Numerical Interpolation

Newton's Interpolation Formula

There are three possible Newton's Interpolation techniques

a. Newton's forward Interpolation technique

b. Newton's backward Interpolation technique

c. Newton's divided difference technique

Newton's forward and backward interpolation (also called as **Newton Gregory technique**) are only available for equal intervals (that is, there is equal spacing in x data points).

- Out of this, Newton's forward interpolation technique is to be used when the x - data point is **near the beginning**. That is, if you have four data points, say x = 2 , 4 , 6 , 8 and 10. If you are looking for value of y at x = 3 which is near the beginning of the table, then Newton's forward interpolation works best.
- Newton's backward difference technique is to be used when the x - data points is **near the finish**. For the above example, if we are looking for y at x = 9 then backward difference works best.
- So both the forward and the backward differences techniques are for **equal spacing** of x. But when the spacing between x is **not equal** enough, then we use the Newton's divided difference technique.
- For example, if the x - data points are something like 1, 2, 4, 9, 20, then we use the Newton's divided difference technique.

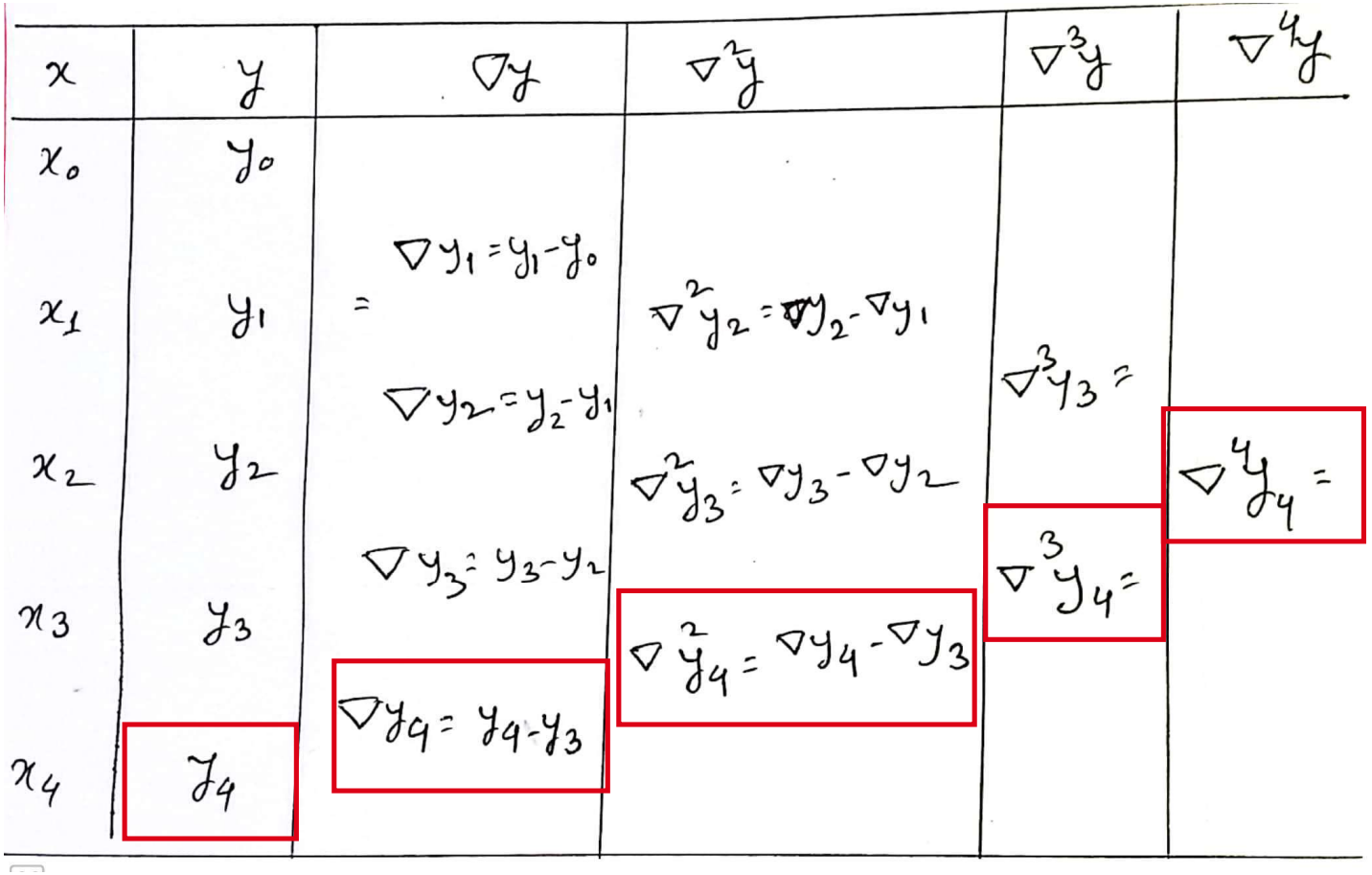
Another difference is that

a. In the forward and the backward interpolation technique, the divided difference operator is defined as only the difference in y.

b. But in Newton's divided difference technique, the divided difference operator is defined as (difference in y) / (difference in x).

