 2.1642 - \frac{0.8079}{12.051}$$

∴ *x*2=2.0971

3*rd* iteration :

$f(x2) = f(2.0971) = 2.0971^3 - 2 \cdot 2.0971 - 5 = 0.0289$

$\text{now}, f'(x2) = f'(2.0971) = 3 \cdot 2.0971^2 - 2 = 11.1939$

$x3 = x2 - f(x2)/f'(x2)$

$x3 = 2.0971 - 0.0289/11.1939$

$\therefore x3 = 2.0946$

4*th* iteration :

$$f(x3) = f(2.0946) = 2.0946^3 - 2 \cdot 2.0946 - 5 = 0$$

$$\text{now}, f'(x3) = f'(2.0946) = 3 \cdot 2.0946^2 - 2 = 11.1615$$

- $x4 = x3 - f(x3)/f'(x3)$

$$x4 = 2.0946 - 0/11.1615$$

$\therefore$ *x*4=2.0946

n	x_0	$f(x_0)$	$f'(x_0)$	x_1	Update
1	2.5	5.625	16.75	2.1642	$x_0 = x_1$
2	2.1642	0.8079	12.051	2.0971	$x_0 = x_1$
3	2.0971	0.0289	11.1939	2.0946	$x_0 = x_1$
4	2.0946	0	11.1615	2.0946	$x_0 = x_1$

Answer: Approximate root of the equation *x*3-2*x*-5=0 using Newton Raphson mehtod is 2.0946 (After 4 iterations).

Example-2: Find a root of an equation $f(x)=3x-\cos x-1$ using Newton Raphson method.

Solution: Here $3x - \cos(x) - 1 = 0$

Let $f(x) = 3x - \cos(x) - 1$

- $d/dx(3x - \cos(x) - 1) = 3 + \sin(x)$

$\therefore f'(x) = 3 + \sin(x)$

Here,

x	0	1
f(x)	-2	1.4597

- Here $f(0)= -2 <0$ and $f(1) =1.4597 >0$

  $\therefore$ Root lies between 0 and 1

$$x0 = (0 + 1)/2 = 0.5$$

$$x0 = 0.5$$

1*st* iteration :

$$f(x0) = f(0.5) = 3 \cdot 0.5 - \cos(0.5) - 1 = -0.3776$$

$$f'(x0) = f'(0.5) = 3 + \sin(0.5) = 3.4794$$

$$x1 = x0 - \frac{f(x0)}{f'(x0)}$$

$$x1 = 0.5 - (-0.3776)/3.4794$$

$$x1 = 0.6085$$

2*nd* iteration :

$$f(x1) = f(0.6085) = 3 \cdot 0.6085 - \cos(0.6085) - 1 = 0.0051$$

$$f'(x1) = f'(0.6085) = 3 + \sin(0.6085) = 3.5717$$

$$x2 = x1 - \frac{f(x1)}{f'(x1)}$$

$$x2 = 0.6085 - 0.0051/3.5717$$

*x*2=0.6071

3*rd* iteration :

$$f(x2) = f(0.6071) = 3 \cdot 0.6071 - \cos(0.6071) - 1 = 0$$

$$f'(x2) = f'(0.6071) = 3 + \sin(0.6071) = 3.5705$$

$$x3 = x2 - f(x2)/f'(x2)$$

$$x3 = 0.6071 - 0/3.5705$$

*x*3=0.6071

n	x_0	$f(x_0)$	$f'(x_0)$	x_1	Update
1	0.5	-0.3776	3.4794	0.6085	$x_0 = x_1$
2	0.6085	0.0051	3.5717	0.6071	$x_0 = x_1$
3	0.6071	0	3.5705	0.6071	$x_0 = x_1$

Approximate root of the equation $3x-\cos(x)-1=0$ using Newton Raphson mehtod is 0.6071 (After 3 iterations)

Finding Inverse of a number by Newton-Raphson Method

The **inverse** of a number "a" can be considered as the root of the function $\mathrm{x} = \frac{1}{a}$ which can be solved by N-R method. The formula is -

$$x_{n+1} = 2x_n - a(x_n)^2$$

F(x)=$\frac{1}{x} - a$

F'(x)= - $\frac{1}{x\text{^}2}$

The N-R formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example: Find the inverse of 13.

Answer: Here, a=13. Assuming Xo=0.1

$$x_1 = 2x_0 - a(x_o)^2$$
$$x_1 = 2 * 0.1 - 13 * (0.1)^2 = 0.07$$
$$x_2 = 2 * 0.07 - 13 * (0.07)^2 = 0.0763$$
$$x_3 = 2 * 0.0763 - 13 * (0.0763)^2 = 0.07692$$
$$x_4 = 2 * 0.07692 - 13 * (0.07692)^2 = 0.076923$$

Ans: 0.076923.

Finding Inverse square root of a number by Newton-Raphson Method

X=$\frac{1}{\sqrt{a}}$. The formula:

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{1}{ax_n}\right]$$

Example: Find the inverse square root of 15. correct up to 3 decimal places.

Answer: Here, a=15. Assuming Xo=0.1

$x_1 = \frac{1}{2}\left[x_0 + \frac{1}{ax_0}\right] = \frac{1}{2}\left[0.1 + \frac{1}{15*0.1}\right]$ = 0.3833

$x_2 = \frac{1}{2}\left[x_1 + \frac{1}{ax_1}\right]$ = 0.2786

$x_3 = \frac{1}{2}\left[x_2 + \frac{1}{ax_2}\right]$ = 0.2589

……

- **Example: Find the cube root of 10 by N-R method. correct up to 3 decimal places.//(**Finding nth root of a real number using newton raphson method.)

Answer: here, $\sqrt[3]{10} = x$;

$$x^3 - 10 = 0$$
$$f(x) = x^3 - 10$$
$$f'(x) = 3x^2$$

Formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

x_n	$f(x) = x^3 - 10$	$f'(x) = 3x^2$	x_{n+1}
2	-2	12	2.16667
2.16667	0.1713009	14.083337	2.154503
2.154503	……	…….	…..