Secant method

Secant Method (Improved form of Regula falsi method)

- The *regula falsi*, aka. false position method, is a bracketing algorithm. It iterates through intervals that always contain a root whereas the secant method is basically Newton's method without explicitly computing the derivative at each iteration.
- The secant is faster but may not converge at all. Regula falsi is slower but as long as the initial interval contains a root, the last interval will also do.
- No need to check for signs.
- Secant method keeps the most recent 2 estimates.

Secant method Steps (Rule)

Step-1: Find points x0 and x1 such that x0<x1 and f(x0)·f(x1)<0.

Step-2: find next value using

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

Step-3: If f(x2)=0 then x2 is an exact root,
else x0=x1 and x1=x2

Step-4: Repeat steps 2 & 3 until f(xn)=0 or |f(xn)|≤Accuracy

Example: Find a root of an equation $f(x)=2x^3-2x-5$ using Secant method. Desired Accuracy 0.001

Solution: Here, $2x^3-2x-5=0$
Let $f(x)=2x^3-2x-5$

Here,

x	0	1	2
f(x)	-5	-5	7

So, root lies between……

1*st* iteration :

Here $f(1)= -5 <0$ and $f(2)= 7 >0$

$\therefore$ Now, Root lies between $x0=1$ and $x1=2$

$$\therefore x2 = x0 - f(x0) \cdot \frac{x1-x0}{f(x1)-f(x0)}$$

$$x2 = 1 - (-5) \cdot \frac{2-1}{7-(-5)}$$

$$x2 = 1.41667$$

$$\therefore f(x2) = f(1.41667) = -2.14699$$

2*nd* iteration :

Here $x1$=2 and $x2 = 1.41667$

$f(x1) = f(2) = 7\ and\ f(x2) = f(1.41667) = -2.14699$

$$\therefore x3 = x1 - f(x1) \cdot \frac{x2 - x1}{f(x2) - f(x1)}$$

$$x3 = 2 - (7) \cdot \frac{1.41667 - 2}{-2.14699 - 7}$$

$$x3 = 1.55359$$

$$\therefore f(x3) = f(1.55359) = 2 \cdot 1.55359\text{^}3 - 2 \cdot 1.55359 - 5 = -0.60759$$

3*rd* iteration :

$x2 = 1.41667$ and $x3 = 1.55359$
$f(x2) = f(1.41667) = -2.14699\ and\ f(x3) = f(1.55359) = -0.60759$

$$\therefore x4 = x2 - f(x2) \cdot \frac{x3 - x2}{f(x3) - f(x2)}$$

$$x4 = 1.41667 - (-2.14699) \times \frac{1.55359 - 1.41667}{-0.60759 - (-2.14699)}$$

$$x4 = 1.60763$$

$$\therefore f(x4) = f(1.60763) = 2 \cdot 1.607633 - 2 \cdot 1.60763 - 5 = 0.09449$$

n	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	Update
1	1	-5	2	7	1.41667	-2.14699	$x_0 = x_1$ $x_1 = x_2$
2	2	7	1.41667	-2.14699	1.55359	-0.60759	$x_0 = x_1$ $x_1 = x_2$
3	1.41667	-2.14699	1.55359	-0.60759	1.60763	0.09449	$x_0 = x_1$ $x_1 = x_2$
4	1.55359	-0.60759	1.60763	0.09449	1.60036	-0.00324	$x_0 = x_1$ $x_1 = x_2$
5	1.60763	0.09449	1.60036	-0.00324	1.6006	-0.00002	$x_0 = x_1$ $x_1 = x_2$

Approximate root of the equation $2x\text{^}3 - 2x - 5 = 0$ using Secant method is 1.6006