Newton's Backward Interpolation Method

Backward Difference table



Newton's Backward Interpolation Method

Given data

X	Y
X_o	Y_o
X_1	Y_1
X_2	Y_2
X_3	Y_3
X_4	Y_4
X_5	Y_5
.....	.....
X_n	Y_n

Here,

h= interval

$$\mathrm{P} = \frac{X - X_n}{h}$$

Newton's Backward Difference formula

$$p = \frac{x - x_n}{h}$$

$$y(x) = y_n + p\,\nabla y_n + \frac{p(p+1)}{2!} \cdot \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \cdot \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \cdot \nabla^4 y_n + \ldots$$

- Example-1: Find Solution using Newton's Backward Difference formula for x = 1925.

x	f(x)
1891	46
1901	66
1911	81
1921	93
1931	101

- Solution: The value of table for x and y

x	1891	1901	1911	1921	1931
y	46	66	81	93	101

- Newton's backward difference table is

x	y	∇y	∇2y	∇3y	∇4y
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

- The value of x for which we need to find the $f(x)$: $x=1925$

$h = x1 - x0 = 1901 - 1891 = 10$
$p = x - xn/h = (1925 - 1931)/10 = -0.6$

- Newton's backward difference interpolation formula is

$$y(x) = yn + p\nabla yn + \left[\frac{p(p+1)}{2!} \cdot \nabla^2 yn\right] + \left[\frac{p(p+1)(p+2)}{3!} \cdot \nabla^3 yn\right] + \left[\frac{p(p+1)(p+2)(p+3)}{4!} . \nabla^4 yn\right]$$

$y(1925)=101+(-0.6)\times 8+[-0.6(-0.6+1)/2 \times -4]+[-0.6(-0.6+1)(-0.6+2)/6 \times -1]+[-0.6(-0.6+1)(-0.6+2)(-0.6+3)/24 \times -3]$

$y(1925)=101- 4.8+ 0.48+ 0.056+ 0.1008$

$y(1925)=96.8368$

Ans: $y(1925)=96.8368$

Example-2: Find Solution using Newton's Backward Difference formula

- x f(x)
- 0 1
- 1 0
- 2 1
- 3 10

When, x = 4

- Solution: The value of table for x and y

x	0	1	2	3
y	1	0	1	10

- Newton's backward difference table is

x	y	∇y	∇2y	∇3y
0	1			
		-1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

- The value of x for which we need to find the $f(x)$: x=4

h= $x1$-$x0$ =1-0 =1

p= x-xn/h = 4-3/1 =1

- Newton's backward difference interpolation formula is

$$y(x) = yn + p\nabla yn + \left[\frac{p(p+1)}{2!}\cdot\nabla^2 yn\right] + \left[\frac{p(p+1)(p+2)}{3!}\cdot\nabla^3 yn\right] + \left[\frac{p(p+1)(p+2)(p+3)}{4!}.\nabla^4 yn\right]$$

$y(4) = 10 + 1 \times 9 + [1(1+1)/2 \times 8] + [1(1+1)(1+2)/6 \times 6]$

$y(4) = 10+9+8+6$

$y(4) = 33$

Solution of newton's backward interpolation method $y(4) = 33$

Newton’s Divided Interpolation Formula

Newton's Divided Difference Interpolation formula.

$x = x_0, x_1, x_2, x_3, x_4, \ldots\ldots x_n$

$y = y_0, y_1, y_2, y_3, y_4 \ldots\ldots y_n.$

The divided difference operator: denoted as - 'Δ' or 'first-order' or $f(x_0, x_1 \cdots x_n)$

and defined as:

$$\begin{cases} \Delta y_0 = \dfrac{y_1 - y_0}{x_1 - x_0} \\ \Delta y_1 = \dfrac{y_2 - y_1}{x_2 - y_1} \\ \Delta y_2 = \dfrac{y_3 - y_2}{x_3 - x_2} \end{cases} \quad \Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} \quad \Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} \quad \Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2} \quad \Delta^3 y_0 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} \quad \Delta^3 y_1 = \frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1} \quad \Delta^3 y_2 = \frac{\Delta^2 y_3 - \Delta^2 y_2}{x_5 - x_2}$$

x1

or,

$$\begin{cases} f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{y_1 - y_0}{x_1 - x_0} \\ f(x_0, x_1, x_2) = \dfrac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} \end{cases}$$

or

formation of divided difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0}$			
x_1	y_1		$\Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$		
		$\Delta y_1 = \frac{y_2 - y_1}{x_2 - x_1}$		$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}$	
x_2	y_2		$\Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$		$\frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0}$
		$\Delta y_2 = \frac{y_3 - y_2}{x_3 - x_2}$		$\frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}$	
x_3	y_3		$\Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$		
		$\Delta y_3 = \frac{y_4 - y_3}{x_4 - x_3}$			
x_4	y_4				

"Unequal Intervals"

$$y = f(x) = y_0 + (x - x_0)\,\Delta y_0 + (x - x_0)(x - x_1)\,\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)\,\Delta^3 y_0 + \cdots\cdots$$

Or can be written as

Newton's Divided Difference Interpolation formula
$y(x) = y_0 + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots$

Example 1: Find solution using newton's divided interpolation formula. For x = 301

x	f(x)
300	2.4771
304	2.4829
305	2.4843
307	2.4871

Solution: Newton's divided difference table is

x	y	1st order	2nd order
300	2.4771		
		0.00145	
304	2.4829		0
		0.0014	
305	2.4843		0
		0.0014	
307	2.4871		

The value of x at you want to find the $f(x)$:x=301

using Newton's divided difference interpolation formula

$y(301)=2.4771+(301-300)\times0.00145+(301-300)(301-304)\times0$

$y(301)=2.4771+(1)\times0.00145+(1)(-3)\times0$

$y(301)=2.4771+0.00145+0$

$y(301)=2.47858$

Solution of divided difference interpolation method $y(301)=$**2.47858**

Example 2: Find solution using newton's divided interpolation formula.
For x = 2.7

x	f(x)
2	0.69315
2.5	0.91629
3	1.09861